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Research Paper

Logistic Model and NewForecast of China's Population

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Abstract:*The Logistic model is one of the classic models for studying population ecology, and it is an improvement of the Malthus model. It has a strong application background and important practical value in the fields of natural science and social science. This paper uses Logistic model to study some problems of China's population. Firstly, we fit trends in the birth rate r and the death population H. Secondly, we study the relationship between r and H. Finally, we predict the size of China's population in the future.According to the fitting of the obtained parameters r and H, we obtain that the estimated error is smaller than the previous one, which can better reflect the change law of China's population and the trend of population change in the future.*
Key words: *Logistic model; population forecast; parameter estimation; least squares principle*

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I. INTRODUCTION

The Logistic model is another classic model of population ecology after the Malthus classic model. British statistician T.R. Malthus[1] proposed the famous Malthus model in his book "The Principle of Population",

$$\left.\begin{array}{l}\frac{dN}{dt}=rN(t)\\ N(t)|_{t=t_0}=N_0\end{array}\right\}\rightarrow(1)$$

In the model, r is a constant representing the birth rate and $N(t)$ is the population size. The Malthus model assumes that population growth is infinite. But in reality, the population will not grow indefinitely due to the influence of resources and the environment.

Considering the competition of populations, the competition term "$-KN$^2" is added to the Malthus model. The new model is

$$\left.\begin{array}{c}\frac{dN}{dt}=rN-KN^2(or=rN\left(1-\frac{N}{C}\right))\\ N|_{t=t_0}=N_0\end{array}\right\}\rightarrow(2)$$

where r and K $(C=r/K)$ are called life coefficients, and this is the Logistic growth retardation model proposed by the Belgian mathematician P. F. Verhulst[2].

Logistic model occupies a very important position in the study of ecological population.In order to make the model have a wider range of applications and be more in line with the actual situation, many scholars have proposed a variety of improvement methods for the model.

Hutchinson[3] proposed a logistic model with time delay. Smith[4] improved the model by linking the population model to the use of food. Hallam and Clark[5] differentiated the equilibrium state and environmental capacity of population development, and improved the model. Cui and G.J. Lawson[6] introduced the parameters representing the utilization efficiency of nutrients by the population based on the relationship between nutrition and growth, and proposed the Cui-Lawson model. Li and Zhao [7]proposed an adaptive logistic model with nonlinear density constraints.

In 2012, M.F. Laham[8] et al. studied which catching strategy for fish can obtain the maximum profit in the long-term, and introduced the harvesting function into the Logistic growth model to obtain an improved model,

$$\left.\begin{aligned}\frac{dN(t)}{dt}=rN(t)\left(1-\frac{N(t)}{N_m}\right)-H(t),\\ N(t_0)=t_0,\end{aligned}\right\}\rightarrow(3)$$

In 2016, Alfred[9] studied the capture strategy of wrasse farming and proposed three different forms of the harvest function in the Logistic model with the harvest function:

A) Constant harvest growth model

$$dx(t)/dt=rx(t)(1-x(t)/C)-h\ \rightarrow(4)$$

B) Proportional Harvest Growth Model

$$dx(t)/dt=rx(t)(1-x(t)/C)-hx(t)\rightarrow(5)$$

C) Cycle Harvest Growth Model

$$dx(t)/dt=rx(t)(1-x(t)/C)-h(1+\sin 2\pi t\ )\rightarrow(6)$$

Optimal catches to protect wrasse from extinction were estimated for each model, providing a basis for developing fishing strategies.

In 2020, Chen and Xiao[10] improved the model (3) again and established the models

$$\frac{\mathrm{dx(t)}}{\mathrm{dt}}=r(t)x(t)\left(1-\frac{x(t)}{C}\right)-H\big(t,x(t)\big)\rightarrow(7)$$

On the premise that both $r(t)$ and $H(t,w(t))$ are non-zero constant functions, and $h=kr$, they studied the future population changes in China and obtained good prediction results.

This paper studies the prediction of China's population when r and H in Model (4) are non-valued functions. In population forecasting, r is the birth rate and H is the number of deaths. Assuming that r is a constant value function in a certain period of time, the overall r is a function of time.

II. MODEL SOLUTION

Solving the constant harvest growth model,

$$\left.\begin{aligned}\frac{dx(t)}{dt}=rx(t)\left(1-\frac{x(t)}{C}\right)-h\\ x(0)=x_0\end{aligned}\right\}\rightarrow(8)$$

r and h are constants in this model, representing $r(t)$ and $H(t,x(t))$, respectively.

In order to solve the differential equation in a way of separating variables, let $x(t)=y(t)+\delta$, the model is transformed into

$$\left.\begin{aligned}\frac{dy(t)}{dt}=r(y(t)+\delta)\left(1-\frac{y(t)+\delta}{C}\right)-h\\ y_0=x_0-\delta\end{aligned}\right\}\rightarrow(9)$$

That is

$$\left.\begin{aligned}\frac{dy(t)}{dt}=r(y(t)+\delta)\left(1-\frac{y(t)+2\delta}{C}\right)-(\frac{r\delta^2}{C}-r\delta+h)\\ y_0=x_0-\delta\end{aligned}\right\}\rightarrow(10)$$

let $\frac{r\delta^2}{C}-r\delta+h=0$, get

$$\mathrm{y(t)}=\frac{y_0(C-2\delta)}{(C-2\delta-y_0)e^{-\frac{C-2\delta}{C}\int_0^t rdt}+y_0}\rightarrow(11)$$

Substitute $x=y+\delta$ to get the solution

$$\mathrm{x(t)}=\frac{(\mathrm{x}_0-\delta)(C-2\delta)}{(C-\delta-x_0)e^{-\frac{C-2\delta}{C}\int_0^t rdt}+(x_0-\delta)}+\delta\rightarrow(12)$$

where $\delta=\frac{C}{2}\pm\sqrt{\frac{C^2}{4}-\frac{hC}{r}}$.

To minimize residuals:

$$\mathrm{Q}=\min\sum_{t=1}^{n}\big(x_t-x(t)\big)^2\ \rightarrow(13)$$

Since there are C, h, r in the residual, the above formula is converted into

$$\left.\begin{array}{l}\frac{\partial Q}{\partial r}=0\\ \frac{\partial Q}{\partial C}=0\\ \frac{\partial Q}{\partial h}=0\end{array}\right\} \rightarrow (14)$$

Using the principle of least squares, solve the parameters C, h, r.

III. PARAMETER ESTIMATION

The parameters C, h, r are estimated using the principle of least squares, and the initial values of the parameters need to be given to make the parameter results converge faster. In this paper, the differential term is converted into a difference scheme to solve and determine the initial value.

According to Taylor's formula

$$u(t+h)=u(t)+u^{'}(t)h+\frac{1}{2!}u^{''}(t)h^2+\cdots+\frac{1}{n!}u^{(n)}(t)h^n+o(h^{n+1}) \rightarrow (15)$$

when the step size h tends to 0, there is $u(t+h)=u(t)+u'(t)h$. Consider here the expression when the model step size $h=1$:

$$\frac{dx}{dt}=rx(t)\left(1-\frac{x(t)}{C}\right)-h=x(t+1)-x(t) \rightarrow (16)$$

it is necessary to establish three equations to solve the parameters in Equation 16. In order to make the parameter results more holistic and representative, select t=1, 35, 71. According to the China's population data in Table 1.

TABLE 1 China's population data[11]

time t	year	Population/billion	time t	year	Population/billion	time t	year	Population/billion
t=0	1950	5.5196	t=24	1974	9.0859	t=48	1998	12.4761
t=1	1951	5.63	t=25	1975	9.242	t=49	1999	12.5786
t=2	1952	5.7482	t=26	1976	9.3717	t=50	2000	12.6743
t=3	1953	5.8796	t=27	1977	9.4974	t=51	2001	12.7627
t=4	1954	6.0266	t=28	1978	9.6259	t=52	2002	12.8453
t=5	1955	6.1465	t=29	1979	9.7542	t=53	2003	12.9227
t=6	1956	6.2828	t=30	1980	9.8705	t=54	2004	12.9988
t=7	1957	6.4653	t=31	1981	10.0072	t=55	2005	13.0756
t=8	1958	6.5994	t=32	1982	10.1654	t=56	2006	13.1448
t=9	1959	6.7207	t=33	1983	10.3008	t=57	2007	13.2129
t=10	1960	6.6207	t=34	1984	10.4357	t=58	2008	13.2802
t=11	1961	6.5859	t=35	1985	10.5851	t=59	2009	13.345
t=12	1962	6.7296	t=36	1986	10.7507	t=60	2010	13.4091
t=13	1963	6.9172	t=37	1987	10.93	t=61	2011	13.4916
t=14	1964	7.0499	t=38	1988	11.1026	t=62	2012	13.5922
t=15	1965	7.2538	t=39	1989	11.2704	t=63	2013	13.6726
t=16	1966	7.4542	t=40	1990	11.4333	t=64	2014	13.7646
t=17	1967	7.6368	t=41	1991	11.5823	t=65	2015	13.8326
t=18	1968	7.8534	t=42	1992	11.7171	t=66	2016	13.9232
t=19	1969	8.0671	t=43	1993	11.8517	t=67	2017	14.0011

t=20	1970	8.2992	t=44	1994	11.985	t=68	2018	14.0541
t=21	1971	8.5229	t=45	1995	12.1121	t=69	2019	14.1008
t=22	1972	8.7177	t=46	1996	12.2389	t=70	2020	14.1212
t=23	1973	8.9211	t=47	1997	12.3626	t=71	2021	14.126

The initial values of parameters C=17.402, h=0.2492, r=0.0954 were obtained by running the program.

Appendixa

```
data=[0  5.5196;15.63;2        5.7482;3        5.8796;4        6.0266;5  6.1465;66.2828;76.4653;8
         6.5994;96.7207;10     6.6207;11       6.5859;12       6.7296;13       6.9172;14
         7.0499;15    7.2538;16    7.4542;17    7.6368;18    7.8534;19    8.0671;20
         8.2992;21    8.5229;22    8.7177;23    8.9211;24    9.0859;25    9.242;26
         9.3717;27    9.4974;28    9.6259;29    9.7542;30    9.8705;31    10.0072;32
         10.1654;33   10.3008;34   10.4357;35   10.5851;36   10.7507;37   10.93;38
         11.1026;39   11.2704;40   11.4333;41   11.5823;42   11.7171;43   11.8517;44
         11.985;45    12.1121;46   12.2389;47   12.3626;48   12.4761;49   12.5786;50
         12.6743;51   12.7627;52   12.8453;53   12.9227;54   12.9988;55   13.0756;56
         13.1448;57   13.2129;58   13.2802;59   13.345;60    13.4091;61   13.4916;62
         13.5922;63   13.6726;64   13.7646;65   13.8326;66   13.9232;67   14.0011;68
         14.0541;69   14.1008;70   14.1212;71   14.126];
syms r C H
t=[1,35,71];
for i=1:3
   x1=data(t(i),2);
   x2=data(t(i)+1,2);
   eq(i)=r*x1*(1-x1/C)-H-(x2-x1);
end
[C,H,r]=solve([eq(1)==0,eq(2)==0,eq(3)==0])
C=double(C)
H=double(H)
r=double(r)
```

Using the block diagram designed by Simulink

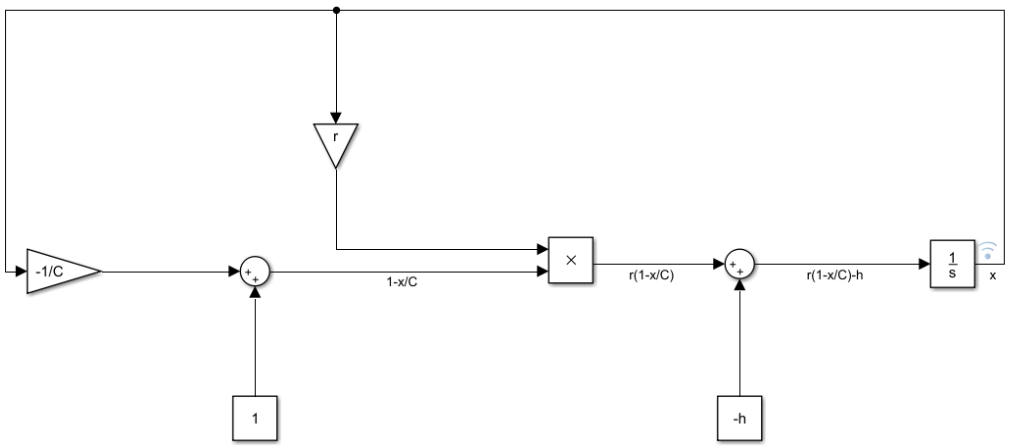

Diagram 1

the parameter estimation results are obtained. The initial state $x(0)$ takes the population of 5.5196 in 1950, and more accurate parameter results are obtained by fitting, $C = 18.24719, h = 0.25569, r = 0.092625$.

This paper mainly studies the parameters r and h, and the environmental capacity C is kept constant. In order to obtain more data and predict the parameters more accurately, this paper selects every 5 years as a fitting time period. And from the previous assumptions, in each time period, the parameters h and r are constant values.

After fitting the population data of each time period separately, the fitting effect is shown in Figure 1. The residual error is 0.078278 small and the error is within the acceptable range.

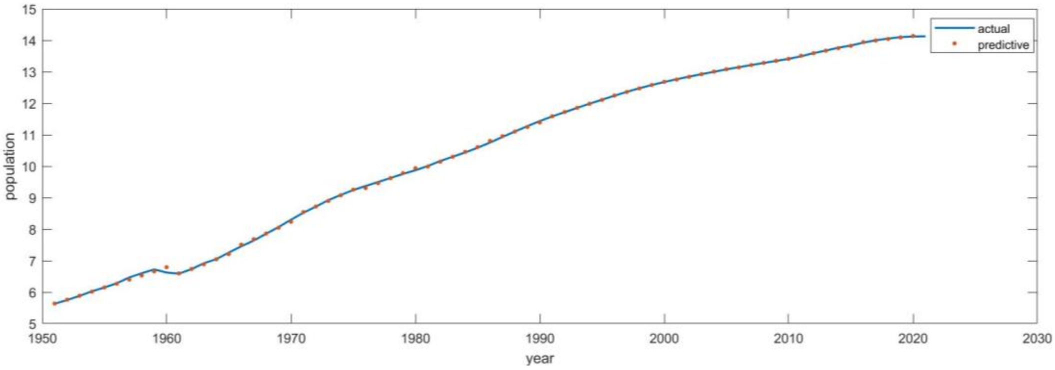

Figure 1 Fitting effect

The specific parameter results are shown in table 2. In the table, T is the time period label, each time period is 5 groups of adjacent data, and $x(0)$ is uniformly fixed at 5.5196.

TABLE 2 Parameter estimation results

period of time T	r	H
1	0.099736	0.269417257
2	0.031238	1.11E-14
3	0.208204	0.738830114
4	0.167303	0.56708065
5	0.111368	0.328947353
6	0.054247	0.088677692
7	0.077656	0.193057367
8	0.097543	0.277889467
9	0.089628	0.242100537
10	0.097201	0.274243659
11	0.117737	0.362202392
12	0.109987	0.330227314
13	0.065796	0.146589716
14	0.0887	0.237763126

IV. PARAMETRIC ANALYSIS AND PREDICTION

In order to observe the parameter changes more intuitively, draw relevant images, and as shown in Figure 2

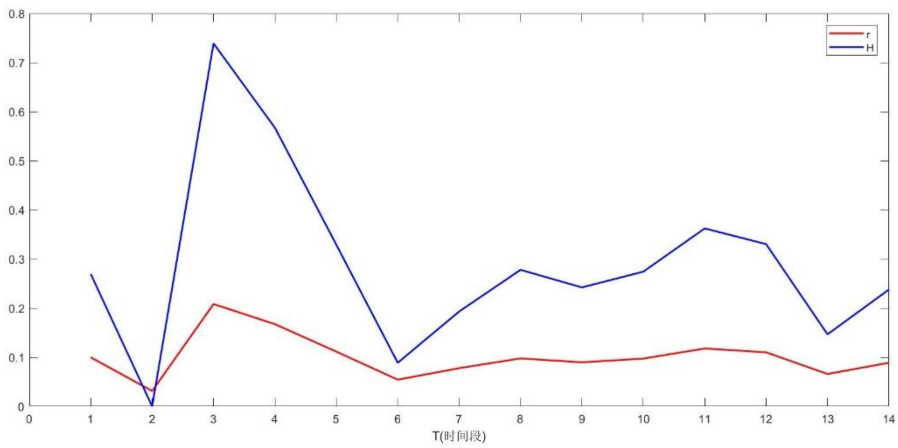

Figure 2 r, h trend

A preliminary observation of Figure 2 shows that the data r value fluctuates around a constant, which coincides with the assumption of previous scholars that r is a constant. Both r and H show the same trend of increase and decrease at the turning point of data change, and it is speculated that there may be a linear relationship.

Taking r as the abscissa and h as the ordinate to draw a scatter plot, there is an obvious linear relationship. The $H(r)$ function is established by linear fitting, and the result is shown in Figure 3.

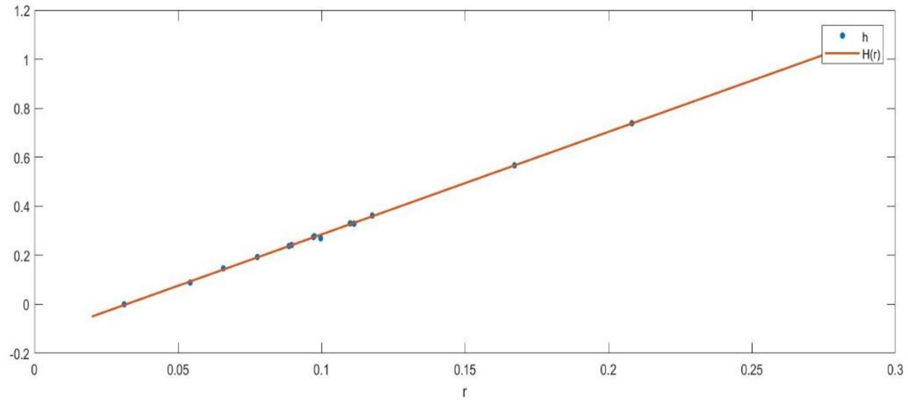

Figure 3 r, H linear fit

The function represented by the solid red line in the figure is

$$H = 4.185 * r - 0.1336 \rightarrow (17)$$

The linear relationship shown by r and h is consistent with the assumption that H is proportional to r ($H = kr$) when calculating the population in the literature[10]. To a certain extent, it proves the accuracy of the parameter piecewise fitting results in this paper.

For the changing trend of r, its clear functional form cannot be obtained temporarily. The forecast function of the Excel table will be used to forecast r in the short term in the future. For the first prediction, the data source uses the existing 14 data of the parameter r, and each subsequent prediction adds the previous prediction result on the basis of the previous prediction data source. h is calculated by the function (17). The prediction results are shown in table 3.

TABLE 3r, H trend forecast values

period of time T	15	16	17	18	19	20
r	0.07276	0.06629	0.07163	0.06865	0.07383	0.06468
H	0.17091	0.14383	0.16617	0.15371	0.17537	0.13707

The variation trend of r is shown in Figure 4. And the red line in the figure is the future trend of parameter r, which is a reasonable development in the future.

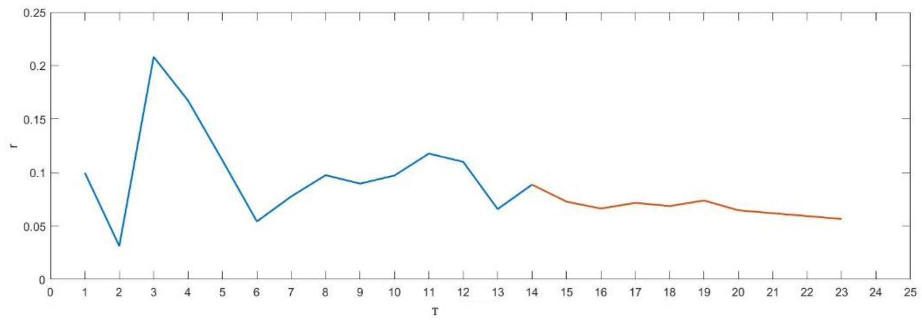

Figure 4 r forecast trend

V. CONCLUSION

Use the predicted values of the parameters and formula (12) to predict the future population of China, and draw Figure 5.

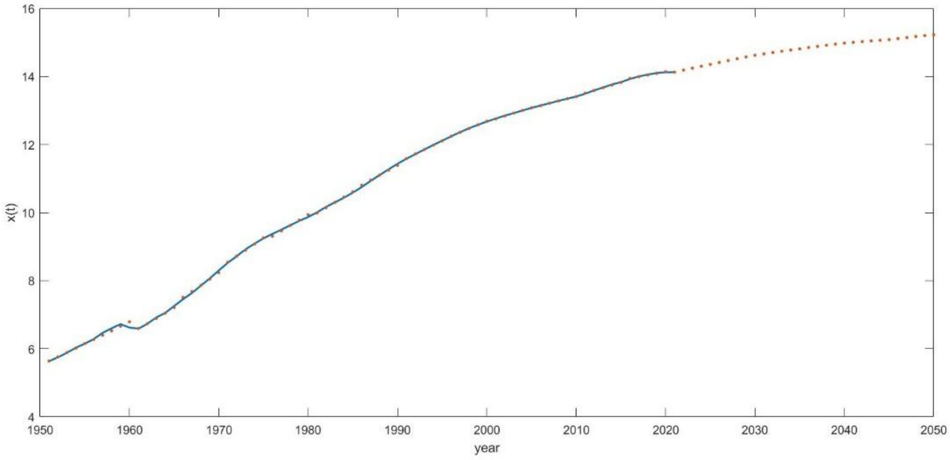

Figure 5 Predicted population

It can be seen from the figure that the population will increase slowly in the future, which is in line with the development of China's current population. The specific data are shown in Table 4.

TABLE 4 China Population Forecast

year	Population/billion	year	Population/billion
2022	14.18602	2031	14.66259
2023	14.24367	2032	14.70188
2024	14.29901	2033	14.73949
2025	14.35212	2034	14.77547
2026	14.41027	2035	14.8099
2027	14.46622	2036	14.84692
2028	14.52005	2037	14.88238
2029	14.57181	2038	14.91633
2030	14.62156	2039	14.94884

In this paper, it is assumed that the parameter growth rate r is a function related to time, which has a certain reference value for the prediction of future population.

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