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Research Paper

Partition block coordinate statistics on Γ_1 non-deranged permutations

M.S.Magami and Ibrahim M.
Department of Mathematics,
Usmanu Danfodiyo University, Sokoto,
P.M.B 2346 Sokoto State Nigeria

ABSTRACT

In this paper we study some partition block coordinate statistics of a permutations called Γ_1 non deranged permutations. We compute and redefined some statistics with respect to Γ_1 non deranged permutations; we also showed that left opener bigger block $lobTC(\omega_i)$ is equidistributed with right opener bigger block $robTC(\omega_i)$

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I. INTRODUCTION

Permutation statistic has a long history and has grown at rapid pace in the last few decades the subject originated in early 19h century by the work of Euler (1913) until Macmahon (1915) extensive study which become an established discipline of mathematic. In the last three decades much progress hs been made in discovering and analyzing new statistics see for example [Foata,(1976);Foata,(1984);Rawlings,(1981);Simon and Stanton,(1994); Stanley,(1976)]. Ibrahim *et al*(2016) studied the representation of Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ via group character, and also established that the character of every $\omega_i \in G_P^{\Gamma_1}$ is equal to one if $\omega_i = e$ otherwise p. Aremu *et al*(2017) studied the Fuzzy ideal of function *f* Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ and established that it is one side fuzzy (only right fuzzy but not left) also the α-level cut of *f* coincides with $G_P^{\Gamma_1}$ if $\alpha = \frac{1}{p}$. Ibrahim *et al*(2017) studied ascent on Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ in which recursion formula for generating Ascent number, Ascent bottom and Ascent top was develop and also observe that $Asc(\omega_i)$ union $Asc(\omega_{p-1})$ is equal to $Asc(\omega_1)$ .Garba and Ibrahim (2019) established that inversion number and major index are not equidistributed in Γ_1-non deranged permutations and also established that the difference between sum of the major index and sum of the inversion number is equal to sum of descent number in Γ_1-non deranged permutations. Ibrahim and Muhammad (2019) studied standard representation of Γ_1-non deranged permutations and also identified relation to ascent block by partitioning the permutation set in which a recursion formula for generating maximum number of block and minimum number of block were develop and it is also observed $ar(\omega_i)$ that is equidistributed with $asc(\omega_i)$ for any arbitrary permutation group. More recently Ibrahim and Ibrahim (2019) established that in Γ_1-non deranged permutations, the radius of a graph of any ω_1 is zero, the graph of any $\omega_i \in G_p^{\Gamma_1}$ is null, and by restricting 1, the graph of ω_{p-1} is complete.

Hence we will in this paper show that left opener bigger block $lobTC(\omega_i)$ is equidistributed with right opener bigger block $robTC(\omega_i)$

II. PRELIMINARIES

Definition 2.1 (Aremu *et,al* 2017)
Let Γ be a non empty set of prime cardinality greater or equal to 5 such that $\Gamma \subset \mathbb{N}$ A bijection ω on Γ of the form

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & (1+i)_{mp} & (1+2i)_{mp} & & & & (1+(p-1)i)_{mp} \end{pmatrix}$$

is called a Γ_1-non deranged permutation. We denoted G_p to be the set of all Γ_1-non deranged permutations.
Definition 2.2 (Aremu *et,al* 2017)
The pair G_p and the natural permutation composition forms a group which is denoted as $G_p{}^{\Gamma_1}$.This is a special permutation group which fixes the first element of Γ.
Definition 2.3 (Ibrahim *et,al* 2017)
An ascent of permutation $P = \begin{pmatrix} 1 & 2 & 3 & . & . & . & n \\ p_1 & p_2 & p_3 & . & . & . & p_n \end{pmatrix}$ is any positive i < *n*(where i and *n* are positive integers) where the current value is less than the following one, that is i is an ascent of a permutation p if $p_i < p_{i+1}$.The ascent set of p, denoted as Asc(p), is given by $Asc(p) = \{i \in [n-1] : p(i) < p(i+1)\}$ the ascent number of *p*, denoted as asc(p), is defined as the number of ascent and is given by $asc(p) = |Asc(p)|$.

Definition 2.4
An ascent block of a permutation $\omega_i = a_1 a_2 \ldots a_n$ is the sub word obtained by putting dashes between a_i and a_{i+1} whenever $a_i < a_{i+1}$

Definition 2.5
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ let ω_i be the index of the block counting from left to right containing i and j integer such that $i \in B_j$ for $1 \le i \le n$. Then the Left opener smaller of the partition is defined as $los_i = \#\{j\epsilon\ open(\pi): j < i, \omega_j < \omega_i\}$

Definition 2.6
For any permutation π

$$losTC(\pi) = \sum_{i\epsilon T \cup c(\pi)} los_{i(\pi)}$$

Definition 2.7
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ for $1 \le i \le n$. The Right opener smaller of the partition is defined as $ros_i = \#\{j\epsilon\ open(\pi): j < i, \omega_j > \omega_i\}$

Definition 2.8
For any permutation π

$$rosTC(\pi) = \sum_{i\epsilon T \cup c(\pi)} ros_{i(\pi)}$$

Definition 2.9
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ for $1 \le i \le n$. The Left opener bigger of the partition is defined as $lob_i = \#\{j\epsilon\ open(\pi): j > i, \omega_j < \omega_i\}$

Definition 2.10
For any permutation π

$$lobTC(\pi) = \sum_{i\epsilon T \cup c(\pi)} lob_{i(\pi)}$$

Definition 2.11
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ for $1 \leq i \leq n$. The Right opener bigger of the partition is defined as $rob_i = \#\{j\epsilon\ open(\pi): j > i, \omega_j > \omega_i\}$

Definition 2.12
For any permutation π

$$robTC(\pi) = \sum_{i\epsilon T\cup c(\pi)} rob_{i(\pi)}$$

Definition 2.13
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ for $1 \leq i \leq n$. The Left closer smaller of the partition is defined as $lcs_i = \#\{j\epsilon\ clos(\pi): j < i, \omega_j < \omega_i\}$

Definition 2.14
For any permutation π

$$lcsTC(\pi) = \sum_{i\epsilon T\cup c(\pi)} lcs_{i(\pi)}$$

Definition 2.15
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ for $1 \leq i \leq n$. The Right closer smaller of the partition is defined as $rcs_i = \#\{j\epsilon\ clos(\pi): j < i, \omega_j > \omega_i\}$

Definition 2.16
For any permutation π

$$rcsTC(\pi) = \sum_{i\epsilon T\cup c(\pi)} rcs_{i(\pi)}$$

Definition 2.17
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ for $1 \leq i \leq n$. The Left closer bigger of the partition is defined as $lcb_i = \#\{j\epsilon\ clos(\pi): j > i, \omega_j < \omega_i\}$

Definition 2.18
For any permutation π

$$lcbTC(\pi) = \sum_{i\epsilon T\cup c(\pi)} lcb_{i(\pi)}$$

Definition 2.19
Given a partition $\pi = B_1/B_2/B_2/\ldots/B_k \epsilon O\rho_n^k$ for $1 \leq i \leq n$. The Right closer bigger of the partition is defined as $lcb_i = \#\{j\epsilon\ clos(\pi): j > i, \omega_j < \omega_i\}$

Definition 2.20
For any permutation π

$$lcbTC(\pi) = \sum_{i\epsilon T\cup c(\pi)} lcb_{i(\pi)}$$

III. MAIN RESULTS

In this section, we present some partition block coordinate results of subgroup $G_p^{\Gamma_1}$ of S_p (Symmetry group of prime order with $p \geq 5$).

Proposition 3.1
$Let\ \omega_{P-1}\epsilon G_p^{\Gamma_1}$. Then the
$rosTC(\omega_{P-1}) = p - 2$
Proof:
The block of $\omega_{P-i}\ is = 1/p/p-1/p-2/p-3/\ldots/3/2$, only the first block is proper, the $ros(\omega_{P-1}) = 0p-2/p-3/p-4/p-5/\ldots/1/0$ and by finding the non-opener of the $ros(\omega_{P-1})$, we have only $p-2$ to be the non-opener and hence we have $rosTC(\omega_{P-1}) = p-2$. □

Proposition 3.2

$Let\ \omega_{P-1} \epsilon G_P^{\Gamma_1}$. Then the

$rcsTC(\omega_{P-1}) = p - 2$

Proof:

The proof of this proposition can be likening to that of proposition 3.1, we have the $rcs(\omega_{P-1}) = 0\ p-2/p-3/p-4/p-5/ \ldots/1/0$,and hence we conclude that $rcsTC(\omega_{P-1}) = p - 2$. □

Proposition 3.3

For any $\omega_2 \epsilon G_P^{\Gamma_1}$ .Then the

$losTC(\omega_2) = \frac{p-3}{2}$

Proof:

Any $\omega_2 \epsilon G_p^{\Gamma_1}$ contain two improper ascent block, is seen that the $|los(\omega_2)| = (\frac{p-1}{2})$ and it is seen that the block of $losTC(\omega_2)$ contains an opener with the index 1, by subtracting 1 from $|los(\omega_2)|$ we have $losTC(\omega_2) = \frac{p-1}{2} - 1$ this show that $losTC(\omega_2) = \frac{p-3}{2}$ □

Corollary 3.4

For any $\omega_2 \epsilon G_P^{\Gamma_1}$. Then the

$lcbTC(\omega_2) = rcbTC(\omega_2)$

Proof:

It is obvious that

$$|lcb(\omega_2) = |rcb(\omega_2)| = \frac{P-1}{2}$$

Just like the proof of proposition 3.3, it is seen that both the block of $lcb(\omega_2)$ and $rcb(\omega_2)$ contains an opener with the index 1 and hence by subtracting 1 from $\frac{P-1}{2}$ we have $lcbTC(\omega_2) = rcbTC(\omega_2) = \frac{p-3}{2}$. □

Lemma 3.5

Let $\omega_2 \epsilon G_P^{\Gamma_1}$

$rcsTC(\omega_2) = 1$

Proof:

It is obvious that from any block of $\omega_2 \epsilon G_P^{\Gamma_1}$ that the letter p appears in the first block and since it is the largest letter in the permutation, hence it records index 1 when finding the right close smaller of any permutation, while other blocks record 0. Thus since all the opener record zero, the result follows. □

Proposition 3.6

Let $\omega_{p-1} \epsilon G_P^{\Gamma_1}$

$rosTC(\omega_{p-1}) = rcsTC(\omega_{p-1})$

Proof:

It follows from proposition 3.1 and proposition 3.2 □

Proposition 3.7

Let $\omega_2 \epsilon G_P^{\Gamma_1}$

$$rosTC(\omega_2) = \frac{p-1}{2}$$

Proof:

Let the $rosTC(\omega_2)$ with respect to $p \geq 5$ be T_p, it is in the form

$$T_5 = 2 \qquad (3.1)$$
$$T_7 = 3 \qquad (3.2)$$
$$T_{11} = 5 \qquad (3.3)$$
$$\vdots = \vdots \qquad (3.4)$$

Multiplying (3.1) to (3.3) by 2 we have

$$2T_5 = 4$$
$$2\,T_7 = 6$$
$$2T_{11} = 10$$
$$\vdots = \vdots$$

We can rewrite this as

$$2T_5 = 2\left(\frac{5-1}{2}\right)$$
$$2\,T_7 = 2\left(\frac{7-1}{2}\right)$$
$$2T_{11} = 2\left(\frac{11-1}{2}\right)$$
$$\vdots = \vdots$$
$$2rosTC(\omega_2) = 2T_p = 2\left(\frac{p-1}{2}\right)$$

This implies that

$$2rosTC(\omega_2) = 2\left(\frac{p-1}{2}\right)$$
$$rosTC(\omega_2) = \left(\frac{p-1}{2}\right)$$ □

Proposition 3.8

For any $\omega_i \epsilon G_P^{\Gamma_1}$

$lobTC(\omega_i) = 0$

Proof:

Every arbitrary $\omega_i \in G_P^{\Gamma_1}$ has the its strict closer and the transients of the block to be greater than the subsequent openers when finding the left opener bigger and this make it possible for every non opener in the $rob_i(\omega_i)$ equal to zero. Hence $lobTC(\omega_i) = robTC(\omega_i) = 0$ □

Lemma 3.9

For any $\omega_i \epsilon G_P^{\Gamma_1}$

$lobTC(\omega_i) = robTC(\omega_i) = 0$

Proof:

Just like proposition 3.8 Every arbitrary $\omega_i \in G_P^{\Gamma_1}$ has the its strict closer and the transients of the block to be greater than the subsequent openers when finding the right opener bigger and every non opener in the $rob_i(\omega_i)$ equal to zero. Hence $lobTC(\omega_i) = robTC(\omega_i) = 0$ □

Lemma 3.10

For any $\omega_i \epsilon G_P^{\Gamma_1}$

$lobTC(\omega_i)$ is equidistributed to $robTC(\omega_i)$

Proof:

The proof follows from lemma 3.10 □

IV. CONCLUSION

This paper has provided very useful theoretical properties of this scheme called Γ_1-non deranged permutations in relation to the partition block coordinate statistics. We have shown that left opener bigger block $lobTC(\omega_i)$ is equidistributed with right opener bigger block $robTC(\omega_i)$.

V. RECOMMENDATION

Further researches should be conducted on Γ_1-non deranged permutations in relation to the other permutation statistics such as record, anti-record ,fixed point, cycle valley and others in other to find new algebraic and combinatorial results.

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