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Research Paper

Micro g^*-closed sets in Micro Topological Spaces

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ABSTRACT: *The main objective of this paper is to initiate a new class of sets called Micro g^*-closed sets in Micro topological spaces. Further we derive the interrelations between Micro g^*-closed sets with already existing Micro closed sets in Micro topological spaces. Also we derive its fundamental properties.*
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KEYWORDS: *Micro topological spaces, Micro closed set, Micro g-closed set, Micro g^*- closed set, Micro $T^*_{1/2}$- space.*

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I. INTRODUCTION

The concept of rough set theory was studied by Pawlak [8] and he introduced the notion of lower approximation, upper approximation and boundary region of a subset of the universe. Carmel Richard [4] introduced the concept of Nano topology. The Micro topology was introduced by Sakkraiveeranan Chandrasekar [9] and he also studied the concepts of Micro pre-open and Micro semi-open sets. Ibrahim [6,7] introduced Micro β-open sets and Micro g-closed sets in Micro topological spaces. Recently, Anandhi and Balamani [1,2] initiated the concept of Micro α-generalized closed sets in Micro topological spaces and also they have studied the properties of Micro separation axioms related to Micro α-generalized closed sets in Micro topological spaces. In this paper we have introduced a new class of Micro closed sets called Micro g^*-closed sets and studied its properties in Micro topological spaces. Further we have derived dependency and independency relations between Micro g^*- closed sets with already existing various Micro closed sets. Later we have defined and analysed Micro $T^*_{1/2}$- space.

II. PRELIMINARIES

Definition 2.1[8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibilty relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be in indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = U_{x\in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = U_{x\in U}\{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 2.3[9] Let $(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ and $\mu_R(X)$ satisfies the following axioms:

1. U and $\phi \in \mu_R(X)$
2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$
3. The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4[9] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then the Micro closure of a set A is denoted by $Mic\text{-}cl(A)$ and is defined as $Mic\text{-}cl(A) = \cap \{K: K$ is Micro closed in U and $A \subseteq K\}$. The Micro interior of a set A is denoted by $Mic\text{-}int(A)$ and is defined as $Mic\text{-}int(A) = \cup \{K: K$ is Micro open in U and $A \supseteq K\}$.

Definition 2.5 Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then A is said to be a

(i) Micro pre-open (briefly Mic-pre-open) set if $A \subseteq Mic\text{-}int(Mic\text{-}cl(A))$. [9]
(ii) Micro semi-open (briefly Mic-semi-open) set if $A \subseteq Mic\text{-}cl(Mic\text{-}int(A))$. [9]
(iii) Micro α-open (briefly Mic-α-open) set if $A \subseteq Mic\text{-}int(Mic\text{-}cl(Mic\text{-}int(A)))$. [5]
(iv) Micro β-open (briefly Mic-β-open) set if $A \subseteq Mic\text{-}cl(Mic\text{-}int(Mic\text{-}cl(A)))$. [6]

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.6[7] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized closed (briefly Mic-g-closed) if $Mic\text{-}cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Definition 2.7[1] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized α -closed (briefly Mic-$g\alpha$-closed) if Mic- $\alpha cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro α-open in U.

Definition 2.8[1] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro α-generalized closed (briefly Mic-αg-closed) if Mic-$\alpha cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Definition 2.9[3] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro semi-generalized closed (briefly Mic-sg-closed) if $Mic\text{-}scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro semi-open in U.

Definition 2.10[3] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized semi-closed (briefly Mic-gs-closed) if $Mic\text{-}scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Remark 2.11

(i) A is a Micro closed set if and only if $A = Mic\text{-}cl(A)$. [9]
(ii) Every Micro closed set is Micro α-closed. [5]
(iii) Every Micro closed (open) set is Micro g-closed (open). [7]

III. MICRO g^*-CLOSED SETS

In this section a new class of Micro generalized closed sets called Micro g^*-closed sets in Micro topological spaces is introduced and its interrelations with already existing Micro closed sets are obtained.

Definition 3.1 Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro g^*-closed if $Mic\text{-}cl(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro g-open in U.

Proposition 3.2 Every Micro closed set is Micro g^*-closed but not conversely.

Proof: Let A be a Micro closed set and $A \subseteq L$, where L is Micro g-open in U. By Remark 2.11 (i), $Mic\text{-}cl(A) = A$. Since $A \subseteq L \Rightarrow Mic\text{-}cl(A) \subseteq L$ where L is Micro g-open. Hence A is Micro g^*-closed.

Example 3.3 Let $U = \{1,2,3,4\}$, $U/R = \{\{1\},\{3\},\{2,4\}\}$. Let $X = \{2,4\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{2,4\}\}$. Let $\mu = \{1\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{1\}, \{2,4\}, \{1,2,4\}, U\}$. Micro closed sets are $\phi, \{3\}, \{1,3\}, \{2,3,4\}, U$. Micro g^*-closed sets are $\phi, \{3\}, \{1,3\}, \{2,3,\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}, U$. Here the subset $\{2,3\}$ is Micro g^*-closed, but not Micro closed.

Proposition 3.4 Every Micro g^*-closed set is Micro g -closed but not conversely.

Proof: Let A be a Micro g^*-closed set and L be any Micro open set containing A in U. Since every Micro open set is Micro g-open and A is Micro g^*-closed , $Mic\text{-}cl(A) \subseteq L$. Hence A is Micro g-closed.

Example 3.5 Let $U = \{a,b,c\}, U/R = \{\{c\},\{a,b\}\}$. Let $X = \{a,b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a,b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{c\}, \{a,b\}, U\}$. Micro g^*-closed sets are $\phi, \{c\}, \{a,b\}, U$. Micro g-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, U$. Here the subset $\{a\}$ is Micro g-closed but not Micro g^*-closed.

Proposition 3.6 Every Micro g^*-closed set is Micro αg-closed but not conversely.

Proof: Let A be a Micro g^*-closed set and L be any Micro open set containing A in U. Since every Micro open set is Micro g-open and A is Micro g^*-closed, Mic-$cl(A) \subseteq L$. For every subset A of U, Mic-$\alpha cl(A) \subseteq$ Mic-$cl(A)$ and so Mic-$\alpha cl(A) \subseteq L$. Hence A is Micro αg-closed.

Example 3.7 Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$. Micro g^*-closed sets are $\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, U$. Micro αg-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$. Here the subset $\{a, d\}$ is Micro αg-closed but not Micro g^*-closed.

Proposition 3.8 Every Micro g^*-closed set is Micro gs-closed but not conversely.

Proof: Let A be a Micro g^*-closed set and L be any Micro open set containing A in U. Since every Micro open set is Micro g-open and A is Micro g^*-closed, Mic-$cl(A) \subseteq L$. For every subset A of U, Mic-$scl(A) \subseteq$ Mic-$cl(A)$. Thus Mic-$scl(A) \subseteq L$. Hence A is Micro gs-closed.

Example 3.9 Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{b\}, \{a, b\}, U\}$. Micro g^*-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Micro gs-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, U$. Here the subset $\{a\}$ is Micro gs-closed but not Micro g^*-closed.

Remark 3.10 The following example shows that Micro g^*-closed set is independent from Micro pre-closed set.

Example 3.11 Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, U\}$. Micro pre-closed sets are $\phi, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, U$. Micro g^*-closed sets are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{c\}$ is Micro pre-closed but not Micro g^*-closed and the subset $\{a, b, d\}$ is Micro g^*-closed but not Micro pre-closed.

Remark 3.12 The following example shows that Micro g^*-closed set is independent from Micro semi-closed set.

Example 3.13 Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b, c\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b, c\}\}$. Let $\mu = \{a, b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a, b\}, \{a, b, c\}, U\}$. Micro semi-closed sets are $\phi, \{c, \}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, U.$ Micro g^*-closed sets are $\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U.$ Here the subset $\{c\}$ is Micro semi-closed but not Micro g^*-closed and the subset $\{a, d\}$ is Micro g^*-closed but not Micro semi-closed.

Remark 3.14 The following example shows that Micro g^*-closed set is independent from Micro α-closed set.

Example 3.15 Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Let $X = \{b, d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{b\}, \{b, d\}, U\}$. Micro α-closed sets are $\phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{a, c, d\}, U$. Micro g^*-closed sets are $\phi, \{a, c\}, \{a, b, c\}, \{a, c, d\}, U$. Here the subset $\{a\}$ is Micro α-closed but not Micro g^*-closed and the subset $\{a, b, c\}$ is Micro g^*-closed but not Micro α-closed.

Remark 3.16 The following example shows that Micro g^*-closed set is independent from Micro β-closed set.

Example 3.17 Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Micro β-closed sets are $\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, U$. Micro g^*-closed sets are $\phi, \{a, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{a, b\}$ is Micro β-closed but not Micro g^*-closed and the subset $\{b, c, d\}$ is Micro g^*-closed but not Micro β-closed.

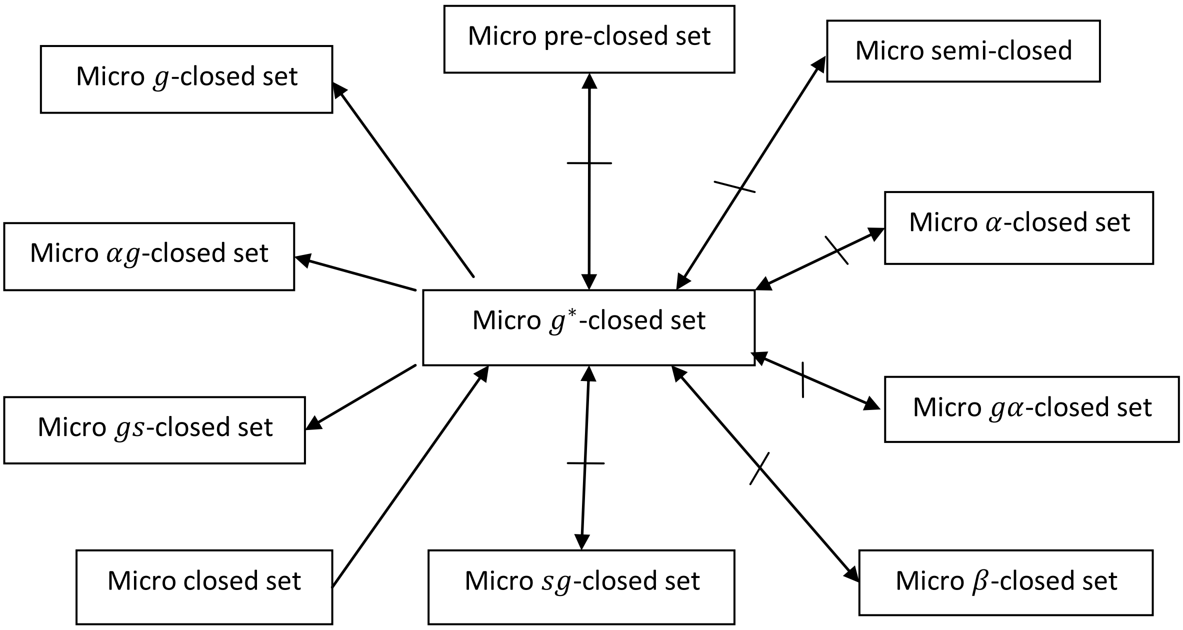
Remark 3.18 The following example shows that Micro g^*-closed set is independent from Micro $g\alpha$-closed set.

Example 3.19 Let $U = \{a, b, c\}$, $U/R = \{\{c\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$. Micro $g\alpha$-closed sets are $\phi, \{b\}, \{c\}, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Here the subset $\{b\}$ is Micro $g\alpha$-closed but not Micro g^*-closed and the subset $\{a, c\}$ is Micro g^*-closed but not Micro $g\alpha$-closed.

Remark 3.20 The following example shows that Micro g^*-closed set is independent from Micro sg-closed set.

Example 3.21 Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, U\}$. Micro g^*-closed sets are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Micro sg-closed sets are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subset $\{b, d\}$ is Micro g^*-closed but not Micro sg-closed and the subset $\{a, c, d\}$ is Micro sg-closed but not Micro g^*-closed.

Remark 3.22 The following diagram shows the dependency and independency relations of Micro g^*-closed sets with already existing Micro closed sets in Micro topological spaces.



IV. PROPERTIES OF MICRO g^*-CLOSED SETS

In this section we analyze some of the fundamental properties of Micro g^*-closed sets.

Theorem 4.1 If A and B be are Micro g^*-closed subsets of $(U, \tau_R(X), \mu_R(X))$ then $A \cup B$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A and B be any two Micro g^*-closed sets in $(U, \tau_R(X), \mu_R(X))$ and L be any Micro g open set containing $A \cup B$. Then $A \subseteq L$ and $B \subseteq L$. Since A and B are Micro g^*-closed sets, $Mic\text{-}cl(A) \subseteq L$ and $Mic\text{-}cl(B) \subseteq L$. Always $Mic\text{-}cl(A \cup B) \subseteq Mic\text{-}cl(A) \cup Mic\text{-}cl(B) \subseteq L$. Hence $A \cup B$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Remark 4.2 The intersection of any two Micro g^*-closed sets in $(U, \tau_R(X), \mu_R(X))$ need not be a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$ as seen from the following example.

Example 4.3 Let $U = \{a, b, c, d\}$, $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Micro g^*-closed sets are $\phi, \{a, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, U$. Here the subsets $\{a, b, d\}$ and $\{b, c, d\}$ are Micro g^*-closed sets but their intersection $\{a, b, d\} \cap \{b, c, d\} = \{b, d\}$ is not Micro g^*-closed.

Theorem 4.4 Let A and B be subsets of $(U, \tau_R(X), \mu_R(X))$ such that $A \subseteq B \subseteq Mic\text{-}cl(A)$. If A is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$, then B is also a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A and B be subsets of $(U, \tau_R(X), \mu_R(X))$ such that $A \subseteq B \subseteq Mic\text{-}cl(A)$. Suppose that A is a Micro g^*-closed set. Let L be a Micro g-open set of $(U, \tau_R(X), \mu_R(X))$ such that $B \subseteq L$. Then $A \subseteq L$ and since A is Micro g^*-closed, $Mic\text{-}cl(A) \subseteq L$. Also since $B \subseteq Mic\text{-}cl(A)$, $Mic\text{-}cl(B) \subseteq Mic\text{-}cl(Mic\text{-}cl(A)) = Mic\text{-}cl(A) \subseteq L$. Hence $Mic\text{-}cl(B) \subseteq L$. Therefore B is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$.

Theorem 4.5 If A is both Micro g-open and Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$, then A is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A be Micro g-open and Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Then by the definition of Micro g^*-closed set, $Mic\text{-}cl(A) \subseteq A$. Always $A \subseteq Mic\text{-}cl(A)$. Therefore $Mic\text{-}cl(A) = A$. Hence A is Micro closed.

Theorem 4.6 Let A be a Micro g^*-closed set of $(U, \tau_R(X), \mu_R(X))$ if and only if $Mic\text{-}cl(A) - A$ does not contain any non empty Micro g-closed set.

Proof: (Necessity) Let F be a Micro g-closed set of $(U, \tau_R(X), \mu_R(X))$ such that $F \subseteq Mic\text{-}cl(A) - A$. Then $A \subseteq U - F$. Since A is Micro g^*-closed and $U - F$ is Micro g-open, $Mic\text{-}cl(A) \subseteq U - F$. This implies $F \subseteq U - Mic\text{-}cl(A)$. So $F \subseteq (U - Mic\text{-}cl(A) \cap (Mic\text{-}cl(A) - A) \subseteq (U - Mic\text{-}cl(A)) \cap Mic\text{-}cl(A) = \phi$. Therefore, $F = \phi$.

Sufficiency: Suppose that $Mic\text{-}cl(A) - A$ contains no non empty Micro g-closed set. Let $A \subseteq L$ and L be Micro g-open. If Mic- $cl(A)$ is not a subset of L then $Mic\text{-}cl(A) \cap L^c$ is a non empty Micro g-closed subset of $Mic\text{-}cl(A) - A$ which is a contradiction. Therefore, $Mic\text{-}cl(A) \subseteq L$ and hence A is Micro g^*-closed.

Theorem 4.7 Let A be a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Then $Mic\text{-}cl(A) - A$ contains no non empty Micro closed set.

Proof: Suppose that A is a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Let F be a Micro closed set contained in $Mic\text{-}cl(A) - A$ i.e., $F \subseteq Mic\text{-}cl(A) - A$. Now F^c is Micro open and hence Micro g-open in U such that $A \subseteq F^c$. Since A is Micro g^*-closed, $Mic\text{-}cl(A) \subseteq F^c$. Thus $F \subseteq [Mic\text{-}cl(A)]^c$. Also $F \subseteq Mic\text{-}cl(A) - A \Rightarrow F \subseteq Mic\text{-}cl(A)$. Therefore $F \subseteq [Mic\text{-}cl(A)]^c \cap [Mic\text{-}cl(A)] = \phi$. Hence $F = \phi$.

Theorem 4.8 If $Mic\text{-}cl(\{x\}) \cap A \neq \phi$ holds for every $x \in Mic\text{-}cl(A)$, then $Mic\text{-}cl(A) - A$ does not contain a non empty Micro closed set.

Proof: Suppose there exists a non empty Micro closed set F such that $F \subseteq Mic\text{-}cl(A) - A$. Let $x \in F$. Then $x \in Mic\text{-}cl(A)$. It follows that $F \cap A = Mic\text{-}cl(A) - A \cap A \supseteq Mic - cl\ (\{x\}) \cap A \neq \phi$. Hence $F \cap A \neq \phi$, which is a contradiction. Thus $F = \phi$.

Theorem 4.9 Let A be a Micro g^*-closed set in $(U, \tau_R(X), \mu_R(X))$. Then A is a Micro closed set if and only if $Mic\text{-}cl(A) - A$ is Micro g-closed.

Proof: (Necessity) Suppose that A is a Micro g^*-closed set. Let A be a Micro closed subset of $(U, \tau_R(X), \mu_R(X))$. Then $Mic\text{-}cl(A) = A$. Therefore $Mic\text{-}cl(A) - A = \phi$ is Micro g-closed.

Sufficiency: Let $Mic\text{-}cl(A) - A$ be a Micro g-closed set. Since A is Micro g^*-closed, by Theorem 4.7 $Mic\text{-}cl(A) - A$ contains no non empty Micro closed set which implies $Mic\text{-}cl(A) - A = \phi$. That is $Mic\text{-}cl(A) = A$. Hence A is Micro closed.

Theorem 4.10 Let $(U, \tau_R(X), \mu_R))$ be a Micro topological space. Then for each $x \in U$, either $\{x\}$ is Micro g-open or $U - \{x\}$ is Micro g^*-closed.

Proof: Let $x \in U$ and suppose that $\{x\}$ is not Micro g-closed in $(U, \tau_R(X), \mu_R))$. Then $U - \{x\}$ is not Micro g-open in U. Hence U is the only Micro g-open set containing $U - \{x\}$. That is $U - \{x\} \subseteq U$. Therefore $Mic\text{-}cl$ $(U - \{x\}) \subseteq U$ which implies that $U - \{x\}$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 4.11 Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. Then U is said to be Micro $T^*_{1/2}$-space if every Micro g^*-closed set in U is Micro closed in U.

Example 4.12 Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, U\}$. Micro closed sets are $\phi, \{b, c\}, U$. Micro g^*-closed sets are $\phi, \{b, c\}, U$. Hence U is a Micro $T^*_{1/2}$-space.

Theorem 4.13 For a space $(U, \tau_R(X),\ \mu_R(X))$ the following conditions are equivalent

(i) $(U, \tau_R(X),\ \mu_R(X))$ is a Micro $T^*_{1/2}$-space

(ii) For each $x \in U$, $\{x\}$ is either Micro g-closed or Micro-open.

Proof: (i) $\Rightarrow$ (ii) Let $x \in U$ and suppose $\{x\}$ is not a Micro g-closed set of $(U, \tau_R(X),\ \mu_R(X))$. Then $U - \{x\}$ is not Micro g- open. Hence U is the only Micro g-open set containing $U - \{x\}$. So $U - \{x\}$ is a Micro g^*-closed set in $(U, \tau_R(X),\ \mu_R(X))$. Since $(U, \tau_R(X),\ \mu_R(X))$ is a Micro $T^*_{1/2}$-space, $U - \{x\}$ is Micro closed or equivalently $\{x\}$ is Micro open in $(U, \tau_R(X),\ \mu_R(X))$.

(ii) $\Rightarrow$ (i) Let A be a Micro g^*-closed set in $(U, \tau_R(X),\ \mu_R(X))$ and $x \in Mic\text{-}cl(A)$. We show that $x \in A$ for the following two cases.

Case 1: Assume that $\{x\}$ is Micro open. Then $U - \{x\}$ is Micro closed. If $x \notin A$, then $A \subseteq U - \{x\}$. Since $x \in Mic\text{-}\ cl(A)$, we have $x \in U - \{x\}$, which is a contradiction. Hence $x \in A$.

Case 2: Assume that $\{x\}$ is Micro g-closed and $x \notin A$. Then $Mic\text{-}\ cl(A) - A$ contains a non-empty Micro g-closed set $\{x\}$. This contradicts Theorem 4.6 as A is a Micro g^*-closed set. Therefore $x \in A$.

V. CONCLUSION

In this article, we have introduced Micro g^*-closed sets in Micro topological spaces. Further the fundamental properties of the defined sets are examined. This research can be extended for Micro g^*-continuous maps and Micro g^*-irresolute maps in the near future.

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