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Research Paper

Nano $g\alpha^*$-continuous functions in Nano topological spaces

P.Anbarasi Rodrigo[*1], P.Subithra[*2]
[1]*Assistant Professor, Department of Mathematics, St.Mary's College (Autonomous),*
(Affiliated to ManonmaniamSundaranar University, Abishekapatti, Tirunelveli)
Thoothukudi-1, TamilNadu, India
[2]*Research Scholar, Reg.No. 21212212092003,*
Department of Mathematics, St.Mary's College (Autonomous),
(Affiliated to ManonmaniamSundaranar University, Abishekapatti, Tirunelveli)
Thoothukudi-1, TamilNadu, India

[1]*anbu.n.u@gmail.com*
[2]*p.subithra18@gmail.com*

Abstract: *The aim of this paper is to introduce and study the concept of new class of functions called nano $g\alpha^*$-continuous function and nano $g\alpha^*$-irresolute function in nano topological spaces. Some of the basic properties of nano $g\alpha^*$-continuous functions and nano $g\alpha^*$-irresolute functions are analysed.*
Keywords: *$\mathbb{N}g\alpha^*$-closed sets, $\mathbb{N}g\alpha^*$-continuous functions, $\mathbb{N}g\alpha^*$-irresolute functions.*

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I. Introduction

Continuity of functions is one of the core concepts of topology. In general, a continuous function is one, for which small changes in the input results in small changes in the output. The concept of nano topology was introduced by M.LellisThivagar and Carmel Richard, which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. He has defined nano closed sets, nano interior and nano closure of a set in nano topological spaces. He has also introduced nano continuous functions, nano open mappings, nano closed mappings and nano homeomorphisms in nano topological spaces. $\mathbb{N}g\alpha^*$-closed set is introduced by P.Subithra and P.Anbarasi Rodrigo in nano topological spaces. The aim of this paper is to introduce and study the concept of new class of functions called nano $g\alpha^*$-continuous function and nano $g\alpha^*$-irresolute function in nano topological spaces. Some of the basic properties of nano $g\alpha^*$-continuous functions and nano $g\alpha^*$-irresolute functions are analysed.

II. Preliminaries

In this section, we recall some basic definitions and results in nano topological spaces which are useful in the sequel.

Definition 2.1:[5] Let $\mathbb{U}$ be a non-empty finite set of objects called the universe and $\mathbb{R}$ be an equivalence relation on $\mathbb{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair ($\mathbb{U}$,$\mathbb{R}$) is said to be the approximation space. Let $\mathbb{X} \subseteq_{\mathbb{N}} \mathbb{U}$. Then

1) The lower approximation of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be for certain classified as $\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{L}_{\mathbb{R}}(\mathbb{X})$.

$$\mathbb{L}_{\mathbb{R}}(\mathbb{X}) = \bigcup_{x\in\mathbb{U}} \{\mathbb{R}(x) : \mathbb{R}(x) \subseteq_{\mathbb{N}} \mathbb{X}\}$$

2) The upper approximation of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be possibly classified as $\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{U}_{\mathbb{R}}(\mathbb{X})$.

$$\mathbb{U}_{\mathbb{R}}(\mathbb{X}) = \bigcup_{x\in\mathbb{U}} \{\mathbb{R}(x) : \mathbb{R}(x) \cap \mathbb{X} \neq \phi\}$$

3) The boundary region of $\mathbb{X}$ with respect to $\mathbb{R}$ is the set of all objects, which can be classified neither as $\mathbb{X}$ nor as not –$\mathbb{X}$ with respect to $\mathbb{R}$ and it is denoted by $\mathbb{B}_{\mathbb{R}}(\mathbb{X})$.

$\mathbb{B}_{\mathbb{R}}(\mathbb{X}) = \mathbb{U}_{\mathbb{R}}(\mathbb{X}) - \mathbb{L}_{\mathbb{R}}(\mathbb{X})$.

Definition 2.2:[5] Let $\mathbb{U}$ be the universe, $\mathbb{R}$ be an equivalence relation on $\mathbb{U}$ and $\tau_{\mathbb{R}}(\mathbb{X}) = \{\mathbb{U}, \phi, \mathbb{U}_{\mathbb{R}}(\mathbb{X}), \mathbb{L}_{\mathbb{R}}(\mathbb{X}), \mathbb{B}_{\mathbb{R}}(\mathbb{X})\}$ where $\mathbb{X} \subseteq_{\mathbb{N}} \mathbb{U}$. Then $\mathbb{R}(\mathbb{X})$ satisfies the following axioms:

1) $\mathbb{U}$ and $\phi \in \tau_{\mathbb{R}}(\mathbb{X})$,
2) The union of the elements of any sub collection of $\tau_{\mathbb{R}}(\mathbb{X})$ is in $\tau_{\mathbb{R}}(\mathbb{X})$,
3) The intersection of the elements of any finite sub collection of $\tau_{\mathbb{R}}(\mathbb{X})$ is in $\tau_{\mathbb{R}}(\mathbb{X})$.

That is, $\tau_{\mathbb{R}}(\mathbb{X})$is a topology on $\mathbb{U}$ called the nano topology on $\mathbb{U}$ with respect to $\mathbb{X}$. We call $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ as the nano topological space(NTS). The elements of $\tau_{\mathbb{R}}(\mathbb{X})$ are called as nano open sets. The complement of nano-open sets is called nano closed sets.

Definition 2.3:[5] If $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is a NTS with respect to $\mathbb{X}$ and if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{U}$, then

- The nano interior of $\mathbb{S}$ is defined as the union of all nano open subsets of $\mathbb{S}$ and it is denoted by $\mathbb{N}int(\mathbb{S})$. That is, $\mathbb{N}int(\mathbb{S})$ is the largest open subset of $\mathbb{S}$.
- The nano closure of $\mathbb{S}$ is defines as the intersection of all nano closed sets containing $\mathbb{S}$ and it is denoted by $\mathbb{N}cl(\mathbb{S})$. That is, $\mathbb{N}cl(\mathbb{S})$ is the smallest nano closed set containing $\mathbb{S}$.

Definition 2.4: A subset $\mathbb{S}$ of a NTS $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is called;

1) Nano pre-open[5] if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}int(\mathbb{N}cl(\mathbb{S}))$
2) Nano semi-open[5] if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}cl(\mathbb{N}int(\mathbb{S}))$
3) Nano α-open[5] if $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{N}int(\mathbb{N}cl(\mathbb{N}int(\mathbb{S})))$
4) Nano regular-open[5] if $\mathbb{S} = \mathbb{N}int(\mathbb{N}cl(\mathbb{S}))$

The complements of the above-mentioned sets are called their respective closed sets.

Definition 2.5: A subset $\mathbb{S}$ of a NTS $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ is called

1) $\mathbb{N}g$-closed[2] if $\mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
2) $\mathbb{N}gs$-closed[4] if $\mathbb{N}scl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
3) $\mathbb{N}\alpha g$-closed[8] if $\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is nano open in $\mathbb{U}$.
4) $\mathbb{N}g^*$-closed[11] if $\mathbb{N}cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{F}$, whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}$ and $\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.

Definition 2.6: A function $\mathbb{f} : (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is called

1) $\mathbb{N}$-continuous[6] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.
2) $\mathbb{N}p$-continuous[12] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}p$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.
3) $\mathbb{N}\alpha$-continuous[7] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}\alpha$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.
4) $\mathbb{N}r$-continuous[10] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}r$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.
5) $\mathbb{N}g$-continuous[3] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.
6) $\mathbb{N}g^*$-continuous[9] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g^*$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.
7) $\mathbb{N}\alpha g$-continuous[9] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}\alpha g$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.
8) $\mathbb{N}gs$-continuous[10] if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}gs$-closed in $\mathbb{U}$ for every nano closed set $\mathbb{S}$ in $\mathbb{V}$.

Definition 2.7:[1] A Nano generalized α^*(in short, $\mathbb{N}g\alpha^*$) closed set is a subset $\mathbb{S}$ of a $NTS(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$ if$\mathbb{N}\alpha cl(\mathbb{S}) \subseteq_{\mathbb{N}} \mathbb{N}int^*(\mathbb{F})$whenever $\mathbb{S} \subseteq_{\mathbb{N}} \mathbb{F}and\mathbb{F}$ is $\mathbb{N}g$-open in $\mathbb{U}$.

Result 2.8:[1]

a) Every nano closed set is $\mathbb{N}g\alpha^*$-closed.
b) Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}g$-closed.
c) Every $\mathbb{N}g^*$-closed set is $\mathbb{N}g\alpha^*$-closed.
d) Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}\alpha g$-closed.
e) Every $\mathbb{N}g\alpha^*$-closed set is $\mathbb{N}gs$-closed.

III. Nano Generalized α^*-Continuous Functions

Definition 3.1: A function $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ is called Nano generalized α^*-continuous (in short, $\mathbb{N}g\alpha^*$-continuous) if $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$ for every nano closed set $\mathbb{S}$ in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. That is, if the inverse image of every nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$ then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous.

Instance 3.2: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{p},{r},{q,s}}. Let $\mathbb{X}$={p,q} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ ,$\mathbb{U}$,{p},{q,s},{p,q,s}}and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{r},{p,r},{q,r},{r,s},{p,q,r}, {p,r,s},{q,r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{q},{r},{p,s}}. Let $\mathbb{Y}$={p,r} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{r},{p,s},{p,r,s}} and nano closed set={ ϕ,$\mathbb{V}$,{q},{q,r},{p,q,s}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=p, $\mathbb{f}$(q)=s, $\mathbb{f}$(r)=q, $\mathbb{f}$(s)=r then $\mathbb{f}^{-1}$(p)=p, $\mathbb{f}^{-1}$(q)=r, $\mathbb{f}^{-1}$(r)=s, $\mathbb{f}^{-1}$(s)=q. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous.

Theorem 3.3: A function $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ is $\mathbb{N}g\alpha^*$-continuous iff the inverse image of every nano-open set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ is $\mathbb{N}g\alpha^*$-open set in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}g\alpha^*$-continuous function. Let $\mathbb{S}$ be any nano-open set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then $\mathbb{S}^c$is nano-closed in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Since $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous, $\mathbb{f}^{-1}(\mathbb{S}^c)$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. That is $\mathbb{U}$-$\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}^{-1}(\mathbb{S})$ is$\mathbb{N}g\alpha^*$-open in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Conversely, let us assume that the inverse image of every nano-open set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ is $\mathbb{N}g\alpha^*$-open set in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Let $\mathbb{T}$ be a nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then $\mathbb{T}^c$is nano-open in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then by our assumption, $\mathbb{f}^{-1}(\mathbb{T}^c)$ is $\mathbb{N}g\alpha^*$-open in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. That is $\mathbb{U}$-$\mathbb{f}^{-1}(\mathbb{T})$ is $\mathbb{N}g\alpha^*$-open in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}^{-1}(\mathbb{T})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Thus the inverse image of every nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous

Theorem 3.4: Every $\mathbb{N}$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is nano closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. By result 2.8 a), $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous.

Remark 3.5: The invert of the preceding theorem may not be true as seen in the succeeding instance.

Instance 3.6: Let $\mathbb{U}$={p,q,r} with $\mathbb{U}/\mathbb{R}$={{p,r},{q}}. Let $\mathbb{X}$={p,r} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{p,r}}, nano closed set={ ϕ ,$\mathbb{U}$,{q}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{q},{p,q},{q,r}}. Let $\mathbb{V}$={p,q,r} with $\mathbb{V}/\mathbb{R}$={{r},{p,q}}. Let $\mathbb{Y}$={r} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{r}} and nano closed set={ ϕ,$\mathbb{V}$, {p,q}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=r, $\mathbb{f}$(q)=p, $\mathbb{f}$(r)=q then $\mathbb{f}^{-1}$(p)=q, $\mathbb{f}^{-1}$(q)=r, $\mathbb{f}^{-1}$(r)=p. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous but not $\mathbb{N}$-continuous since $\mathbb{f}^{-1}$({p,q})={q,r} is not nano closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 3.7: Every $\mathbb{N}\alpha$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}\alpha$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is $\mathbb{N}\alpha$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Since every $\mathbb{N}\alpha$-closed set is $\mathbb{N}g\alpha^*$-closed, $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous.

Remark 3.8: The invert of the preceding theorem may not be true as seen in the succeeding instance.

Instance 3.9: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{p},{r},{q,s}}. Let $\mathbb{X}$={r,s} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{r},{q,s},{q,r,s}}, $\mathbb{N}\alpha$-closed set={ ϕ ,$\mathbb{U}$,{p},{p,r},{p,q,s}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{p},{p,q},{p,r},{p,s},{p,q,r},{p,q,s},{p,r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{p},{q},{r,s}}. Let $\mathbb{Y}$={q,s} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{q},{r,s},{q,r,s}} and nano closed set={ ϕ,$\mathbb{V}$, {p},{p,q},{p,r,s}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=p, $\mathbb{f}$(q)=r, $\mathbb{f}$(r)=s, $\mathbb{f}$(s)=q then $\mathbb{f}^{-1}$(p)=p, $\mathbb{f}^{-1}$(q)=s, $\mathbb{f}^{-1}$(r)=q, $\mathbb{f}^{-1}$(s)=r. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous but not $\mathbb{N}\alpha$-continuous since $\mathbb{f}^{-1}$({p,r,s})={p,q,r} is not $\mathbb{N}\alpha$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 3.10: Every $\mathbb{N}g\alpha^*$-continuous function is $\mathbb{N}p$-continuous

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}g\alpha^*$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Since every $\mathbb{N}g\alpha^*$-closed set is $\mathrm{N}p$-closed, $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}p$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}p$-continuous.

Remark 3.11: The invert of the preceding theorem may not be true as seen in the succeeding instance.

Instance 3.12: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{q},{p,q},{r,s}}. Let $\mathbb{X}$={q} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{r,s},{p,r,s},{q,r,s}}, $\mathbb{N}p$-closed set={ ϕ ,$\mathbb{U}$,{r},{s},{r,s},{p,r,s},{q,r,s}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{r,s},{p,r,s},{q,r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{r},{p,q,s}}. Let $\mathbb{Y}$={q,s} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{p,q,s}} and nano closed set={ ϕ,$\mathbb{V}$, {r}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=q, $\mathbb{f}$(q)=s, $\mathbb{f}$(r)=p, $\mathbb{f}$(s)=r then $\mathbb{f}^{-1}$(p)=r, $\mathbb{f}^{-1}$(q)=p, $\mathbb{f}^{-1}$(r)=s, $\mathbb{f}^{-1}$(s)=q. Then $\mathbb{f}$ is $\mathbb{N}p$-continuous but not $\mathbb{N}g\alpha^*$-continuous since $\mathbb{f}^{-1}$({r})={s} is not $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 3.13: Every $\mathbb{N}r$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}r$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is $\mathbb{N}r$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Since every $\mathbb{N}r$-closed set is $\mathrm{N}g\alpha^*$-closed, $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous.

Remark 3.14: The invert of the former theorem does not holds as seen in the succeeding instance.

Instance 3.15: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{p},{q,r},{s}}. Let $\mathbb{X}$={q,s} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{s},{q,r},{q,r,s}}, $\mathbb{N}r$-closed set={ ϕ ,$\mathbb{U}$,{p},{p,s},{p,q,r}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{p},{p,q},{p,r},{p,s},{p,q,r},{p,q,s},{p,r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{p},{r},{q,s}}. Let $\mathbb{Y}$={q,r} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{r},{q,s},{q,r,s}} and nano closed set={ ϕ,$\mathbb{V}$, {p},{p,r},{p,q,s}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=p, $\mathbb{f}$(q)=s, $\mathbb{f}$(r)=q, $\mathbb{f}$(s)=r then $\mathbb{f}^{-1}$(p)=p, $\mathbb{f}^{-1}$(q)=r, $\mathbb{f}^{-1}$(r)=s, $\mathbb{f}^{-1}$(s)=q. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous but not $\mathbb{N}r$-continuous since $\mathbb{f}^{-1}$({p})={p} is not $\mathbb{N}r$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 3.16: Every $\mathbb{N}g\alpha^*$-continuous function is $\mathbb{N}g$-continuous

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}g\alpha^*$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. By result 2.8 b), $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}g$-continuous.

Remark 3.17: The invert of the former theorem may not be true as seen in the succeeding instance.

Instance 3.18: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{p,q},{r,s}}. Let $\mathbb{X}$={q,r,s} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{p,q},{r,s}}, $\mathbb{N}g$-closed set={ ϕ ,$\mathbb{U}$,{p},{q},{r},{s},{p,q},{p,r},{p,s},{q,r}, {q,s},{r,s},{p,q,r},{p,q,s},{p,r,s},{q,r,s}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$, {p,q},{r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{p},{r},{q,s}}. Let $\mathbb{Y}$={q,r} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{r},{q,s},{q,r,s}} and nano closed set={ ϕ,$\mathbb{V}$, {p},{p,r},{p,q,s}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=r, $\mathbb{f}$(q)=p, $\mathbb{f}$(r)=s, $\mathbb{f}$(s)=q then $\mathbb{f}^{-1}$(p)=q, $\mathbb{f}^{-1}$(q)=s, $\mathbb{f}^{-1}$(r)=p, $\mathbb{f}^{-1}$(s)=r. Then $\mathbb{f}$ is $\mathbb{N}g$-continuous but not $\mathbb{N}g\alpha^*$-continuous since $\mathbb{f}^{-1}$({p})={q} is not $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 3.19: Every $\mathbb{N}g^*$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}g^*$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is $\mathbb{N}g^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. By result 2.8 c), $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous.

Remark 3.20: The invert of the preceding theorem may not be true as seen in the succeeding instance.

Instance 3.21: Let $\mathbb{U}$={p,q,r} with $\mathbb{U}/\mathbb{R}$={{r},{p,q}}. Let $\mathbb{X}$={r} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{r}}, $\mathbb{N}g^*$-closed set={ ϕ ,$\mathbb{U}$,{p,q}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{p},{q},{p,q}}. Let $\mathbb{V}$={p,q,r} with $\mathbb{V}/\mathbb{R}$={{p,r},{q}}. Let

$\mathbb{Y}$={p,r} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{p,r}} and nano closed set={ ϕ,$\mathbb{V}$, {q}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=q, $\mathbb{f}$(q)=r, $\mathbb{f}$(r)=p then $\mathbb{f}^{-1}$(p)=r, $\mathbb{f}^{-1}$(q)=p, $\mathbb{f}^{-1}$(r)=q. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous but not $\mathbb{N}g^*$-continuous since $\mathbb{f}^{-1}$({q})={p} is not $\mathbb{N}g^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 3.22: Every $\mathbb{N}g\alpha^*$-continuous function is $\mathbb{N}\alpha g$-continuous.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}g\alpha^*$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. By result 2.8 d), $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}\alpha g$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}\alpha g$-continuous.

Remark 3.23: The invert of the former theorem may not be true as seen in the succeeding instance.

Instance 3.24: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{r},{q,s}}. Let $\mathbb{X}$={p,r} $\subseteq_{\mathbb{N}}\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{r}}, $\mathbb{N}\alpha g$-closed set={ ϕ ,$\mathbb{U}$,{p},{q},{s},{p,q},{p,r},{p,s},{q,r}, {q,s},{r,s}, {p,q,r},{p,q,s},{p,r,s},{q,r,s}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{p},{q},{s}, {p,q},{p,s},{q,s}, {p,q,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{p},{q,r},{s}}. Let $\mathbb{Y}$={q,s} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{s},{q,r},{q,r,s}} and nano closed set={ ϕ,$\mathbb{V}$, {p},{p,s},{p,q,r}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=r, $\mathbb{f}$(q)=p, $\mathbb{f}$(r)=s, $\mathbb{f}$(s)=q then $\mathbb{f}^{-1}$(p)=q, $\mathbb{f}^{-1}$(q)=s, $\mathbb{f}^{-1}$(r)=p, $\mathbb{f}^{-1}$(s)=r. Then $\mathbb{f}$ is $\mathbb{N}\alpha g$-continuous but not $\mathbb{N}g\alpha^*$-continuous since $\mathbb{f}^{-1}$({p,s})={q,r} is not $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 3.25: Every $\mathbb{N}g\alpha^*$-continuous function is $\mathbb{N}gs$-continuous.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be a $\mathbb{N}g\alpha^*$-continuous function. Let $\mathbb{S}$ be any nano closed set in $(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$. Then the inverse image of $\mathbb{S}$ under the map $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. By result 2.8 e), $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}gs$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}\alpha g$-continuous.

Remark 3.26: The invert of the former theorem may not be true as seen in the succeeding instance.

Instance 3.27: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{q},{p,q},{r,s}}. Let $\mathbb{X}$={q} $\subseteq_{\mathbb{N}}\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{p},{q},{p,q}}, $\mathbb{N}gs$-closed set={ ϕ ,$\mathbb{U}$,{p},{q},{r},{s},{p,r},{p,s},{q,r}, {q,s},{r,s},{p,q,r},{p,q,s},{p,r,s},{q,r,s}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{r,s}, {p,r,s},{q,r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{p,q}{r,s}}. Let $\mathbb{Y}$={q,r,s} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{p,q}, {r,s}} and nano closed set={ ϕ,$\mathbb{V}$,{p,q},{r,s}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=q, $\mathbb{f}$(q)=s, $\mathbb{f}$(r)=p, $\mathbb{f}$(s)=r then $\mathbb{f}^{-1}$(p)=r, $\mathbb{f}^{-1}$(q)=p, $\mathbb{f}^{-1}$(r)=s, $\mathbb{f}^{-1}$(s)=q. Then $\mathbb{f}$ is $\mathbb{N}gs$-continuous but not $\mathbb{N}g\alpha^*$-continuous since $\mathbb{f}^{-1}$({p,q})={p,r} is not $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$.

Remark 3.28: The composition of two $\mathbb{N}g\alpha^*$-continuous functions need not be $\mathbb{N}g\alpha^*$-continuous.

Instance 3.29: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{p},{q},{r,s}}. Let $\mathbb{X}$={q,s} $\subseteq_{\mathbb{N}}\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ ,$\mathbb{U}$,{q},{r,s},{q,r,s}}, nano closed set={ ϕ ,$\mathbb{U}$,{p},{p,q},{p,r,s}}and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{p},{p,q},{p,r},{p,s},{p,q,r},{p,q,s},{p,r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{p},{r},{q,s}}. Let $\mathbb{Y}$={p,q} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{p},{q,s},{p,q,s}}, nano closed set={ ϕ,$\mathbb{V}$,{r},{p,r},{q,r,s}} and$\mathbb{N}g\alpha^*$-closed set={ ϕ,$\mathbb{V}$,{r},{p,r},{q,r},{r,s},{p,q,r}, {p,r,s},{q,r,s}}. Let $\mathbb{W}$={p,q,r,s} with $\mathbb{W}/\mathbb{R}$={{p,q},{r,s}}. Let $\mathbb{Z}$={p,q} $\subseteq_{\mathbb{N}}\mathbb{W}$. Then $\lambda_{\mathbb{R}}(\mathbb{Z})$={ ϕ, $\mathbb{W}$,{p,q}}, nano closed set={ ϕ, $\mathbb{W}$,{r,s}} and $\mathbb{N}g\alpha^*$-closed set={ϕ, $\mathbb{W}$,{r},{s}, {r,s},{p,r,s},{q,r,s}}. Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=r, $\mathbb{f}$(q)=s, $\mathbb{f}$(r)=p, $\mathbb{f}$(s)=q then $\mathbb{f}^{-1}$(p)=r, $\mathbb{f}^{-1}$(q)=s, $\mathbb{f}^{-1}$(r)=p, $\mathbb{f}^{-1}$(s)=q. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous. Let $\mathbb{g}:(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))\longrightarrow(\mathbb{W},\lambda_{\mathbb{R}}(\mathbb{Z}))$ be defined by $\mathbb{g}$(p)=r, $\mathbb{g}$(q)=p, $\mathbb{g}$(r)=s, $\mathbb{g}$(s)=q then $\mathbb{g}^{-1}$(p)=q, $\mathbb{g}^{-1}$(q)=s, $\mathbb{g}^{-1}$(r)=p, $\mathbb{g}^{-1}$(s)=r. Then $\mathbb{g}$ is $\mathbb{N}g\alpha^*$-continuous. Let $\mathbb{S}$={p} be nano closed in $(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$. Then $(\mathbb{f}\circ\mathbb{g})^{-1}$ ($\mathbb{S}$)=$\mathbb{g}^{-1}$($\mathbb{f}^{-1}$($\mathbb{S}$))=$\mathbb{g}^{-1}$($\mathbb{f}^{-1}$({p}))=$\mathbb{g}^{-1}$({r}) ={p} which is not $\mathbb{N}g\alpha^*$-closed in $(\mathbb{W},\lambda_{\mathbb{R}}(\mathbb{Z}))$. Hence $(\mathbb{f}\circ\mathbb{g})$:$(\mathbb{W},\lambda_{\mathbb{R}}(\mathbb{Z}))\longrightarrow(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))$ is not $\mathbb{N}g\alpha^*$-continuous.

Theorem 3.30: The composition of $\mathbb{N}g\alpha^*$-continuous function and $\mathbb{N}$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Proof: Let $\mathbb{f}:(\mathbb{U},\tau_{\mathbb{R}}(\mathbb{X}))\longrightarrow(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))$ be $\mathbb{N}g\alpha^*$-continuous function and $\mathbb{g}:(\mathbb{V},\sigma_{\mathbb{R}}(\mathbb{Y}))\longrightarrow(\mathbb{W},\lambda_{\mathbb{R}}(\mathbb{Z}))$ be $\mathbb{N}$-continuous function. Let $\mathbb{S}$ be nano closed set in $(\mathbb{W},\lambda_{\mathbb{R}}(\mathbb{Z}))$. Since $\mathbb{g}$ is $\mathbb{N}$-continuous, $\mathbb{g}^{-1}(\mathbb{S})$ is nano closed in

$(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$. Since $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous, $\mathbb{f}^{-1}(\mathbb{g}^{-1}(\mathbb{S}))$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Thus $(\mathbb{g} \circ \mathbb{f})^{-1}$ ($\mathbb{S}$) is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Hence $(\mathbb{g} \circ \mathbb{f}) :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{W}, \lambda_{\mathbb{R}}(\mathbb{Z}))$ is a $\mathbb{N}g\alpha^*$-continuous function.

IV. Nano Generalized α^*-Irresolute Functions

Definition 4.1: A function $\mathbb{f} :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is called Nano generalized α^*-irresolute (in short, $\mathbb{N}g\alpha^*$-irresolute) function if the inverse image of every $\mathbb{N}g\alpha^*$-closed set in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$.

Instance 4.2: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{p},{q,r},{s}}. Let $\mathbb{X}$={q,s} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ ,$\mathbb{U}$,{s},{q,r},{q,r,s}}and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{p},{p,q},{p,r},{p,s},{p,q,r}, {p,q,s},{p,r,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{q},{r},{p,s}}. Let $\mathbb{Y}$={p,r} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{r},{p,s},{p,r,s}} and $\mathbb{N}g\alpha^*$-closed set={ ϕ,$\mathbb{V}$,{q},{p,q},{q,r},{q,s},{p,q,r}, {p,q,s},{q,r,s}}. Let $\mathbb{f} :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=q, $\mathbb{f}$(q)=p, $\mathbb{f}$(r)=r, $\mathbb{f}$(s)=s then $\mathbb{f}^{-1}$(p)=q, $\mathbb{f}^{-1}$(q)=p, $\mathbb{f}^{-1}$(r)=r, $\mathbb{f}^{-1}$(s)=s. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-irresolute.

Theorem 4.3: Every $\mathbb{N}g\alpha^*$-irresolute function is $\mathbb{N}g\alpha^*$-continuous.

Proof: Let Let$\mathbb{f} :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be $\mathbb{N}g\alpha^*$-irresolute function. Then the inverse image of every $\mathbb{N}g\alpha^*$-closed set in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Let $\mathbb{S}$ be a nano closed set in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$. By result 2.8 a), $\mathbb{S}$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$. Since $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-irresolute function, $\mathbb{f}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Hence $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous.

Remark 4.4: The invert of the preceding theorem may not be true as seen in the succeeding instance.

Instance 4.5: Let $\mathbb{U}$={p,q,r,s} with $\mathbb{U}/\mathbb{R}$={{r},{q,s}}. Let $\mathbb{X}$={p,r} $\subseteq_{\mathbb{N}}$ $\mathbb{U}$. Then $\tau_{\mathbb{R}}(\mathbb{X})$={ϕ,$\mathbb{U}$,{r}} and $\mathbb{N}g\alpha^*$-closed set={ϕ ,$\mathbb{U}$,{p},{q},{s},{p,q},{p,s},{q,s},{p,q,,s}}. Let $\mathbb{V}$={p,q,r,s} with $\mathbb{V}/\mathbb{R}$={{r},{p,q,s}}. Let $\mathbb{Y}$={q,s} $\subseteq_{\mathbb{N}}\mathbb{V}$. Then $\sigma_{\mathbb{R}}(\mathbb{Y})$={ϕ,$\mathbb{V}$,{p,q,s}}, nano closed set={ ϕ,$\mathbb{V}$,{r}} and $\mathbb{N}g\alpha^*$-closed set={ϕ, $\mathbb{V}$,{r},{p,r},{q,r},{r,s},{p,q,r},{p,r,s}, {q,r,s}}. Let $\mathbb{f} :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be defined by $\mathbb{f}$(p)=r, $\mathbb{f}$(q)=s, $\mathbb{f}$(r)=p, $\mathbb{f}$(s)=q then $\mathbb{f}^{-1}$(p)=r, $\mathbb{f}^{-1}$(q)=s, $\mathbb{f}^{-1}$(r)=p, $\mathbb{f}^{-1}$(s)=q. Then $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-continuous but not $\mathbb{N}g\alpha^*$-irresolute since $\mathbb{f}^{-1}$({p,r})={p,r} is not $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$.

Theorem 4.6: The composition of two $Ng\alpha^*$-irresolute functions is $Ng\alpha^*$-irresolute.

Proof: Let $f :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ and $g :(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y})) \longrightarrow (\mathbb{W}, \lambda_{\mathbb{R}}(\mathbb{Z}))$ be two $\mathbb{N}g\alpha^*$-irresolute functions. Let $\mathbb{S}$ be a $\mathbb{N}g\alpha^*$-closed set in $(\mathbb{W}, \lambda_{\mathbb{R}}(\mathbb{Z}))$.Since $\mathbb{g}$ is $\mathbb{N}g\alpha^*$-irresolute, $\mathbb{g}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$. Also, since $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-irresolute, $\mathbb{f}^{-1}(\mathbb{g}^{-1}(\mathbb{S}))$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Thus $(\mathbb{g} \circ \mathbb{f})^{-1}$ ($\mathbb{S}$) is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Hence $(\mathbb{g} \circ \mathbb{f}) : (\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{W}, \lambda_{\mathbb{R}}(\mathbb{Z}))$ is a $\mathbb{N}g\alpha^*$-irresolute function.

Theorem 4.7: The composition of $\mathbb{N}g\alpha^*$-irresolute function and $\mathbb{N}g\alpha^*$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Proof: Let $\mathbb{f} :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$ be $\mathbb{N}g\alpha^*$-irresolute function and $\mathbb{g} :(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y})) \longrightarrow (\mathbb{W}, \lambda_{\mathbb{R}}(\mathbb{Z}))$ be $\mathbb{N}g\alpha^*$-continuous function. Let $\mathbb{S}$ be nano closed set in $(\mathbb{W}, \lambda_{\mathbb{R}}(\mathbb{Z}))$. Since $\mathbb{g}$ is $\mathbb{N}g\alpha^*$-continuous, $\mathbb{g}^{-1}(\mathbb{S})$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{V}, \sigma_{\mathbb{R}}(\mathbb{Y}))$. Since $\mathbb{f}$ is $\mathbb{N}g\alpha^*$-irresolute, $\mathbb{f}^{-1}(\mathbb{g}^{-1}(\mathbb{S}))$ is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Thus $(\mathbb{g} \circ \mathbb{f})^{-1}$ ($\mathbb{S}$) is $\mathbb{N}g\alpha^*$-closed in $(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X}))$. Hence $(\mathbb{g} \circ \mathbb{f}) :(\mathbb{U}, \tau_{\mathbb{R}}(\mathbb{X})) \longrightarrow (\mathbb{W}, \lambda_{\mathbb{R}}(\mathbb{Z}))$ is a $\mathbb{N}g\alpha^*$-continuous function.

Corollary4.8: The composition of $\mathbb{N}g\alpha^*$-irresolute function and $\mathbb{N}$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Corollary4.9: The composition of $\mathbb{N}g\alpha^*$-irresolute function and $\mathbb{N}\alpha$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

Corollary4.10: The composition of $\mathbb{N}g\alpha^*$-irresolute function and $\mathbb{N}g^*$--continuous function is $\mathbb{N}g\alpha^*$-continuous.

Corollary4.11: The composition of $\mathbb{N}g\alpha^*$-irresolute function and $\mathbb{N}r$-continuous function is $\mathbb{N}g\alpha^*$-continuous.

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