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Research Paper

A New Proof for Fermat's Last Theorem using Ramanujam-Nagell Equation

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Abstract

Fermat's Last Theorem states that it is impossible to find positive integers A, B and C satisfying the equation

$$A^n + B^n = C^n$$

where n is any integer > 2.
Taking the proofs of Fermat for the index $n = 4$, and Euler for $n = 3$, it is sufficient to prove the theorem for $n = p$, any prime > 3 [1].
We hypothesize that all r, s and t are non-zero integers in the equation

$$r^p + s^p = t^p$$

and establish contradiction.
Just for supporting the proof in the above equation, we have another equation

$$x^3 + y^3 = z^3$$

Without loss of generality, we assert that both x and y as non-zero integers; z^3 a non-zero integer; z and z^2 irrational.
We create transformation equations to the above two equations through parameters, into which we have incorporated the Ramanujam-Nagell equation. Solving the transformed equations we prove the theorem.

Keywords: Transformed Fermat's Equations through Parameters.

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I. Introduction

Around 1637, Pierre-de-Fermat, the French mathematician wrote in the margin of his book that the equation $A^n + B^n = C^n$ has no solution in integers A, B and C, if n is any integer > 2. Fermat stated in the margin of a book that he himself had found a marvelous proof of the theorem, but the margin was too narrow to contain it. His proof is available only for the index $n = 4$, using infinite descent method.

Many mathematicians like Sophie Germain, E.E. Kummer had proved the theorem for particular cases. Number theory has been developed leaps and bounds by the immense contributions by a lot of mathematicians. Finally, after 350 years, the theorem was completely proved by Prof. Andrew Wiles, using highly complicated mathematical tools and advanced number theory[2], [3].

Here we are trying an elementary proof.

II. Assumptions

1) We initially hypothesize that all r, s and t are non-zero integers satisfying the equation
$$r^p + s^p = t^p$$
where p is any prime > 3, with $gcd(r, s, t) = 1$ and establish a contradiction in this proof. Since both s and t cannot simultaneously be squares, we can have $\sqrt{st}$ irrational.

2) Just for supporting the proof in the above equation, we have another equation.
$$x^3 + y^3 = z^3; \qquad gcd(x, y, z^3) = 1$$
Without loss of generality, we can have both x and y as non-zero integers, z^3 a non-zero integer; both z and z^2 irrational. Since we prove the theorem only in the equation $r^p + s^p = t^p$ for all possible integral values of r, s and t we have the choice in having $x = 37$; $y = 144$; $z^3 = 37^3 + 144^3 = 19 \times 181 \times 883$.

3) By trial and error we have created the transformation equations to $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$
using parameters called a, b, c, d, e and f. Creation of such transformation equations could be done in thousands of ways, but finding a proof is most difficult and rare. Every time the rational terms in equation (B) we derive from the transformed equations got cancelled out on both sides. After enormous random trials, the formulation of transformed equations was achieved to bring out the results for proving the theorem.

4) Into the transformed equations we have incorporated the Ramanujam-Nagell equation
$$2^n = 7 + \ell^2,$$
which has just five solutions given by
$$\begin{aligned} 2^3 &= 7 + 1^2 \\ 2^4 &= 7 + 3^2 \\ 2^5 &= 7 + 5^2 \\ 2^7 &= 7 + 11^2 \\ \text{and} \quad 2^{15} &= 7 + 181^2 \end{aligned}$$
In this proof we use only the solution $2^{15} = 7 \times 181^2$, where $\ell = 181$.

Proof. By random trials, we have created the following equations

$$\left(a\sqrt{37z^3}+b\sqrt{2^{n/2}}\right)^2+\left(\frac{c\sqrt{883}+d\sqrt{\ell^{5/3}}}{\sqrt{2^{3n/2}}}\right)^2=\left(e\sqrt{3}+f\sqrt{19}\right)^2$$

and

$$\left(\frac{a\sqrt{19}-b\sqrt{st}}{\sqrt{\ell}}\right)^2+\left(\frac{c\sqrt{37}-d\sqrt{r}}{\sqrt{7^{5/3}}}\right)^2=\left(\frac{e\sqrt{7^{1/3}}-f\sqrt{37z}}{\sqrt{\ell^{7/3}}}\right)^2 \qquad \text{(A)}$$

as the transformed equations of $x^3+y^3=z^3$ and $r^p+s^p=t^p$ respectively through the parameters a, b; c, d; e and f.

We may have

$$a\sqrt{37z^3}\ +\ b\sqrt{2^{n/2}}=\sqrt{x^3} \qquad \text{(i)}$$
$$a\sqrt{19}-b\sqrt{st}=\sqrt{\ell r^p} \qquad \text{(ii)}$$
$$c\sqrt{883}+d\sqrt{\ell^{5/3}}=\sqrt{y^3 2^{3n/2}} \qquad \text{(iii)}$$
$$c\sqrt{37}-d\sqrt{r}=\sqrt{s^p 7^{5/3}} \qquad \text{(iv)}$$
$$e\sqrt{3}+f\sqrt{19}=\sqrt{z^3} \qquad \text{(v)}$$
$$\text{and} \qquad e\sqrt{7^{1/3}}-f\sqrt{37z}=\sqrt{t^p\ell^{7/3}} \qquad \text{(vi)}$$

Solving simultaneously (i) and (ii); (iii) and (iv); (v) and (vi), we get

$$a=\left(\sqrt{stx^3}+\sqrt{2^{n/2}\ell r^p}\right)\Big/\left(\sqrt{37stz^3}+\sqrt{19\times 2^{n/2}}\right)$$
$$b=\left(\sqrt{19x^3}-\sqrt{37\times z^3\ell r^p}\right)\Big/\left(\sqrt{37stz^3}+\sqrt{19\times 2^{n/2}}\right)$$
$$c=\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\Big/\left(\sqrt{883r}+\sqrt{37\ell^{5/3}}\right)$$
$$d=\left(\sqrt{37\times 2^{3n/2}y^3}-\sqrt{883\times 7^{5/3}s^p}\right)\Big/\left(\sqrt{883r}+\sqrt{37\ell^{5/3}}\right)$$
$$e=\left(z^2\sqrt{37}+\sqrt{19t^p\ell^{7/3}}\right)\Big/\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)$$
$$\text{and} \qquad f=\left(\sqrt{7^{1/3}z^3}-\sqrt{3t^p\ell^{7/3}}\right)\Big/\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)$$

From (i) and (iii), we get

$$\sqrt{2^{n/2}}\times\sqrt{2^{3n/2}}=\left(\sqrt{x^3}-a\sqrt{37z^3}\right)\left(c\sqrt{883}+d\sqrt{\ell^{5/3}}\right)\Big/\left(b\sqrt{y^3}\right)$$
$$\text{i.e.,}\quad 2^n=\Big\{(c)\sqrt{883x^3}+(d)\sqrt{\ell^{5/3}x^3}-(ac)\sqrt{37\times 883z^3}$$
$$-(ad)\sqrt{37z^3\ell^{5/3}}\Big\}\Big/\left(b\sqrt{y^3}\right)$$

From (iii) and (vi), we get

$$\sqrt{\ell^{5/3}} \times \sqrt{\ell^{7/3}} = \left(\sqrt{2^{3n/2}y^3} - c\sqrt{883}\right)\left(e\sqrt{7^{1/3}} - f\sqrt{37z}\right) \Big/ \left(d\sqrt{t^p}\right)$$

$$\text{i.e.,} \quad \ell^2 = \Big\{(e)\sqrt{2^{3n/2}7^{1/3}y^3} - (f)\sqrt{2^{3n/2} \times 37y^3z} \\ -(ce)\sqrt{883 \times 7^{1/3}} + (cf)\sqrt{883 \times 37z}\Big\} \Big/ \left(d\sqrt{t^p}\right)$$

Substituting the above equivalent values of 2^n, 7 and ℓ^2 in the Ramanujam-Nagell equation $2^n = 7 + \ell^2$ after multiplying both sides by $\left\{(bde)\sqrt{y^3s^pt^p}\right\}$ we get the equation

$$\left\{(de)\sqrt{s^pt^p}\right\}\left\{(c)\sqrt{883x^3} + (d)\sqrt{\ell^{5/3}x^3} - (ac)\sqrt{37 \times 883z^3} - (ad)\sqrt{37z^3\ell^{5/3}}\right\} \\ = \left\{(bd)\sqrt{y^3t^p}\right\}\left((c)\sqrt{37t^p\ell^{7/3}} + 37(cf)\sqrt{z} - (d)\sqrt{rt^p\ell^{7/3}} - (df)\sqrt{37rz}\right) \\ + \left\{(be)\sqrt{y^3s^p}\right\}\Big\{(e)\sqrt{2^{3n/2}7^{1/3}y^3} - (f)\sqrt{2^{3n/2}37y^3z} \\ - (ce)\sqrt{883 \times 7^{1/3}} + (cf)\sqrt{883 \times 37z}\Big\} \tag{B}$$

We will work out all rational terms available in equation (B), after multiplying both sides by

$$\left(\sqrt{37stz^3} + \sqrt{19 \times 2^{n/2}}\right)\left(\sqrt{883r} + \sqrt{37\ell^{5/3}}\right)^2\left(\sqrt{3 \times 37z} + \sqrt{19 \times 7^{1/3}}\right)^2$$

For freeing from denominators on the parameters a, $,b$, c, d, e and f and again multiplying both sides by

$$\left\{\sqrt{19 \times 37 \times z^5t\ell^{4/3}}\right\}$$

for getting some rational terms.

I term in LHS of Equation (B), after multiplying by the respective terms, and substituting for $\{(cd)e\}$

$$= \sqrt{883x^3s^pt^p}\left(\sqrt{37stz^3} + \sqrt{19 \times 2^{n/2}}\right)\left(\sqrt{3 \times 37z} + \sqrt{19 \times 7^{1/3}}\right) \\ \times \sqrt{19 \times 37z^5t\ell^{4/3}}\left(\sqrt{2^{3n/2}y^3r} + \sqrt{7^{5/3}\ell^{5/3}s^p}\right)\Big(\sqrt{37 \times 2^{3n/2}y^3} \\ - \sqrt{883 \times 7^{5/3}s^p}\Big)\left(z^2\sqrt{37} + \sqrt{19t^p\ell^{7/3}}\right)$$

There is no rational part in this term.

II term in LHS of equation (B), after multiplying by the respective terms, and substituting for $\{d^2e\}$

$$
\begin{aligned}
&= \sqrt{s^p t^p \ell^{5/3} x^3}\left(\sqrt{37stz^3}+\sqrt{19\times 2^{n/2}}\right)\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)\\
&\quad\times \sqrt{19\times 37z^5t\ell^{4/3}}\left(z^2\sqrt{37}+\sqrt{19\times \ell^{7/3}t^p}\right)\left\{\left(37y^3\sqrt{2^{3n}}\right)\right.\\
&\quad\quad \left.+\left(883\times 7^{5/3}s^p\right)-2\sqrt{37\times 2^{3n/2}y^3}\sqrt{883\times 7^{5/3}s^p}\right\}
\end{aligned}
$$

There is no rational part in this term.

III term in LHS of equation (B), after multiplying by the respective terms, and substituting for $\{a(cd)e\}$ is

$$
\begin{aligned}
&= \left(-\sqrt{s^pt^p}\right)\sqrt{37\times 883z^3}\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)\left(\sqrt{stx^3}+\sqrt{2^{n/2}\ell r^p}\right)\\
&\quad\times \sqrt{19\times 37z^5t\ell^{4/3}}\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\left(\sqrt{37\times 2^{3n/2}y^3}\right.\\
&\quad\quad \left.-\sqrt{883\times 7^{5/3}s^p}\right)\left(z^2\sqrt{37}+\sqrt{19t^p\ell^{7/3}}\right)
\end{aligned}
$$

On multiplying by,

$$
\begin{aligned}
&\left\{\left(-\sqrt{s^pt^p}\right)\sqrt{37\times 883z^3}\sqrt{19\times 7^{1/3}}\sqrt{2^{n/2}\ell r^p}\sqrt{7^{5/3}\ell^{5/3}s^p}\right.\\
&\quad\left.\sqrt{19\times 37z^5t\ell^{4/3}}\left(\sqrt{37\times 2^{3n/2}y^3}\right)\left(z^2\sqrt{37}\right)\right\}
\end{aligned}
$$

We get

$$\left\{-\left(2^n\times 7\times 19\times 37^2z^6\right)\left(\ell^2s^p\sqrt{t^{p+1}}\right)\sqrt{883y^3r^p}\right\}$$

where $y=12^2$; this term will be discussed later on.

IV term in LHS of equation (B), after multiplying by the respective terms, and substituting for $\{ad^2e\}$ is

$$
\begin{aligned}
&= \left(-\sqrt{s^pt^p}\right)\left(\sqrt{37z^3\ell^{5/3}}\right)\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)\sqrt{19\times 37z^5t\ell^{4/3}}\\
&\quad\times\left(\sqrt{stx^3}+\sqrt{2^{n/2}\ell r^p}\right)\left\{\left(37y^3\sqrt{2^{3n}}\right)+\left(883\times 7^{5/3}s^p\right)\right.\\
&\quad\quad\left.-\left(2\sqrt{37\times 2^{3n/2}y^3}\sqrt{883\times 7^{5/3}s^p}\right)\right\}\left(z^2\sqrt{37}+\sqrt{19t^p\ell^{7/3}}\right)
\end{aligned}
$$

On multiplying by

$$
\begin{aligned}
&\left\{\left(-\sqrt{s^pt^p}\right)\sqrt{37z^3\ell^{5/3}}\sqrt{19\times 7^{1/3}}\sqrt{19\times 37z^5t\ell^{4/3}}\sqrt{2^{n/2}\ell r^p}\right.\\
&\quad\left.\left(-2\sqrt{37\times 2^{3n/2}y^3}\right)\sqrt{883\times 7^{5/3}s^p}\right)\right\}\left(z^2\sqrt{37}\right)
\end{aligned}
$$

We get

$$\left\{\left(2^{n+1}\times 7\times 19\times 37^2\ell^2z^6s^p\sqrt{t^{p+1}}\right)\sqrt{883y^3r^p}\right\}$$

This term will be discussed later on.

I term in RHS of equation (B), after multiplying by the respective terms, and substituting for $\{b(cd)\}$ is

$$
\begin{aligned}
&= \left(t^p\sqrt{37y^3\ell^{7/3}}\right)\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)^2\sqrt{19\times 37z^5t\ell^{4/3}}\\
&\quad\left(\sqrt{19x^3}-\sqrt{37\times \ell r^pz^3}\right)\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\\
&\quad\quad\times\left(\sqrt{37\times 2^{3n/2}y^3}-\sqrt{883\times 7^{5/3}s^p}\right)
\end{aligned}
$$

There is no rational part in this term.

II term in RHS of equation (B), after multiplying by the relevant terms, and substituting for $\{b(cd)f\}$

$$= 37\sqrt{y^3zt^p}\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)\sqrt{19\times 37z^5t\ell^{4/3}}$$
$$\times\left(\sqrt{19x^3}-\sqrt{37\times r^pz^3\ell}\right)\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)$$
$$\times\left(\sqrt{37\times 2^{3n/2}y^3}-\sqrt{883\times 7^{5/3}s^p}\right)\left(\sqrt{7^{1/3}z^3}-\sqrt{3t^p\ell^{7/3}}\right)$$

Rational part in this term

$$=\Big\{37\sqrt{y^3zt^p}\sqrt{19\times 7^{1/3}}\sqrt{19\times 37z^5t\ell^{4/3}}\sqrt{19x^3}\sqrt{7^{5/3}\ell^{5/3}s^p}$$
$$\times\left(-\sqrt{883\times 7^{5/3}s^p}\right)\sqrt{7^{1/3}z^3}\Big\}$$
$$=\left[-\left(7^2\times 19\times 37\ell z^3\right)\sqrt{37x^3y^3}\left(s^p\sqrt{t^{p+1}}\right)\sqrt{19\times 883\times \ell z^3}\right]$$

Also on multiplying by

$$\Big\{(37\sqrt{y^3zt^p})\sqrt{19\times 7^{1/3}}\sqrt{19\times 37z^5t\ell^{4/3}}\left(-\sqrt{37r^pz^3\ell}\right)\times\sqrt{7^{5/3}\ell^{5/3}s^p}$$
$$\times\left(-\sqrt{883\times 7^{5/3}s^p}\right)\sqrt{7^{1/3}z^3}\Big\}$$

We get

$$\left\{\left(19\times 37^2z^6\right)\left(7^2\ell^2s^p\sqrt{t^{p+1}}\right)\left(\sqrt{883\times y^3r^p}\right)\right\}$$

This term will be discuss later on.

III term in RHS of Equation (B), after multiplying by the respective terms, and substituting for $\{bd^2\}$

$$=-\left(t^p\sqrt{y^3r\ell^{7/3}}\right)\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)^2\left(\sqrt{19x^3}-\sqrt{37\times \ell r^pz^3}\right)$$
$$\times\sqrt{19\times 37z^5t\ell^{4/3}}\Big\{\left(37y^3\sqrt{2^{3n}}\right)+\left(883\times 7^{5/3}s^p\right)$$
$$-2\left(\sqrt{37\times 2^{3n/2}y^3}\times\sqrt{883\times 7^{5/3}s^p}\right)\Big\}$$

There is no rational part in this term.

IV term in RHS of Equation (B), after multiplying by the respective terms, and substituting for $\{bd^2f\}$

$$=\left(-\sqrt{37y^3t^prz}\right)\left(\sqrt{3\times 37z}+\sqrt{19\times 7^{1/3}}\right)\sqrt{19\times 37z^5t\ell^{4/3}}$$

$$\times\left(\sqrt{19x^3}-\sqrt{37\times \ell r^pz^3}\right)\Big\{\left(37y^3\sqrt{2^{3n}}\right)+\left(883\times 7^{5/3}s^p\right)$$
$$-\left(2\sqrt{37\times 2^{3n/2}y^3}\right)\left(\sqrt{883\times 7^{5/3}s^p}\right)\Big\}\left(\sqrt{7^{1/3}z^3}-\sqrt{3t^p\ell^{7/3}}\right)$$

There is no rational part in this term.

V term in RHS of Equation (B), after multiplying by the respective terms, and substituting for $\{be^2\}$

$$=\left(y^3\sqrt{2^{3n/2}s^p\left(7^{1/3}\right)}\right)\left(\sqrt{883r}+\sqrt{37\ell^{5/3}}\right)^2\sqrt{19\times 37z^5t\ell^{4/3}}$$
$$\times\left(\sqrt{19x^3}-\sqrt{37\times \ell r^pz^3}\right)\left(z^2\sqrt{37}+\sqrt{19t^p\ell^{7/3}}\right)^2$$

There is no rational part in this term.

VI term in RHS of Equation (B), after multiplying by the respective terms, and substituting for $\{b(ef)\}$

$$= \left(-y^3\sqrt{2^{3n/2}\times 37s^pz}\right)\left(\sqrt{883r}+\sqrt{37\ell^{5/3}}\right)^2\sqrt{19\times 37z^5t\ell^{4/3}} \\ \times\left(\sqrt{19x^3}-\sqrt{37\times \ell r^pz^3}\right)\left(z^2\sqrt{37}+\sqrt{19t^p\ell^{7/3}}\right)\left(\sqrt{7^{1/3}z^3}-\sqrt{3t^p\ell^{7/3}}\right)$$

There is no rational part in this term.

VII term in RHS of Equation (B), after multiplying by the respective terms, and substituting for $\{bce^2)\}$

$$= \left(-\sqrt{883\times 7^{1/3}y^3s^p}\right)\left(\sqrt{883r}+\sqrt{37\ell^{5/3}}\right)\sqrt{19\times 37z^5t\ell^{4/3}} \\ \left(\sqrt{19x^3}-\sqrt{37\times \ell r^pz^3}\right)\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right) \\ \times\left(z^2\sqrt{37}+\sqrt{19t^p\ell^{7/3}}\right)^2$$

On multiplying by

$$\left\{\left(-\sqrt{883\times 7^{1/3}y^3s^p}\right)\sqrt{37\ell^{5/3}}\sqrt{19\times 37z^5t\ell^{4/3}}\left(-\sqrt{37\times \ell r^pz^3}\right) \\ \times\sqrt{7^{5/3}\ell^{5/3}s^p}\left(2z^2\sqrt{19\times 37t^p\ell^{7/3}}\right)\right\}$$

We get,

$$\left\{\left(2\times 7\times \ell^4\right)\left(s^p\sqrt{t^{p+1}}\right)(19\times 37^2z^6)\sqrt{883\times y^3r^p}\right\}$$

where $y = 12^2$. This term will be discussed later on.

VIII term in RHS of Equation (B), after multiplying by the respective terms, and substituting for $\{bc(ef)\}$

$$= \left(\sqrt{y^3s^p}\sqrt{883\times 37z}\right)\left(\sqrt{883r}+\sqrt{37\ell^{5/3}}\right)\sqrt{19\times 37z^5t\ell^{4/3}} \\ \times\left(\sqrt{19x^3}-\sqrt{37\times \ell r^pz^3}\right)\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right) \\ \times\left(z^2\sqrt{37}+\sqrt{19t^p\ell^{7/3}}\right)\left(\sqrt{7^{1/3}z^3}-\sqrt{3t^p\ell^{7/3}}\right)$$

Rational part in this term

$$= \left\{\sqrt{y^3s^p}\sqrt{883\times 37z}\sqrt{37\ell^{5/3}}\sqrt{19\times 37z^5t\ell^{4/3}}\sqrt{19x^3}\sqrt{7^{5/3}\ell^{5/3}s^p} \\ \times\sqrt{19t^p\ell^{7/3}}\sqrt{7^{1/3}z^3}\right\}$$

$$= \left\{(7\times 19\times 37z^3\ell^3)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{37x^3y^3}\sqrt{19\times 883\times \ell z^3}\right\}$$

[Since $x = 37;\ y = 12^2;\ z^3 = 19\times 181\times 883$ and $\ell = 181$]

Also on multiplying by,

$$\left\{\sqrt{y^3s^p}\sqrt{883\times 37z}\left(\sqrt{37\ell^{5/3}}\right)\sqrt{19\times 37z^5t\ell^{4/3}}\left(-\sqrt{37\times \ell r^pz^3}\right) \\ \times\sqrt{7^{5/3}\ell^{5/3}s^p}\sqrt{19t^p\ell^{7/3}}\sqrt{7^{1/3}z^3}\right\}$$

We get

$$\left\{ -(7\times \ell^4)(19\times 37^2 z^6)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{883y^3r^p}\right\}$$

Case (i):

Sum of all rational terms on LHS of Equation (B), not having $\sqrt{883y^3r^p}$ as a factor = Nil.

Sum of all rational terms on RHS of Equation (B) not having $\sqrt{883y^3r^p}$ as a factor

$$= -(7^2\times 19\ell z^3)\sqrt{37x^3y^3}\left(s^p\sqrt{t^{p+1}}\right)\sqrt{19\times 883\times \ell z^3} \qquad \text{(vide II term)}$$

$$+ (7\times 19\times 37\ell^3 z^3)\sqrt{37x^3y^3}\left(s^p\sqrt{t^{p+1}}\right)\sqrt{19\times 883\times \ell z^3} \qquad \text{(vide VIII term)}$$

$$= -(7\times 19\times 37\ell z^3)\sqrt{37x^3y^3}\left(s^p\sqrt{t^{p+1}}\right)\sqrt{19\times 883\ell z^3}\left(7-\ell^2\right)$$

Equating the rational terms on both sides of Equation (B), after dividing both sides by

$$\left\{(-7\times 19\times 37\ell z^3)\sqrt{37x^3y^3}\sqrt{19\times 883\ell z^3}\left(7-\ell^2\right)\right\}$$

we get

$$\left(s^p\sqrt{t^{p+1}}\right) = 0$$

i.e., Either $s = 0$ or $t = 0$. This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$.

Case (ii):

Sum of such rational terms on LHS of Equation (B), which contain $\sqrt{883y^3r^p}$ as a factor

$$= (2^n\times 7\times 19\times 37^2\ell^2 z^6)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{883y^3r^p}$$

(combining III & IV terms)

Sum of such rational terms on RHS of Equation (B), containing $\sqrt{883y^3r^p}$ as a factor

$$= (19\times 37^2 z^6)(7\times \ell^2)\left(s^p\sqrt{t^{p+1}}\right)\sqrt{883y^3r^p}(7+\ell^2)$$

(combining III, VII & VIII terms)

which gets cancelled with LHS rational terms. $(\because \quad 7+\ell^2 = 2^n)$

III. Conclusion

Since Equation (B) in this proof was derived directly from the transformation equations of Fermat's equations for the index 3 and p where p is any prime > 3, the result $st = 0$, we have obtained on solving the transformed equations should reflect upon the Fermat's equation $r^p + s^p = t^p$, thus proving that only a trivial solution exists in the equation $r^p + s^p = t^p$.

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