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Research Paper

Making a Non-Singular Matrix Singular

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ABSTRACT: *Given any non-singular square matrix and its inverse, the matrix can be made singular by replacing any single element, column, or all elements.*

KEYWORDS: *matrix, singular, nonsingular, non-singular, inverse, square, algorithm, fast, time complexity, efficient*

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I. INTRODUCTION

Let A be a non-singular $n \times n$ matrix and B be its inverse. An element in matrix A can be referred to by a_{ij}, where i and j are i-th row and j-th column number of A, respectively. An element in matrix B can be similarly referred to by b_{rc}, where r and c are r-th row and c-th column number of B, respectively.

II. FORMULATION

Method 1 to make A singular is by replacing any single element, a_{ij}, with $a_{ij} - \frac{1}{b_{ji}}$. A becomes singular using this simple method. If b_{ji} is 0, another element a_{ij} must be chosen.

Method 2 to make A singular is by replacing every element in column j of array A, with the reciprocal of the sum of all elements in row j of array B. This can be written as

$$a_{iJ} - \frac{1}{\sum_{c \in C} b_{jc}}$$

Where C = {1, …, n}, i = {1, …, n}, and J remains constant. This method requires more replacement than Method 1, but nonetheless changes A from non-singular to singular. If $\sum_{c \in C} b_{jc}$ is 0, another column a_{iJ} must be chosen.

Method 3 requires the most calculation. A can be made singular by adding the reciprocal sum of every element in B to every element in A. This can be accomplished using the following formula:

$$a_{ij} - \frac{1}{\sum_{r=1}^{n} \sum_{c=1}^{n} b_{rc}}$$

Where i = {1, …, n}, j = {1, …, n}, and $\sum_{r=1}^{n} \sum_{c=1}^{n} b_{rc} \neq 0$.

Further information about the derivation and proof of the formula $-\frac{1}{b_{ji}}$ introduced in Method 1 can be found in [1].

III. TYPICAL EXAMPLES

Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0.8 & 1.8 \\ 3 & -0.8 & 2.8 \\ -1 & 0.2 & 1.2 \end{bmatrix}$, and $A^{-1} = B$. Assume A is non-singular. The following matrices are all A, made singular using the methods described above.

Using Method 1

$$\begin{bmatrix} 2+\frac{1}{2} & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3-\frac{1}{3} & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4+1 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4 \\ 4-\frac{1}{0.8} & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3+\frac{1}{0.8} & 1 \\ 1 & 2 & 4 \end{bmatrix},$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1-\frac{1}{0.2} \\ 1 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1-\frac{1}{1.8} & 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2-\frac{1}{2.8} & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4-\frac{1}{1.2} \end{bmatrix}$$

Using Method 2

$$\begin{bmatrix} 2-\frac{1}{0.6} & 3 & 4 \\ 4-\frac{1}{0.6} & 3 & 1 \\ 1-\frac{1}{0.6} & 2 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3-\frac{1}{5} & 4 \\ 4 & 3-\frac{1}{5} & 1 \\ 1 & 2-\frac{1}{5} & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4-\frac{1}{0.4} \\ 4 & 3 & 1-\frac{1}{0.4} \\ 1 & 2 & 4-\frac{1}{0.4} \end{bmatrix}$$

Using Method 3

$$\begin{bmatrix} 2-\frac{1}{6} & 3-\frac{1}{6} & 4-\frac{1}{6} \\ 4-\frac{1}{6} & 3-\frac{1}{6} & 1-\frac{1}{6} \\ 1-\frac{1}{6} & 2-\frac{1}{6} & 4-\frac{1}{6} \end{bmatrix}$$

IV. CONCLUSION

Given any non-singular square matrix and its inverse, using the method 1, 2 or 3 above, the matrix can be made singular by replacing any single element, column, or all elements.

REFERENCES

[1]. F. C. Chang, "Inversion of a perturbed matrix," Appl.Math.Letters, vol.19, pp.163-173, 2006.

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