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Research Paper

Examining the Impact of Debt Maturity Time, Volatility and Expected Return on Credit Spreads Produced by Merton and MKMV Models

George Jumbe [1], Ravi Gor[2]
[1]Research Scholar, Department of Mathematics, Gujarat University
ORCID: 0000-0003-0452-4453
[2]Department of Mathematics, Gujarat University
Corresponding Author: George Jumbe

ABSTRACT: *This study examines how the Merton and Moody's KMV (MKMV) models' output of credit spreads changes over time, with particular attention paid to volatility and expected asset returns. We first determined the yield on the hazardous loan and then the credit spread in order to compare the outcomes of the Merton and MKMV approaches. We used the balance sheet for Apple Inc. (AAPL), which spanned the time period from September 29, 2019, to September 29, 2022. Results indicate that the MKMV strategy employing AAPL data produces considerable credit spreads as compared to the Merton technique.*
KEY WORDS: *Credit Spread, Volatility, Debt Maturity Time, Merton Model, MKMV Model.*

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I. INTRODUCTION

A credit spread or a yield spread or a bond credit spread, is the difference in yield between bonds of a similar maturity but with different credit quality. Consequently, the spread is the fluctuation in returns caused by different credit qualities. The credit spread serves to mitigate the exposure to credit risk, which is linked to structural characteristics (assets, liabilities). An increasing credit spread may be of worry because it could mean that the borrower wants money more urgently and urgently. While a widening spread indicates worsening economic conditions, a falling spread indicates increasing creditworthiness. [11].

The credit spread is also an options strategy, which entails the purchase of one option and the sale of another option with the same class (puts and calls), expiration, and striking price. It is intended to generate income when the spread between the two options gets smaller [2]. Bond credit spreads are a reliable indicator of the state of the economy, when they broaden (bad) or narrow (good). A decreasing bond credit spread may indicate better economic circumstances and reduced risk in general. A growing bond credit spread often portends deteriorating economic conditions and more risk in general. Variations in credit spreads are frequently brought on by changes in the economy (inflation), changes in liquidity, and demand for investments in specific markets [5]. The credit spread option involves selling or writing a high-premium option and simultaneously buying a lower premium option. When a position is opened, a premium is credited to the trader's or investor's account since the premium received from the writing option is higher than the premium paid for the long option [10]. The biggest profit that traders or investors can make while employing a credit spread strategy is the net premium, which is also the trader's maximum profit. The credit spread is a measure of how much riskier one financial product is than another (a compound of yield and spread) [6].

The credit spread options has two strategies for credit spread; the bull put spread, where the trader expects the underlying security to go up, and the bear call spread, where the trader expects the underlying security to go down. When the spreads decrease and make it possible to compare a corporate bond with an interest rate at which there is no risk, the credit spread yields a profit [4]. The Merton and Moody's KMV (Kealhofer, McQuown and Vasicek) (MKMV) methodologies are used in this study to examine the impact of expected return, volatility, and debt maturity time on credit spreads.

II. THE MERTON MODEL

Merton (1974) models a company's capital structure (equity and debt), linking it to credit risk on the presumption that default will be triggered if the value of the assets fall below the value of its liabilities at maturity [12]. This is based on Black and Scholes' (1973) option pricing theory [1].

Assume that the value of the business's assets A_t at time t is made up of equity E_t and debt D_t in the form of zero-coupon bonds with face values K that mature at time $T > t$. The following equation determines the capital structure [7]:

$$A_t = E_t + D_t \qquad (1)$$

By choosing a debt maturity T, any debt is translated into a zero-coupon bond. when $A_T > K$, the company's debt holders are fully repaid the amount $A_T - K$ and shareholder equity is still worth something. On the other hand, if $A_T < K$ the company collapses because of its debt. In this case, debt holders would have first priority over shareholders for the remaining asset, leaving shareholders with nothing. The equity value at a given point in time T can be stated as follows:

$$E_T = \max\left(A_T - K, 0\right) \qquad (2)$$

This is the payoff of a European call option that was written on an underlying asset A and had a maturity T and strike price of K. The movements of the asset price A are described by the following geometric Brownian motion (GBM) process with risk-neutral dynamics [14].

$$dA_t = rA_t dt + \sigma_A A_t dW \qquad (3)$$

where W is a standard Brownian motion, r is the risk-free interest rate and σ_A is the asset's return volatility in the risk-neutral measure. Applying the Black-Scholes assumptions, we get the following equations for the European call and put options:

$$E = CALL = AN\left(d_1\right) - Ke^{-rT} N\left(d_2\right) \qquad (4)$$

for the call option value, and

$$E = PUT = Ke^{-rT} N\left(-d_2\right) - AN\left(-d_1\right) \qquad (5)$$

for the put option.

where $N(.)$ is the standard normal cumulative distribution probability function,

$$d_1 = \frac{\ln\left(\frac{A}{K}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A \sqrt{T}}, \qquad (6)$$

$$d_2 = \frac{\ln\left(\frac{A}{K}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A \sqrt{T}} = d_1 - \sigma_A \sqrt{T} \qquad (7)$$

The debt's value is established by $A-E$. Under risk-neutral circumstances, the probability that the business may fail on its debt is given by $N(-d_2)$. In this case, a credit default is caused by the shareholders' call option maturing out-of-money at T, with the risk-neutral probability given by:

$$P(A_T < K) = N(-d_2), \tag{8}$$

2.1 Asset price A and asset volatility σ_A

Asset price A and asset volatility σ_A are un-observable in the equity market. Only equity value E and equity volatility σ_E are observed in the equity market. Since the equity is an option on firm value is also a function of asset price A and asset volatility σ_A. To obtain the asset price and asset volatility, we use another geometric Brownian motion for equity E and use Ito's Lemma to demonstrate that instantaneous volatilities satisfy the following equation [9]:

$$\sigma_E = \frac{A\sigma_A}{E}\frac{\partial E}{\partial A} \tag{9}$$

using Black-Scholes equation, it can be shown that $\frac{\partial E}{\partial A} = N(d_1)$, then (9) we becomes:

$$\sigma_E = \frac{A\sigma_A}{E}N(d_1) \tag{10}$$

$$E\sigma_E = A\sigma_A N(d_1) \tag{11}$$

where $N(d_1)$ is essentially the delta of equity with respect to firm value. The price of an equity E and the volatility σ_E of its return are observed in the equity market. Finally, (4) and (11), can be solved simultaneously for A and σ_A.

2.2 Distance to Default (DD) by Merton Model

The foundation for assessing credit risk is distance to distance (DD). The DD is the number of standard deviations between the expected asset value at maturity T and the debt threshold K. It reflects how far a firm's asset value is from the value of obligations that would trigger a default and it is a standard index that evaluates a company's creditworthiness and enables comparisons between different companies and over time. There are fewer likelihood of defaults when values of DD are high because the corporation is more likely to repay debts on schedule. . After determining the values of asset price A and asset volatility σ_A, we can calculate the distance to default (DD), so that we can be able to calculate the probability of default (PD). The DD is given by [3]:

$$DD = \frac{\log A + \left(r - \sigma_A^2/2\right)T - \log(K)}{\sigma_A\sqrt{T}} \tag{12}$$

2.3 Probability of Default (PD) by Merton Model

The likelihood of default (PD) is the chance that the asset value will fall below the debt threshold at the end of the time horizon T and is given by:

$$PD = 1 - N(DD) = N(-DD) \qquad (13)$$

2.4 Determination of Credit spread by the Merton model

Creditors are frequently at risk of default, but they can entirely protect themselves by buying a European put option with the identical underlying asset A_t and strike price K. Such a put option will profit $K - A_T$ if $A_T < K$ and will be worthless if $A_T > K$. Creditors will be assured a payoff of K at time T if these two positions (debt and put option) are combined, creating a risk-free position [17]:

$$D_t + P_t = Ke^{-rT} \qquad (14)$$

where P_t represents the put option price at time t, which is calculated using a European put option and the Black-Scholes formula. The price difference between the debt's current hazardous (risky) and riskless values is determined by the value of the put option, hence the market value of the debt D may be calculated using the equation:

$$D = Ke^{-rT} - PUT \qquad (15)$$

The PUT determines the price difference between risky and riskless bonds, and a higher value of the PUT causes a wider interest rate spread. Given that the firm's value is more erratic (volatile), the spread on hazardous debt must increase along with the value of the put option. As the interest rate on risk-free debt rises, the spread on risky debt decreases.

The credit spread is the discrepancy (difference) between the yield on the hazardous loan and the risk-free rate. As our definition, let's take the loan's market price at time zero D. Assets have a value equal to the sum of their two sources of funding, equity and debt:

$$D = A - E \qquad (16)$$

where;

$$E = AN(d_1) - Ke^{-rT}N(d_2)$$

Substituting E to (10) we get:

$$\begin{aligned} D &= A - AN(d_1) + Ke^{-rT}N(d_2) \\ &= A\big(1 - N(d_1)\big) + Ke^{-rT}N(d_2) \\ &= AN(-d_1) + Ke^{-rT}N(d_2) \qquad (17) \end{aligned}$$

The yield to maturity for the debt can then be defined as:

$$D = Ke^{-yT} \qquad (18)$$

Comparing right sides of equations (11) and (16) we get:

$$Ke^{-yT} = AN(-d_1) + Ke^{-rT}N(d_2)$$

$$y = -\frac{1}{T}\ln\left(\frac{A}{K}N(-d_1) + e^{-rT}N(d_2)\right) \qquad (19)$$

The same result can be gained from the fundamental formula on rate of return with continuous compounding given by:

$$y = \frac{1}{T}\ln\left(\frac{K}{D}\right) = \frac{1}{T}\ln\left(\frac{K}{AN(-d_1) + Ke^{-rT}N(d_2)}\right) \qquad (20)$$

The credit spread by the Merton model can be finally obtained by reducing the yield rate with the risk – free rate:

$$s = y - r = \frac{1}{T}\ln\left(\frac{K}{AN(-d_1) + Ke^{-rT}N(d_2)}\right) - r \qquad (21)$$

III. THE MKMV MODEL

KMV Corporation was established in 1989 by Oldrich Vasicek, John McQuown, and Stephen Kealhofer [8], [13] and [15]. A significant supplier of tools for quantitative credit research to lenders, investors, and businesses, KMV was bought by Moody's Corporation on February 11, 2002, giving rise to the name MKMV. Based on the Merton Option Pricing Theory, the MKMV Company MKMV Company found that when the value of their assets meets the total book value of their debts, businesses often do not go into default. The asset value when a firm fails is frequently midway between the current or short-term liabilities (STL) and the total or long-term liabilities (LTL), known as the default point.

$$DPT = STL + k \times LTL, \quad 0 \le k \le 1, \qquad (22)$$

Following multiple default firm observations, MKMV Company learned that the most frequent default point is at $k = 0.5$ and that modifications to the default point affect the model's expected accuracy [16].

$$DPT = STL + 0.5 \times LTL \qquad (23)$$

3.1 Determination of Credit spread by the MKMV model

We obtain the equation for yield and credit spread implied by the MKMV model by substituting the default threshold *K* with the MKMV default point DPT from Merton's yield and credit spread equation (20):

$$y = \frac{1}{T}\ln\left(\frac{DPT}{AN(-d_1) + (DPT)e^{-rT}N(d_2)}\right) \qquad (24)$$

and

$$s = \frac{1}{T}\ln\left(\frac{DPT}{AN(-d_1) + (DPT)e^{-rT}N(d_2)}\right) - r \qquad (25)$$

where *r* is the risk free interest rate.

IV. DETERMINATION OF CREDIT SPREADS USING MERTON AND MKMV MODELS

The balance sheet for Apple Inc. (AAPL) for the assets, long-term debt, and short-term liabilities recorded from 2019 September 29 to 2022 September 29 is shown in Table 1. The default point values are calculated using (23). (23) and we use this data to calculate the credit spreads that the Merton and MKMV models generate.

Table 1. Current liabilities, long term liabilities, total asset values and default points

Time (T)	9/29/2019	9/29/2022	9/29/2021	9/29/2022
Total Asset (A)	338,516,000	323,888,000	351,002,000	352,755,000
Total debt (TD)	108,047,000	112,436,000	124,719,000	120,069,000
CL	105,718,000	105,392,000	125,481,000	153,982,000
LTL	142,310,000	153,157,000	162,431,000	148,101,000
DPT	176,873,000	181,970,500	206,696,500	228,032,500

Source (Apple Inc.(AAPL) , https://finance.yahoo.com/quote/AAPL/balance-sheet?p=AAPL).

4.1 Time (T) effects on Credit Spreads

The credit spreads for the Merton and MKMV techniques are shown in Table 2 using the formulas (21) and (25), respectively. The spreads grew from years 1 through 5, then started to decline from years 6 through 10. This is shown in the table. The table shows that, as compared to the Merton technique, the MKMV creates higher spreads (sDPT). The table generally displays the narrowing of spreads for longer maturity times, which denotes the development of profit.

Table 2. Shows the credit spreads for both Merton and MKMV against Time ($\sigma = 0.5$ and $r = 0.21$)

Time (*T*)	1	2	3	4	5	6	7	8	9	10
sTD	0.0015	0.0044	0.0058	0.0063	0.0064	0.0063	0.0060	0.0057	0.0054	0.0051
sSTL	0.0020	0.0053	0.0067	0.0071	0.0071	0.0068	0.0065	0.0062	0.0058	0.0055
sLTL	0.0061	0.0104	0.0112	0.0109	0.0103	0.0096	0.0089	0.0082	0.0076	0.0071
sDPT	0.0208	0.0223	0.0203	0.0180	0.0160	0.0143	0.0129	0.0116	0.0106	0.0097

The credit spread curves are hump-shaped, as seen in Figure 1. On the spread curves, there are peak points. Since the spreads are initially small, the corporation has sufficient assets to cover its liabilities. The spreads then expand swiftly (quickly) before gradually declining over longer maturities, suggesting fluctuations in asset value.

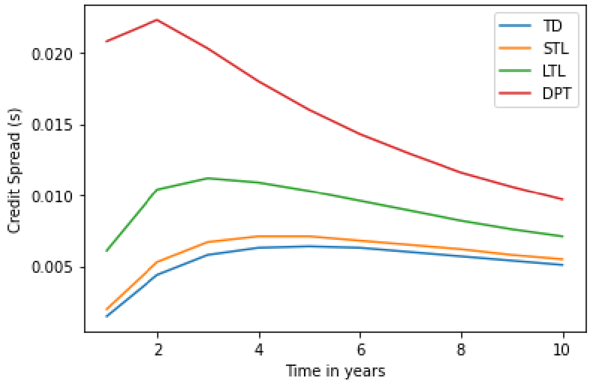

Figure 1. Credit Spreads from Debt Maturity

4.2 Volatility (σ) effects on Credit Spreads

The credit spreads determined by variable volatility are shown in Table 3. Comparing the Merton technique to the MKMV approach, bigger spreads (sDPT) are produced. The table indicates that spreads expand as volatilities rise. The possibility of quickly making more money rises along with volatility, which raises the danger of default.

Table 3. Shows the credit spreads for both Merton and MKMV against Volatility ($T = 1$ and $r = 0.21$)

Volatilty (σ)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
sTD	-5.5511e-17	3.4607e-12	1.0719e-06	0.0001	0.0015	0.0063	0.0166	0.0332	0.0562	0.0853
sSTL	1.3878e-16	1.9347e-11	2.3590e-06	0.0002	0.0020	0.0078	0.0196	0.0379	0.0626	0.0933
sLTL	-5.5511e-17	9.5514e-09	0.0011	1.1003e-03	0.0061	0.0176	0.0364	0.0620	0.0939	0.1310
sDPT	4.9960e-16	5.7028e-06	0.0008	0.0067	0.0208	0.0433	0.0731	0.1089	0.1497	0.1947

Figure 2 illustrates how credit spreads widen as volatility rises. As a result, there will be little liquidity during periods of high volatility because the disparity between the two prices is bigger. Increased default risk and worsening economic conditions are often indicated by widening bond credit spreads.

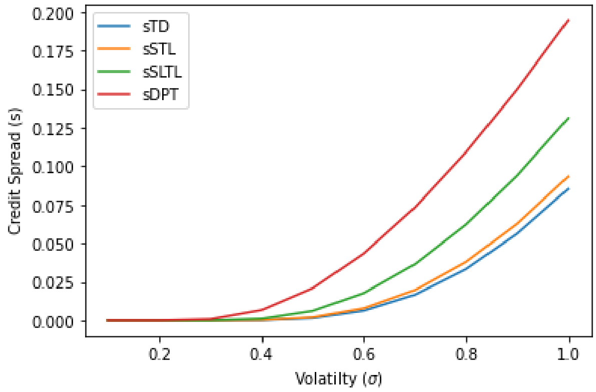

Figure 2. Credit Spreads from Volatilities

4.3 Interest Rates (r) effects on Credit Spreads

The credit spreads determined by various interest rates are shown in Table 4. As interest rates increase from the table, we observe the shrinking of credit spreads. This suggests that credit spreads will narrow in the short term as the risk-free rate rises while widening in the long term. Spreads that are getting narrower show that the company's economic situation is getting better.

Table 4. Shows the credit spreads for both Merton and MKMV against Interest Rates ($T = 1$ and $\sigma = 0.5$)

Rates (r)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
sTD	0.0027	0.0016	0.0009	0.0005	0.0002	0.0001	5.5645e-05	2.5669e-05	1.1418e-05	4.8965e-06
sSTL	0.0036	0.0021	0.0012	0.0006	0.0003	0.0002	8.2607e-05	3.8842e-05	1.7614e-05	7.7017e-06
sLTL	0.0104	0.0064	0.0039	0.0023	0.0013	0.0007	0.0004	0.0002	8.9759e-05	4.2377e-05
sDPT	0.0321	0.0217	0.0142	0.0090	0.0055	0.0033	0.0019	0.0011	0.0006	0.0003

Figure 3 illustrates how credit spreads have shrunk as interest rates have risen. This suggests that the company's financial situation is getting better as interest rates rise. Low interest rates, on the other hand, are beneficial to borrowers whereas high interest rates are solely advantageous to lenders. The required interest rate decreases with decreasing default risk, whereas increases in interest rates are necessary for increasing default risks. Higher interest income is the opportunity cost of assuming less default risk.

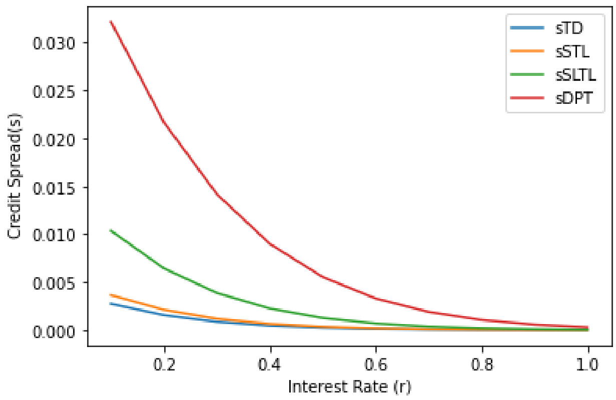

Figure 3. Credit Spreads from Interest Rates

V. CONCLUSION AND SUGGESTION FOR FUTURE RESEARCH

In this research, we have looked into how changes in interest rates, volatility, and debt maturity times affect credit spreads. The credit spreads produced by the Merton and MKMV techniques were compared. The outcomes in each case demonstrate that the spreads produced by the MKMV approach (sDPT) are substantial in comparison to those produced by the Merton approach (sTD, sSTL and sLTL). According to Figure 1, credit spreads appear to be contracting for longer maturities. This suggests that the company's financial situation improves over longer maturities. Figure 2 depicts the widening of credit spreads as volatility increases. This suggests that when volatility increases, corporate economic prospects deteriorate. Figure 3 illustrates how spreads have shrunk as interest rates have risen. This suggests that the company's economic situation improves when interest rates rise. Low interest rates, on the other hand, are beneficial to borrowers whereas high interest rates are solely advantageous to lenders. In general, the Merton approach and the MKMV approach compare favorably. We will look into how changes in interest rates, volatility, and debt maturity times affect a company's likelihood of defaulting on its debt in the future.

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