Quest Journals
Journal of Research in Applied Mathematics
Volume 8 ~ Issue 1 (2022) pp: 22-27
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Some Results on Fixed Points in Partial Metric Spaces

Ashish Nawghare[1] and Haresh Chaudhari[2]
[1]*(Department of Mathematics, Bhusawal Arts, Science and P. O. Nahata Commerce College Bhusawal,)*
[2]*(MGSM's Arts, Science and Commerce College, Chopda)*
Corresponding Author: Ashish Nawghare

ABSTRACT: *Matthews [1], in 1994 introduced the partial metric spaces with the new distance on a nonempty set X, as a part of the study of denotational semantics of dataflow networks. He proved Banach Contraction Theorem [2] in these spaces and also given some applications of Banach contraction principle in program verification. In the study of partial metric space many authors have proved different fixed point theorems in partial metric space using a variety of contractive conditions. The purpose of this paper is to present some fixed point results in partial metric space with a new contractive condition.*
Keywords: *Metric Space, Contraction mappings, Cauchy's sequence, Fixed point.*

***R**eceived 17 Jan, 2022; **R**evised 28 Jan, 2022; **A**ccepted 31 Jan, 2022 © The author(s) 2022.*
P**ublished with open access at **www.questjournals.org

I. INTRODUCTION

The technique in fixed point theory of non-linear analysis has great importance in the development of pure and applied sciences like mathematics, engineering, and computer sciences. Some fields of the mathematical sciences are geometry, integral transforms, differential equations, functional analysis, and topology. One of the fundamental results provided by this theory is the well-known Banach Contraction Theorem (BCT) given by Banach [2] in 1922. In the last few decades, the BCT has been studied in different domains and also generalized on many generalizations of metric spaces with a variety of contractive mappings like Kannan mapping [3], Cyclic mapping [4], Admissible mappings[5] and quasi contraction mapping [6] in these different generalizations of metrics spaces some of them are Cone metric spaces[7], Partial metric spaces [1], G metric space[8], Quasi metric spaces[9], Dislocated metric spaces[10], Dislocated quasi metric space[11], b- metric spaces[12], Dislocated quasi b-metric[13] and many more. For further study of generalizations of BCT in these one can refer to [14-20].

In the study of domain theory, as a part of the study of denotational semantics of dataflow networks; Matthews [1] in 1994 introduced the partial metric space. It has supplied notion of distance in semantics domain. In his study of the denotational semantics of programming languages he constructed a topological model for a programming language, defined as a system of logic, and revealed that the Banach contraction principle can be applied in program verification in Computer Science in the context of partial metric spaces.

After this, many researchers studied fixed point theorems in partial metric spaces. For more details, the reader can refer to [21-25].

II. MATERIAL AND METHODS

In this paper, we use the letters $\mathbb{R}$, $\mathbb{R}^+$and $\mathbb{N}$ to denote the set of real numbers, the set of nonnegative real numbers and the set of natural numbers respectively. First we discuss some definitions and properties of partial metric spaces.

Definition 2.1 [1] Let X be a non-empty set and $p : X \times X \to \mathbb{R}^+$ be such that the following conditions are satisfied for all $x, y, z \in X$

i. $x = y \Leftrightarrow p(x, x) = p(y, y) = p(x, y)$;
ii. $p(x, x) \leq p(x, y)$;
iii. $p(x, y) = p(y, x)$;
iv. $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$

Department of Mathematics, Bhusawal Arts, Science and P. O. Nahata Commerce College Bhusawal, India.

Then p is called a partial metric on X and the pair (X, p) is called a partial metric space.

Remarks 2.1

From definition, in partial metric spaces

1. If p(x, y) = 0 then x = y (from conditions (i) and (ii)); But if x = y then p(x, y) may not be 0.
2. The function $d_p : X \times X \to R^+$ defined as $d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ satisfies the condition of an usual metric on X.
3. Each partial metric p on X generates a T_0 topology τ_p on X, whose base is a family of open p-balls $\{B_p(x, \epsilon) : x \in X, \epsilon > 0\}$ where $B_p(x, \epsilon) = \{y \in X : p(x, y) \leq p(x, x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.

Definition 2.2 [1] Let (X, p) be a partial metric space a sequence $\{x_n\}$ in the partial metric space (X, p) converges to the limit x if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

Definition 2.3 [1] Let (X, p) be a partial metric space a sequence $\{x_n\}$ in the partial metric space (X, p) is called a Cauchy sequence if $p(x, x) = \lim_{m,n\to\infty} p(x_m, x_n)$ exists and is finite.

Definition 2.4 [1] A partial metric space (X, p) is called complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p to a point $x \in X$ such that $p(x, x) = \lim_{m,n\to\infty} p(x_m, x_n)$.

Lemma 2.1 [1] Let (X, p) be a partial metric space. A sequence $\{x_n\}$ is a Cauchy sequence in the partial metric space (X, p) if and only if it is a Cauchy Sequence in the Metric Space (X, d_p).

Lemma 2.2 [1] A partial metric space (X, p) is complete if and only if the metric space (X, d_p) is complete. Moreover, $\lim_{n\to\infty} p(x, x_n) = 0$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n) = \lim_{m,n\to\infty} p(x_m, x_n)$.

Banach [2] given Banach Fixed Point Theorem on complete metric spaces as follows:

Theorem 2.1 [2] If (X, d) is a complete metric space and $T : X \to X$ is a mapping such that,

$d(Tx, Ty) \leq \alpha\, d(x, y)$ For all $x, y \in X$ and some $0 \leq \alpha < 1$ then T has a unique fixed point $x \in X$. Moreover, the Picard sequence of iterates $\{T^n x\}, n \in N$ converges, for every $x \in X$, to x^*.

BCT has been generalized by number of authors we record here some of these well-known Theorems.

Theorem 2.2 Boyd-Wang [14] Let X be complete metric space, and $T : X \to X$. Assume that, there exists a right-continuous function $\phi : [0, \infty) \to [0, \infty)$ such that $\phi(r) < r$ if $r > 0$,

and $d(Tx, Ty) \leq \phi(d(x, y))$ for all $x, y \in X$ then T has a unique fixed point $x_0 \in X$ and $\{T^n x\} \to x_0$ for each $x \in X$.

Theorem 2.3 Caristi [15] Let X be a Complete Metric Space, and let $T : X \to X$. Assume that there exists a lower semi continuous function $\psi : X \to [0, \infty)$ such that,

$d(x, Tx) \leq \psi(x) - \psi(T(x))$ for all $x \in X$ then T has a fixed point in X.

Kannan [3] has obtained the extension of the Banach fixed point theorem known as Kannan fixed point theorem.

Theorem 2.4 Kannan [3] If (X, d) is a complete metric space and $T : X \to X$ is a mapping such that,

$d(Tx, Ty) \leq \frac{\alpha}{2}\{d(x, Tx) + d(y, Ty)\}$ for all $x, y \in X$ and some $0 \leq \alpha < 1$ then T has a unique fixed point $x \in X$.

Moreover, The Picard's Sequence of iterates $\{T^n x\}$ converges, for every $x \in X$ to $x^* \in X$

The contraction mapping used in this theorem is known as Kannan mapping or Kannan contraction.

Theorem 2.5 Ćirić [6] Let $T : M \to M$ be a mapping of a metric space (M, d) into itself such that, $d(Tx, Ty) \leq q \max\{d(x, y); d(x, Tx); d(y, Ty); d(x, Ty); d(y, Tx)\}$ for some $q < 1$ and all $x, y \in M$, then T has a unique fixed point in M.

The contraction mapping used in this theorem is known as quasi-contraction mapping.

In this paper fixed point results in Partial metric spaces are studied. To develop these results we take motivation from R. Krishnakumar and Damodharan [16]. The results obtained for the cone metric spaces are proved for partial metric spaces.

*Corresponding Author: Ashish Nawghare

III. RESULTS

Theorem 3.1 Let (X, p) be a complete partial metric space. Suppose that the self-mapping $T : X \to X$ satisfies the condition:

$$p(Tx, Ty) \le \left(\frac{p(x, Tx) + p(y, Ty)}{p(x, Tx) + p(y, Ty) + k}\right) p(x, y)$$

for all $x,y \in X$, where $k \ge 1$, then T has a unique fixed point in X and $\{T^n x^*\}_{n=1}^{\infty}$ converges to a fixed point, for all $x^* \in X$.

Proof Let $x_0 \in X$ be arbitrary point in X. We define a sequence $\{x_n\}$ in X by denoting

$x_1 = Tx_0, x_2 = Tx_1 = T^2x_0, \dots, x_{n+1} = Tx_n = T^n x_0$.

Consider, $p(x_{n+1}, x_n) = p(Tx_n, Tx_{n-1})$

$$\le \left(\frac{p(x_n, Tx_n) + p(x_{n-1}, Tx_{n-1})}{p(x_n, Tx_n) + p(x_{n-1}, Tx_{n-1}) + k}\right) p(x_n, x_{n-1})$$

$$\le \left(\frac{p(x_n, x_{n+1}) + p(x_{n-1}, x_n)}{p(x_n, x_{n+1}) + p(x_{n-1}, x_n) + k}\right) p(x_n, x_{n-1})$$

We take, $\beta_n = \left(\frac{p(x_n, x_{n+1}) + p(x_{n-1}, x_n)}{p(x_n, x_{n+1}) + p(x_{n-1}, x_n) + k}\right)$

We have, $d(x_{n+1}, x_n) \le \beta_n p(x_n, x_{n-1}) \le \beta_n \beta_{n-1} p(x_{n-1}, x_{n-2}) \le (\beta_n \beta_{n-1} \dots \beta_1) p(x_1, x_0)$

Here, We can observe that, the sequence $\{\beta_n\}$ is the non-increasing sequence, with positive terms.

Therefore, $\beta_1 \beta_2 \dots \beta_n \le \beta_1^n$ and also $\beta_1^n \to 0$ as $n \to \infty$.

It follows that, $\lim_{n\to\infty} (\beta_n \beta_{n-1} \dots \beta_1) = 0$.

Thus,

We get that, $\lim_{n\to\infty} p(x_{n+1}, x_n) = 0$.

Now, for all $m, n \in N$ and $m > n$

Since $p(x_{m-1}, x_{m-1}) \ge 0 \Rightarrow p(x_m, x_n) \le p(x_m, x_{m-1}) + p(x_{m-1}, x_n)$.

We have,

$p(x_m, x_n) \le p(x_m, x_{m-1}) + p(x_{m-1}, x_{m-2}) + \dots + p(x_{n+1}, x_n)$

$p(x_m, x_n) \le (\beta_{m-1}\beta_{m-2}\dots\beta_1) p(x_1, x_0) + (\beta_{m-2}\beta_{m-3}\dots\beta_1) p(x_1, x_0) + \dots + (\beta_n \beta_{n-1}\dots\beta_1) p(x_1, x_0)$

$p(x_m, x_n) \le \beta_1^{m-1} p(x_1, x_0) + \beta_1^{m-2} p(x_1, x_0) + \dots + \beta_1^{n+1} p(x_1, x_0) + \beta_1^n p(x_1, x_0)$

$p(x_m, x_n) \le (\beta_1^{m-1} + \beta_1^{m-2} + \dots + \beta_1^{n+1} + \beta_1^n) p(x_1, x_0)$.

Take $n \to \infty$ we get, $p(x_m, x_n) \to 0$

Thus,

$\{x_n\}$ is Cauchy sequence of real numbers in partial metric space (X, p).

Since, (X, p) is complete partial metric space, $\{x_n\}$ must converge to some point $x^* \in X$, therefore, $x_n \to x^*$ as $n \to \infty$.

$$p(Tx^*, x^*) \le p(Tx^*, Tx_n) + p(Tx_n, x^*) \le \left(\frac{p(x^*, Tx^*) + p(x_n, Tx_n)}{p(x^*, Tx^*) + p(x_n, Tx_n) + k}\right) p(x^*, x_n) + p(x_{n+1}, x^*)$$

$$\le \left(\frac{p(x^*, Tx^*) + p(x_n, x_{n+1})}{p(x^*, Tx^*) + p(x_n, x_{n+1}) + k}\right) p(x^*, x_n) + p(x_{n+1}, x^*)$$

$p(Tx^*, x^*) \le 0$ as $n \to \infty$

Therefore, $p(Tx^*, x^*) = 0 \quad \Rightarrow \quad Tx^* = x^*$

Uniqueness,

Suppose that x_0 and y_0 are two fixed points of T.

Consider, $p(x_0, y_0) = p(Tx_0, Ty_0) \le \left(\frac{p(x_0, Tx_0) + p(y_0, Ty_0)}{p(x_0, Tx_0) + p(y_0, Ty_0) + k}\right) p(x_0, y_0)$.

Thus, $p(x_0, y_0) \le 0 \Rightarrow p(x_0, y_0) = 0 \Rightarrow x^* = y^*$.

Hence, T has unique fixed point x^*.

To prove,

Consider,

$p(T^n x^*, x^*) = p\left(T^{n-1}(Tx^*), x^*\right) = p\left(T^{n-1}x^*, x^*\right) = p\left(T^{n-2}(Tx^*), x^*\right) = \ldots = p\left(Tx^*, x^*\right) = 0$

Thus ,

$p(T^n x^*, x^*) = 0 \Rightarrow T^n x^* = x^*$

Proving that, $T^n x^*$ converges to a fixed point, for all $x^* \in X$.

Corollary 3.1: Let (X, p) be a complete partial metric space. Suppose that the self-mapping $T: X \to X$ satisfies the condition:

$$p(Tx, Ty) \leq \left(\frac{p(x, Tx) + p(y, Ty)}{p(x, Tx) + p(y, Ty) + 1}\right) p(x, y)$$

for all $x, y \in X$, then T has a unique fixed point in X and $\{T^n x^*\}_{n=1}^{\infty}$ Converges to a fixed point, for all $x^* \in X$.

Proof The proof of the corollary immediately follows from the proof of the above theorem 3.1 by putting k=1.

Theorem 3.2 Let (X, p) be a complete partial metric space. Suppose that the self-mapping $T: X \to X$ satisfies the condition:

$$p(Tx, Ty) \leq \left(\frac{p(y, Ty)}{p(x, Tx) + p(y, Ty) + k}\right) p(x, y)$$

for all $x, y \in X$ where $k \geq 1$, then T has a unique fixed point in X and $\{T^n x^*\}_{n=1}^{\infty}$ Converges to a fixed point, for all $x^* \in X$.

Proof Let $x_0 \in X$ be an arbitrary (fix) point in X.

We define a sequence $\{x_n\}$ in X by denoting $x_1 = Tx_0, x_2 = Tx_1 = T^2 x_0, \ldots, x_{n+1} = Tx_n = T^n x_0$.

Consider, $p(x_{n+1}, x_n) = p(Tx_n, Tx_{n-1}) \leq \left(\frac{p(x_{n-1}, Tx_{n-1})}{p(x_n, Tx_n) + p(x_{n-1}, Tx_{n-1}) + k}\right) p(x_n, x_{n-1}) \leq \left(\frac{p(x_{n-1}, x_n)}{p(x_n, x_{n+1}) + p(x_{n-1}, x_n) + k}\right) p(x_n, x_{n-1})$

We take, $\beta_n = \left(\frac{p(x_{n-1}, x_n)}{p(x_n, x_{n+1}) + p(x_{n-1}, x_n) + k}\right)$

We have,

$d(x_{n+1}, x_n) \leq \beta_n p(x_n, x_{n-1}) \leq \beta_n \beta_{n-1} p(x_{n-1}, x_{n-2}) \leq (\beta_n \beta_{n-1} \cdots \beta_1) p(x_1, x_0)$

Here, We can observe that, $\{\beta_n\}$ is the non-increasing sequence, with positive terms.

Therefore, $\beta_1 \beta_2 \cdots \beta_n \leq \beta_1^n$ and also $\beta_1^n \to 0$ as $n \to \infty$.

It follows that,

$\lim_{n\to\infty} (\beta_n \beta_{n-1} \cdots \beta_1) = 0$.

Thus,

We get that, $\lim_{n\to\infty} p(x_{n+1}, x_n) = 0$

Now, for all $m, n \in N$ and $m > n$

Since, $(x_{m-1}, x_{m-1}) \geq 0 \Rightarrow p(x_m, x_n) \leq p(x_m, x_{m-1}) + p(x_{m-1}, x_n)$

We have,

$$p(x_m, x_n) \leq p(x_m, x_{m-1}) + p(x_{m-1}, x_{m-2}) + \ldots + p(x_{n+1}, x_n)$$

$$p(x_m, x_n) \leq p(x_m, x_{m-1}) + p(x_{m-1}, x_{m-2}) + \ldots + p(x_{n+1}, x_n)$$

$$p(x_m, x_n) \leq (\beta_{m-1}\beta_{m-2} \cdots \beta_1) p(x_1, x_0) + (\beta_{m-2}\beta_{m-3} \cdots \beta_1) p(x_1, x_0) + \ldots + (\beta_n \beta_{n-1} \cdots \beta_1) p(x_1, x_0)$$

$$p(x_m, x_n) \leq \beta_1^{m-1} p(x_1, x_0) + \beta_1^{m-2} p(x_1, x_0) + \ldots + \beta_1^{n+1} p(x_1, x_0) + \beta_1^n p(x_1, x_0)$$

$$p(x_m, x_n) \leq (\beta_1^{m-1} + \beta_1^{m-2} + \ldots + \beta_1^{n+1} + \beta_1^n) p(x_1, x_0)$$

Take $n \to \infty$ we get, $p(x_m, x_n) \to 0$

Thus,

We have $\{x_n\}$ is a Cauchy sequence of real numbers in partial metric space (X, p).

Since (X, p) is complete partial metric space, $\{x_n\}$ must converge to some point $x^* \in X$, therefore, $x_n \to x^*$ as $n \to \infty$

$$p(Tx^*, x^*) \leq p(Tx^*, Tx_n) + p(Tx_n, x^*) \leq \left(\frac{p(x_n, Tx_n)}{p(x^*, Tx^*) + p(x_n, Tx_n) + k}\right) p(x^*, x_n) + p(x_{n+1}, x^*)$$

$$\leq \left(\frac{p(x_n, x_{n+1})}{p(x^*, Tx^*) + p(x_n, x_{n+1}) + k}\right) p(x^*, x_n) + p(x_{n+1}, x^*)$$

$p(Tx^*, x^*) \leq 0$ as $n \to \infty$

Therefore,

$$p(Tx^*, x^*) = 0 \Rightarrow Tx^* = x^*$$

Uniqueness,

Suppose that x_0 and y_0 are two fixed points of T .

Consider, $$p(x_0, y_0) = p(Tx_0, Ty_0) \leq \left(\frac{p(y_0, Ty_0)}{p(x_0, Tx_0) + p(y_0, Ty_0) + k}\right) p(x_0, y_0)$$

Thus, $$p(x_0, y_0) \leq 0 \Rightarrow p(x_0, y_0) = 0 \Rightarrow x^* = y^*$$

Hence, T has unique fixed point x^*.

To prove (ii)

Consider,

$$p(T^n x^*, x^*) = p\left(T^{n-1}(Tx^*), x^*\right) = p\left(T^{n-1}x^*, x^*\right) = p\left(T^{n-2}(Tx^*), x^*\right) = \ldots = p(Tx^*, x^*) = 0$$

Thus, $p(T^n x^*, x^*) = 0 \Rightarrow T^n x^* = x^*$

Proving that, $T^n x^*$ converges to a fixed point, for all $x^* \in X$.

Corollary 3.2 Let (X, p) be a complete partial metric space. Suppose that the self-mapping $T:X \rightarrow X$ satisfies the following conditions:

$$p(Tx, Ty) \leq \left(\frac{p(y, Ty)}{p(x, Tx) + p(y, Ty) + 1}\right) p(x, y)$$

For all $x, y \in X$, then T has a unique fixed point in X and $\{T^n x^*\}_{n=1}^{\infty}$ converges to a fixed point, for all $x^* \in X$.

Proof The proof of the corollary immediately follows from the proof of the above theorem 3.3 by putting k=1.

IV. CONCLUSION

In BCT [2], The contraction constant α such that $0 \leq \alpha < 1$ is fixed for any pair of points x, y in Metric Space. Matthews [1] proved BCT in Partial Metric Spaces. R. Krishnakumar and Damodharan in [16] proved the results in Theorem 3.1, Corollary3.2, Theorem 3.3 and Corollary 3.4 in Normal cone metric space in which the contraction constant may change for different pair of points. In this paper we have considered same varying contraction constant in Partial Metric Space and proved the existence and uniqueness of fixed point theorems.

REFERENCES

[1]. Matthews, S.G. Partial Metric Topology. Proceedings of Eighth Summer Conference on General Topology and Applications, Annals of the New York Academy of Sciences. 1994. **728:** p. 183-197.

[2]. Banach S., Surles opérations dans les ensembles abstraits et leur application aux equations intégrales. Fund. Math., 1922. **3**: p.133-181.

[3]. Kannan, R., Some results on fixed points. Bulletin of Calcutta Mathematical Society, 1968. **60**: p. 71-76.

[4]. Kirk, W.A., et. al., Fixed points for mapping satisfying Cyclic contractive conditions. Fixed Point Theory, 2003. **4**: p. 79-89.

[5]. Marija, C., et. al., Some fixed point results on quasi-b-metric-like spaces. Journal of Inequalities and Applications, 2015. **374**: p. 1-17.

[6]. Ćirić., Lj. B., A Generalization of Banach's Contraction Principle. Proceedings of the American Mathematical Society, 1974. **45**: p. 67-273.

[7]. Huang, L.G. and Zhang, X., Cone metric spaces and fixed Point theorems of contractive mappings. Journal of Mathematical analysis and application, 2007. **332**: p. 1468-1476.

[8]. Mustafa, Z. and B. Sims, Some remarks concerning D-metric spaces. Proceedings of International Conference on Fixed Point Theory and Applications, Yokohama, Japan, 2004. p. 189-198.

[9]. Wilson, W. A., On quasi-metric spaces. Am J Math, 1931. **53**(3): p. 675-684.

[10]. Hitzler, P. and Seda A K., Dislocated topologies. J. Electr. Eng., 2000. **51**: p. 3-7.

[11]. Zeyada, F. M. et. al., A generalization of fixed point theorem due to Hitzler and Seda in dislocated quasi-metric space. Arabian Journal of Science Engineering, 2006. **31**(1A): p. 111-114.

[12]. Bakhtin, I. A., The contraction mapping principle in quasi-metric spaces. Funct. Anal. Unianowsk Gos. Ped. Inst., 1989. **30**: p. 26-37.

[13]. Chakkrid, K. and C. Suanoom, Dislocated quasi-b-metric spaces and fixed point theorems for cyclic contractions. Fixed Point Theory and Applications, 2015. **74**: p. 1-12.

[14]. Boyd, D. W. and J. S. W. Wong, On Nonlinear Contractions. Proceedings of the American Mathematical Society, 1969. **20**: p. 458-464.

[15]. Caristi, J, Fixed point theorem for mapping satisfying Inwardness Conditions. Trans. Am. Math. Soc., 1976. **215**: p. 241-251.

[16]. Krishnakumar, R. and D. Dhamodharan, Fixed Point Theorems in Normal cone metric space. International J. of Math. Sci. & Engg. Appls. (IJMSEA), 2016. **10**: p. 213-224.

[17]. Rahman, M. and M. Sarwar, Dislocated quasi b-metric space and Fixed Point Theorems. Electronic Journal of Mathematical Analysis and Applications, 2016. **4**: p. 16-24.

[18]. Mustafa, Z. et. al., Some Fixed Point Theorem for mapping on complete G-metric Spaces. Fixed Point Theory and Applications, 2008. **2008**: p. 1-12.

[19]. Czerwik, S., Contraction mappings in b-metric spaces. Acta Math Inf Univ Ostraviensis, 1993. **1**: p. 5-11.

[20]. Isufati A., Fixed point theorem in dislocated quasi-metric space. Applied Math. Sci., 2010. **4**: p. 217-223.
[21]. Valero, O., On Banach Fixed Point Theorems for Partial Metric Spaces. Applied General Topology, 2005. **6**: p. 229-240.
[22]. Masiha, H. P. et. al., Fixed Point Theorems in Partial Metric Spaces with an Application. University of Nis, Faculty of Sciences and Mathematics, 2013. **27**: p. 617-624.
[23]. Choudhury, B. S. and C. Bandyopadhyay, Suzuki Type Common Fixed Point Theorem in Complete Metric Space and Partial Metric Space. University of Nis, Faculty of Sciences and Mathematics, 2015. **29**: p. 1377-1387.
[24]. Alghamdi, M. A. et. al., On Fixed Point Theory in Partial Metric Spaces. Fixed Point Theory and Applications, 2012. **2012**: p. 1-25.
[25]. Karapınar, E. and S. Romaguera, Nonunique fixed point theorems in Partial Metric Spaces. University of Nis, Faculty of Sciences and Mathematics, 2013. **27**: p. 1305-1314.
[26]. Ran, ACM and MCB Reurings, A Fixed Point Theorem in Partially Ordered Sets and some applications to matrix equations. Proceedings of the American Mathematical Society, 2004. **132**: p. 1435-1443.