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Research Paper

A Precise Proof for Fermat's Last Theorem using Ramanujan-Nagell Equation

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Abstract
Fermat's Last Theorem states that it is impossible to find positive integers A, B and C satisfying the equation

$$A^n + B^n = C^n$$

where n is any integer > 2.

Taking the proofs of Fermat for the index n = 4, and Euler for n = 3, it is sufficient to prove the theorem for n = p, any prime > 3 [1].

We hypothesize that all r, s and t are non-zero integers in the equation

$$r^p + s^p = t^p$$

and establish contradiction in this proof.

Just for supporting the proof in the above equation, we have another equation

$$x^3 + y^3 = z^3$$

Without loss of generality, we assert that both x and y as non-zero integers; z^3 a non-zero integer; z and z^2 irrational.
We create transformed equations to the above two equations through parameters, into which we have incorporated an Ramanujan - Nagell equation. Solving the transformed equations we prove the theorem.
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I. Introduction

Around 1637, Pierre-de-Fermat, the French mathematician wrote in the margin of a book that the equation $A^n + B^n = C^n$ has no solution in integers *A*, *B* and *C*, if n is any integer >2. Fermat stated in the margin of the book that he himself had found a marvelous proof of the theorem, but the margin was too narrow to contain it. His proof is available only for the index *n*=4, using infinite descent method.

Many mathematicians like Sophie Germain, E.E. Kummer had proved the theorem for particular cases. Number theory has been developed leaps and bounds by the immense contributions by a lot of mathematicians. Finally, after 350 years, the theorem was completely proved by Prof. Andrew Wiles, using highly complicated mathematical tools and advanced number theory [2], [3].

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Here we are trying an elementary proof.

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II. Assumptions

1) We initially hypothesize that all *r*, *s* and *t* are non-zero integers satisfying the equation

$$r^p + s^p = t^p$$

where *p* is any prime > 3, with $gcd(r, s, t) = 1$ and establish a contradiction in this proof.

2) Just for supporting the proof in the above equation, we have taken another equation.

$$x^3 + y^3 = z^3; \qquad gcd(x, y, z^3) = 1$$

Without loss of generality, we can have both *x* and *y* as non-zero integers, z^3 a non-zero integer; both *z* and z^2 irrational. Since we prove the theorem only in the equation $r^p + s^p = t^p$ for all possible integral values of *r*, *s* and *t* we have the choice in having $x=3^2\times13$; $y=64$; $z^3= 64^3+117^3=7\times181\times1471$.

3) By trial and error method, we have created the transformation equations to $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$

using parameters called *a*, *b*, *c*, *d*, *e* and *f*. Creation of such transformation equations could be done in thousands of ways, but giving a proof is most difficult and rare. Every time the rational terms in equation (8) we derive from the transformed equations got cancelled out on both sides. After enormous random trials, the formulation of transformed equations was achieved to bring out the results for proving the theorem.

4) Into the transformed equations we have incorporated the Euler's equation

$$2^n = 7 + \ell^2$$

We use only the solution

$$2^{15} = 7 + 181^2$$

In this proof, where $\ell=181$. Hence $z^3=7\times1471\times\ell$.

Proof. We have created the following transformation equations to represent $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$ respectively,

$$\left(a\sqrt{13z^3}+b\sqrt{2^{n/2}}\right)^2+\left(\frac{c\sqrt{1471}+d\sqrt{\ell^{5/3}}}{\sqrt{2^{3n/2}}}\right)=\left(e\sqrt{3}+f\sqrt{7}\right)^2$$

and

$$\left(\frac{a\sqrt{7}-b\sqrt{st}}{\sqrt{\ell}}\right)^2+\left(\frac{c\sqrt{13}-d\sqrt{r}}{\sqrt{7^{5/3}}}\right)^2=\left(\frac{e\sqrt{7^{1/3}}-f\sqrt{13z}}{\sqrt{\ell^{7/3}}}\right)^2 \quad (1)$$

through the parameters called *a*, *b*, *c*, *d*, *e* and *f*.

We may have

$$a\sqrt{13z^3} + b\sqrt{2^{n/2}} = \sqrt{x^3} \quad (2)$$

$$a\sqrt{7} - b\sqrt{st} = \sqrt{\ell r^{p}} \tag{3}$$

$$c\sqrt{1471} + d\sqrt{\ell^{5/3}} = \sqrt{y^{3}2^{3n/2}} \tag{4}$$

$$c\sqrt{13} - d\sqrt{r} = \sqrt{s^{p}7^{5/3}} \tag{5}$$

$$e\sqrt{3} + f\sqrt{7} = \sqrt{z^{3}} \tag{6}$$

and

$$e\sqrt{7^{1/3}} - f\sqrt{13z} = \sqrt{t^{p}\ell^{7/3}} \tag{7}$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$a = \left(\sqrt{x^{3}st} + \sqrt{2^{n/2}r^{p}\ell}\right) \Big/ \left(\sqrt{13stz^{3}} + \sqrt{7\times 2^{n/2}}\right)$$

$$b = \left(\sqrt{7x^{3}} - \sqrt{13\ell r^{p}z^{3}}\right) \Big/ \left(\sqrt{13stz^{3}} + \sqrt{7\times 2^{n/2}}\right)$$

$$c = \left(\sqrt{2^{3n/2}y^{3}r} + \sqrt{7^{5/3}\ell^{5/3}s^{p}}\right) \Big/ \left(\sqrt{1471r} + \sqrt{13\ell^{5/3}}\right)$$

$$d = \left(\sqrt{13\times 2^{3n/2}y^{3}} - \sqrt{1471\times 7^{5/3}s^{p}}\right) \Big/ \left(\sqrt{1471r} + \sqrt{13\ell^{5/3}}\right)$$

$$e = \left(z^{2}\sqrt{13} + \sqrt{7\ell^{7/3}t^{p}}\right) \Big/ \left(\sqrt{39z} + 7^{2/3}\right)$$

and $$f = \left(\sqrt{7^{1/3}z^{3}} - \sqrt{3\times \ell^{7/3}t^{p}}\right) \Big/ \left(\sqrt{39z} + 7^{2/3}\right)$$

From (2) & (4), (5) & (7) and (4) & (7), we get

$$\sqrt{2^{n/2}} \times \sqrt{2^{3n/2}} = \left(\sqrt{x^{3}} - a\sqrt{13z^{3}}\right)\left(c\sqrt{1471} + d\sqrt{\ell^{5/3}}\right) \Big/ \left(b\sqrt{y^{3}}\right)$$

i.e., $$2^{n} = \left\{(c)\sqrt{1471x^{3}} + (d)\sqrt{\ell^{5/3}x^{3}} - (ac)\sqrt{13\times 1471z^{3}} - (ad)\sqrt{13\ell^{5/3}z^{3}}\right\} \Big/ \left(b\sqrt{y^{3}}\right)$$

$$\sqrt{7^{1/3}} \times \sqrt{7^{5/3}} = \left(c\sqrt{13} - d\sqrt{r}\right)\left(\sqrt{t^{p}\ell^{7/3}} + f\sqrt{13z}\right) \Big/ \left(e\sqrt{s^{p}}\right)$$

i.e. $$7 = \left\{(c)\sqrt{13t^{p}\ell^{7/3}} + (13cf)\sqrt{z} - (d)\sqrt{rt^{p}\ell^{7/3}} - (df)\sqrt{13rz}\right\} \Big/ \left(e\sqrt{s^{p}}\right)$$

From

$$\sqrt{\ell^{5/3}} \times \sqrt{\ell^{7/3}} = \left(\sqrt{2^{3n/2}y^{3}} - c\sqrt{1471}\right)\left(e\sqrt{7^{1/3}} - f\sqrt{13z}\right) \Big/ \left(d\sqrt{t^{p}}\right)$$

i.e., $$\ell^{2} = \left\{(e)\sqrt{2^{3n/2}7^{1/3}y^{3}} - (f)\sqrt{2^{3n/2}\times 13y^{3}z} - (ce)\sqrt{1471\times 7^{1/3}} + (cf)\sqrt{13z\times 1471}\right\} \Big/ \left(d\sqrt{t^{p}}\right)$$

Substituting the above equivalent values of 2^n; 7 and ℓ^2 in the Ramanujan-Nagell equation $2^n = 7 + \ell^2$ after multiplying both sides by $\left\{(bde)\sqrt{y^3 s^p t^p}\right\}$, we get

$$\left\{(de)\sqrt{s^p t^p}\right\}\left\{(c)\sqrt{1471x^3}+(d)\sqrt{\ell^{5/3}x^3}-(ac)\sqrt{13\times 1471z^3}-(ad)\sqrt{13\ell^{5/3}z^3}\right\}$$

$$=\left\{(bd)\sqrt{y^3 t^p}\right\}\left\{(c)\sqrt{13t^p\ell^{7/3}}+(13cf)\sqrt{z}-(d)\sqrt{rt^p\ell^{7/3}}-(df)\sqrt{13rz}\right\}$$

$$+\left\{(be)\sqrt{y^3 s^p}\right\}\left\{(e)\sqrt{2^{3n/2}7^{1/3}y^3}-(f)\sqrt{2^{3n/2}13y^3 z}\right.$$

$$\left.-(ce)\sqrt{1471\times 7^{1/3}}+(cf)\sqrt{13z\times 1471}\right\} \quad (8)$$

Let us find out all rational terms and equate them on both sides of Equation (8), after multiplying both sides by

$$\left(\sqrt{13stz^3}+\sqrt{7\times 2^{n/2}}\right)\left(\sqrt{1471r}+\sqrt{13\ell^{5/3}}\right)^2\left(\sqrt{39z}+7^{2/3}\right)^2$$

to be free from denominators on the parameters *a*; *b*; *c*; *d*; *e* and *f* and again multiplying by

$$\left\{\sqrt{7\times 13\times \ell^{4/3}z^5 t}\right\}$$

for getting some rational terms, as worked out below, term by term.

I term in LHS of Equation (8), after multiplying by the respective terms, and substituting for {(*cd*)e}

$$=\sqrt{1471x^3 s^p t^p}\left(\sqrt{13stz^3}+\sqrt{7\times 2^{n/2}}\right)\left(\sqrt{39z}+7^{2/3}\right)\left(\sqrt{7\times 13z^5 t\ell^{4/3}}\right)$$

$$\times\left(\sqrt{2^{3n/2}y^3 r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\left(\sqrt{13\times 2^{3n/2s}y^3}-\sqrt{1471\times 7^{5/3}s^p}\right)\left(z^2\sqrt{13}+\sqrt{7t^p\ell^{7/3}}\right)$$

There is no rational part in this term.

II term in LHS of equation (8), after multiplying by the respective terms, and substituting for {d^2e} is

$$=\sqrt{s^p t^p\ell^{5/3}x^3}\left(\sqrt{13stz^3}+\sqrt{7\times 2^{n/2}}\right)\left(\sqrt{39z}+7^{2/3}\right)\left(\sqrt{7\times 13\ell^{4/3}z^5 t}\right)$$

$$\times\left\{\left(13y^3\sqrt{2^{3n}}\right)+\left(1471\times 7^{5/3}s^p\right)-\left(2\sqrt{13\times 2^{3n/2}y^3}\right)\left(\sqrt{1471\times 7^{5/3}s^p}\right)\right\}\left(z^2\sqrt{13}+\sqrt{7\ell^{7/3}t^p}\right)$$

There is no rational part in this term.

III term in LHS of equation (8), after multiplying by the respective terms, and substituting for {$a(cd)e$} is

$$=\left(-\sqrt{s^p t^p}\right)\sqrt{13\times 1471z^3}\left(\sqrt{39z}+7^{2/3}\right)\sqrt{7\times 13\times \ell^{4/3}z^5 t}\left(\sqrt{stx^3}+\sqrt{2^{n/2}\ell r^p}\right)$$

$$\times\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\left(\sqrt{13\times 2^{3n/2}y^3}-\sqrt{1471\times 7^{5/3}s^p}\right)\left(z^2\sqrt{13}+\sqrt{7t^p\ell^{7/3}}\right)$$

On multiplying by,

$$\left\{\left(-\sqrt{s^pt^p}\right)\sqrt{13\times 1471z^3}\left(7^{2/3}\right)\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{2^{n/2}\ell r^p}\right)\right.$$

$$\left.\times\sqrt{7^{5/3}\ell^{5/3}s^p}\left(\sqrt{13\times 2^{3n/2}y^3}\right)\left(z^2\sqrt{13}\right)\right\}$$

We get

$$\left\{-\left(2^n\times 7^2\right)\left(13^2\times \ell^2\right)\left(z^6s^p\sqrt{t^{p+1}}\right)\left(\sqrt{1471y^3r^p}\right)\right\}$$

where $y = 8^2$; this term will be discussed later on.

IV term in LHS of equation (8), after multiplying by the respective terms, and substituting for $\{ad^2e\}$ is

$$=\left(-\sqrt{s^pt^p}\right)\sqrt{13z^3\ell^{5/3}}\left(\sqrt{39z}+7^{2/3}\right)\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{stx^3}+\sqrt{2^{n/2}\ell r^p}\right)$$

$$\times\left\{\left(13y^3\sqrt{2^{3n}}\right)+\left(1471\times 7^{5/3}s^p\right)-\left(2\sqrt{13\times 2^{3n/2}y^3}\sqrt{1471\times 7^{5/3}s^p}\right)\right\}\left(z^2\sqrt{13}+\sqrt{7t^p\ell^{7/3}}\right)$$

On multiplying by,

$$\left\{\left(-\sqrt{s^pt^p}\right)\sqrt{13\ell^{5/3}z^3}\left(7^{2/3}\right)\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{2^{n/2}\ell r^p}\right)\right.$$

$$\left.\times\left(\sqrt{1471\times 7^{5/3}s^p}\right)\left(-2\sqrt{13\times 2^{3n/2}y^3}\right)\left(z^2\sqrt{13}\right)\right\}$$

We get

$$\left\{\left(2^{n+1}\times 7^2\times 13^2\times \ell^2\right)\left(z^6s^p\sqrt{t^{p+1}}\right)\left(\sqrt{1471y^3r^p}\right)\right\}$$

where $y = 8^2$; this term will be discussed later on.

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{b(cd)\}$ is

$$=\left(t^p\sqrt{13y^3\ell^{7/3}}\right)\left(\sqrt{39z}+7^{2/3}\right)^2\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times \ell r^pz^3}\right)$$

$$\times\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\left(\sqrt{13\times 2^{3n/2}y^3}-\sqrt{1471\times 7^{5/3}s^p}\right)$$

There is no rational part in this term.

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{b(cd)f\}$ is

$$=\left(13\times\sqrt{y^3zt^p}\right)\left(\sqrt{39z}+7^{2/3}\right)\sqrt{7\times13\times\ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times\ell r^p z^3}\right)$$

$$\times\left(\sqrt{2^{3n/2}y^3r+}\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\left(\sqrt{13\times2^{3n/2}y^3}-\sqrt{1471\times7^{5/3}s^p}\right)\left(\sqrt{7^{1/3}z^3}-\sqrt{3\times t^p\ell^{7/3}}\right)$$

(i) Rational part in this term

$$=\left\{\left(13\times\sqrt{y^3zt^p}\right)\left(7^{2/3}\sqrt{7\times13\times\ell^{4/3}z^5t}\right)\sqrt{7x^3}\sqrt{7^{5/3}\ell^{5/3}s^p}\left(-\sqrt{1471\times7^{5/3}s^p}\right)\left(\sqrt{7^{1/3}z^3}\right)\right\}$$

$$=\left[-\left(7^3\times13\ell\right)\left(s^p\sqrt{t^{p+1}}\right)z^3\sqrt{13x^3y^3}\sqrt{1471\times7\ell z^3}\right]$$

(ii) Also on multiplying by

$$\left\{\left(13\times\sqrt{y^3zt^p}\right)\left(7^{2/3}\sqrt{7\times13\times\ell^{4/3}z^5t}\right)\left(-\sqrt{13r^pz^3\ell}\right)\sqrt{7^{5/3}\ell^{5/3}s^p}\right.$$

$$\left.\times\left(-\sqrt{1471\times7^{5/3}s^p}\right)\left(\sqrt{7^{1/3}z^3}\right)\right\}$$

We get

$$\left\{\left(7^3\times13^2\times\ell^2\right)\left(z^6s^p\sqrt{t^{p+1}}\right)\sqrt{1471y^3r^p}\right\}$$

This term will be discussed later on.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bd^2\}$ is

$$=-\left(t^p\sqrt{y^3r\ell^{7/3}}\right)\left(\sqrt{39z}+7^{2/3}\right)^2\sqrt{7\times13\times\ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times\ell r^pz^3}\right)$$

$$\times\left(\sqrt{13\times2^{3n/2}y^3}-\sqrt{1471\times7^{5/3}s^p}\right)^2$$

There is no rational part in this term.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bd^2f\}$ is

$$=\left(-\sqrt{13y^3zt^p}\right)\left(\sqrt{39z}+7^{2/3}\right)\sqrt{7\times13\times\ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times\ell r^pz^3}\right)$$

$$\times\left(\sqrt{13\times2^{3n/2}y^3}-\sqrt{1471\times7^{5/3}s^p}\right)^2\left(\sqrt{7^{1/3}z^3}-\sqrt{3\times t^p\ell^{7/3}}\right)$$

There is no rational part in this term.

V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{be^2\}$ is

$$=\left(y^3\sqrt{2^{3n/2}\times 7^{1/3}s^p}\right)\left(\sqrt{1471r}+\sqrt{13\ell^{5/3}}\right)^2\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times \ell r^p z^3}\right)$$

$$\times\left(z^2\sqrt{13}+\sqrt{7\times t^p\ell^{7/3}}\right)^2$$

There is no rational part in this term.

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{b(ef)\}$ is

$$=\left(-y^3\sqrt{2^{3n/2}\times 13zs^p}\right)\left(\sqrt{1471r}+\sqrt{13\ell^{5/3}}\right)^2\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times \ell r^p z^3}\right)$$

$$\times\left(z^2\sqrt{13}+\sqrt{7\times t^p\ell^{7/3}}\right)\left(\sqrt{7^{1/3}z^3}-\sqrt{3t^p\ell^{7/3}}\right)$$

There is no rational part in this term.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bce^2\}$ is

$$=\left(-\sqrt{1471\times 7^{1/3}y^3s^p}\right)\left(\sqrt{1471r}+\sqrt{13\ell^{5/3}}\right)\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times \ell r^p z^3}\right)$$

$$\times\left(z^2\sqrt{13}+\sqrt{7\times t^p\ell^{7/3}}\right)\left(\sqrt{2^{3n/2}y^3r}-\sqrt{7^{5/3}\ell^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{1471\times 7^{1/3}y^3s^p}\right)\sqrt{13\ell^{5/3}}\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(-\sqrt{13\times \ell r^p z^3}\right)\right.$$

$$\left.\times\sqrt{7^{5/3}\ell^{5/3}s^p}\left(2z^2\sqrt{7\times 13t^p\ell^{7/3}}\right)\right.$$

We get

$$\left\{\left(2\times 7^2\times \ell^4\right)\times 13^2\left(s^p\sqrt{t^{p+1}}\right)\left(z^6\right)\sqrt{1471y^3r^p}\right\}$$

where $y = 8^2$; this term will be discussed later on.

VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bc(ef)\}$ is

$$=\left(\sqrt{1471\times 13y^3zs^p}\right)\left(\sqrt{1471r}+\sqrt{13\ell^{5/3}}\right)\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(\sqrt{7x^3}-\sqrt{13\times \ell r^p z^3}\right)$$

$$\times\left(z^2\sqrt{13}+\sqrt{7\times t^p\ell^{7/3}}\right)\left(\sqrt{2^{3n/2}y^3r}+\sqrt{7^{5/3}\ell^{5/3}s^p}\right)\left(\sqrt{7^{1/3}z^3}-\sqrt{3t^p\ell^{7/3}}\right)$$

(i) Rational part in this term

$$=\left\{\left(\sqrt{1471\times 13y^3zs^p}\right)\sqrt{13\ell^{5/3}}\sqrt{7\times 13\times \ell^{4/3}z^5t}\sqrt{7x^3}\sqrt{7^{5/3}\ell^{5/3}s^p}\sqrt{7^{1/3}z^3}\right\}$$

$$=\left\{\left(7^2\times 13\ell^3\right)\left(s^p\sqrt{t^{p+1}}\right)z^3\sqrt{13x^3y^3}\sqrt{1471\times 7\times \ell z^3}\right\}$$

(Since $x = 3^2 \times 13$; $y = 8^2$; $z^3 = 7 \times 181 \times 1471$, where $l = 181$

(ii) Also on multiplying by,

$$\left\{\left(\sqrt{1471\times 13y^3zs^p}\right)\sqrt{13\ell^{5/3}}\sqrt{7\times 13\times \ell^{4/3}z^5t}\left(-\sqrt{13r^pz^3\ell}\right)\sqrt{7^{5/3}\ell^{5/3}s^p}\left(\sqrt{7t^p\ell^{7/3}}\right)\sqrt{7^{1/3}z^3}\right\}$$

We get

$$\left\{-\left(7^2\times \ell^4\right)\left(13^2s^p\sqrt{t^{p+1}}\right)z^6\sqrt{1471\times y^3r^p}\right\}$$

This term will be discussed later on.

Case (1):

Sum of all rational terms containing $\sqrt{1471\times y^3r^p}$ as a factor on LHS of Equation (8).

$$=\left(2^n\times 7^2\times 13^2\ell^2\right)\left(z^6s^p\sqrt{t^{p+1}}\right)\sqrt{1471\times y^3r^p} \quad \text{(Combining III \& IV terms)}$$

Similar terms on RHS of Equation (8)

$$=\left(13^2\times 7^3\times \ell^2\right)\left(z^6s^p\sqrt{t^{p+1}}\right)\sqrt{1471\times y^3r^p} \quad \text{(vide II term)}$$

$$+\left(13^2\times 7^4\times \ell^2\right)\left(z^6s^p\sqrt{t^{p+1}}\right)\sqrt{1471\times y^3r^p} \quad \text{(combining VI \& VIII terms)}$$

$$=\left(2^n\times 7^2\times 13^2\ell^2\right)\left(z^6s^p\sqrt{t^{p+1}}\right)\sqrt{1471\times y^3r^p} \quad \left(\because\ 7+\ell^2=2^n\right)$$

which gets cancelled with LHS term.

Case (2):

There is no rational terms not containing $\sqrt{1471\times y^3r^p}$ as a factor on LHS of Equation (8).

Similar terms on RHS of Equation (8)

$$=-\left(13\times 7^2\times \ell\right)\left(z^3s^p\sqrt{t^{p+1}}\right)\sqrt{13x^3y^3}\sqrt{1471\times 7\ell z^3}\left(7-\ell^2\right)$$

(combining II & VIII terms)

Equating the rational terms on both sides of Equation (8), after dividing both sides by

$$\left(-7^2\times 13\ell\right)z^3\sqrt{13x^3y^3}\sqrt{1471\times 7\ell z^3}\left(7-\ell^2\right)$$

We get,

$$\left(s^p\sqrt{t^{p+1}}\right)=0$$

That is either $s = 0$ or $t = 0$. This contradicts our initial hypothesis that all r; s; t all non-zero integer in the Fermat equation $r^p + s^p = t^p$ and proves that only a trivial solution exists.

III. Conclusion

Since equation (8) in this proof was derived directly from the transformation equations of $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$, the result $st = 0$, that we have obtained on equating the rational terms on both sides of Equation (B) should reflect on the Fermat's equation $r^p + s^p = t^p$.

The only main hypothesis that we made in this proof, namely, r, s and t are non-zero integers has been shattered by the result $st = 0$, and proves the theorem.

References

[1] Hardy G. H. and Wright E. M., An introduction to the theory of numbers, VI Edition. Oxford University Press, 2008, pp. 261-586.

[2] Lawrence C. Washington, Elliptic Curves, Number Theory and Cryptography. Second Edition, 2003, pp. 445-448.

[3] Andrew Wiles, Modular Elliptic Curves and Fermat's Last Theorem, Annals of Mathematics, 1995; 141(3); 443-551.

[4] Seetharaman P.N, Quest Journal of Research in Applied Mathematics, Volume-8, Issue-10, Page 24-32 (2022).

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