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Research Paper

On Fuzzy Sequencing Problems Using Graphical Method

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ABSTRACT: *In this paper, we proposed a solution procedure to solve fuzzy sequencing problem processing '2' jobs through 'm' machines with processing time being as triangular and trapezoidal fuzzy numbers by using ranking index method has been applied to convert the fuzzy processing time into crisp sequencing problems and solving by existing method. A numerical example has been considered and solved for illustration purpose.*
KEW WORDS: *Membership function, Triangular Fuzzy number, Trapezoidal fuzzy number, Fuzzy sequencing problem, Yager ranking function.*

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I. INTRODUCTION

Sequencing problem is a mathematical way out for finding a series, in which a few jobs or tasks are to be done in an order for which total processing time is minimum. In our daily life decision making problems it is sometimes required to take the decision where the values of parameters are ambiguous. Generally in sequencing problem the processing times are precise valued. But in reality it is observed that the processing times during performance of the job are imprecise. To handle impreciseness the introduction of fuzzy sets by L.A. Zadeh in 1965 as an approach to a mathematical representation of vagueness in everyday language, was realized by many researchers and has successfully been applied in every branch of Mathematics. In this paper we have treated imprecise parameters considering fuzzy numbers. Thus the concept of fuzzy sequencing problem provides an efficient framework which solves real life problems with fuzzy processing times. In this paper sequencing problem processing '2' jobs through 'm' machines has taken into consideration. The processing times are considered to be triangular and trapezoidal fuzzy number. Then the related problem has been converted into crisp equivalent sequencing problem using defuzzification of triangular and trapezoidal fuzzy numbers. Here ranking index method has been used for defuzzification of fuzzy number. The idle time for both the machines and total elapsed time is obtained by solving related crisp sequencing problem using graphical method. Finally to illustrate numerical examples has been solved and results have been presented.

II. PRELIMINARIES

In this section in order to make the exposition self-contained, some basic notions and results used in the sequel are given.

Definition 2.1: Let X is a non-empty set and I the unit interval [0, 1]. A fuzzy set $\tilde{A}$ in X is a mapping from X into I.The fuzzy set 0_X is defined as $0_X\ (x) = 0$, for all $x \in$ X and the fuzzy set 1_X is defined as $1_X\ (x) = 1$, for all$x \in$ X.

Definition 2.2: The support of a fuzzy set$\tilde{A}$ is the crisp set defined by $\tilde{A} = \{x \in X: \mu_A(x) > 0\}$ and the core of a fuzzy set $\tilde{A}$ is the crisp set defined by $\tilde{A} = \{x \in X: \mu_A(x) = 1\}$ and also the boundary of a fuzzy set defined by$\tilde{A} = \{x \in X: 0 < \mu_A(x) < 1\}$.

Definition 2.3: A fuzzy set$\tilde{A}$ in convex set $X = \mathbb{R}^n$ is said to be a convex fuzzy set if and only if its α-level set are convex.

Definition 2.4:A Fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its membership function $\mu_A(x): \mathbb{R} \to [0,1]$ has the following characteristics

(i) There exists an $x \in \mathbb{R}$ such that $\mu_A(x) = 1$. That is A is normal.
(ii) For every $x_1, x_2 \in \mathbb{R}$. It means that A is convex.
(iii)$\mu_A(x)$ is upper semi-continuous.
(iv) sup $(\tilde{A})$ is bounded in $\mathbb{R}$.

Definition 2.5: A fuzzy number $\tilde{A} \in \mathbb{R}$ is said to be a triangular fuzzy numbers, if its membership function is given by

$$\mu_A(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & a_1 \leq x \leq a_2 \\ x - a_3)/(a_2 - a_3) & a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

It is denoted by $\tilde{A} = (a_1, a_2, a_3)$ where$a_1, a_2, a_3 \in \mathbb{R}$, a_2 is core of$\tilde{A}$, $a_1$is left width and $a_2$is right width. The shape of the triangular fuzzy number is usually in the form of triangle and hence it is called so. The parametric form of a triangular fuzzy number is represented by

$$\tilde{A} = [a_1 - a_2(1 - r); a_1 + a_3(1 - r)].$$

Definition 2.6: A positive triangular number is denoted as $\tilde{A} = (a_1, a_2, a_3)$where all$a_1 > 0$.

Definition 2.7: A negative triangular number is denoted as $\widetilde{A} = (a_1, a_2, a_3)$ where all$a_3 < 0$.

Definition 2.8: Two triangular fuzzy numbers$\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are said to be equal if and only if $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$.

Definition 2.9: A trapezoidal fuzzy number$\tilde{A}$ is represented by (a_1, a_2, a_3, a_4) and defined by its membership function is given by

$$\mu_A(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ (a_4 - x)/(a_4 - a_3) & a_3 \leq x \leq a_4 \end{cases}$$

Definition 2.10:A ranking function $\Re: F(\mathbb{R}) \to \mathbb{R}$ where $F(\mathbb{R})$ is the set of fuzzy numbers defined on set of real numbers, maps each fuzzy number into real number, where a natural order exists.

Definition 2.11: The ranking function for a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ denoted by $\Re(\tilde{A})$ is defined by $\Re(\tilde{A}) = \frac{1}{6}(a_1 + 4a_2 + a_3)$.

Definition 2.12: The Yager's ranking function for a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ denoted by $Y(\tilde{A})$ is defined by $Y(\tilde{A}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$.

III. FUZZY SEQUENCING PROBLEMS USING GRAPHICAL METHOD

In this section, we proposed a solution procedure to solve fuzzy sequencing problem processing '2' jobs through 'm' machines with processing time being as triangular and trapezoidal fuzzy numbers by using ranking index method has been applied to convert the fuzzy processing time into crisp sequencing problems and solving by existing graphical method.

Graphical method is used to solve the sequencing problem of processing '2' jobs through 'm' machines when

(i) The technological ordering of each of the jobs through 'n' machines is known in advance.
(ii) Each machine can process only one job at a time.
(iii) The exact processing times on all the 'n' machines are known.

The objective is to find the minimum total elapsed time and optimal sequence.

3.1. Procedure for solving fuzzy sequencing problem using graphical method

The solution procedure given below and it consists of the following steps:

Step 1: Using ranking function $\Re(\tilde{A}) = \frac{1}{6}(a_1 + 4a_2 + a_3)$ for triangular fuzzy numbers and Yager's ranking function $Y(\tilde{A}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$ for trapezoidal fuzzy number, the given fuzzy sequencing problem can be converted into crisp sequencing problem.

Step 2: Use x-axis to represent the processing time for job 1 and the y-axis to represent the processing time on job 2.

Step3: Mark the machine times for the two jobs on their corresponding axes as per the given technological order.

Step 4: In such graph moving horizontally will imply that job 1 is being processed while job 2 remains idle. Moving vertically will imply that job 2 is being processed while job 1 remains idle. Diagonal movement along 45° line to the horizontal that is line with slope 1 will imply that both the jobs 1 and 2 are being processed simultaneously. Since each machine can process only one job at a time overlapping region for the machines should be determined first and movement across then should be avoided.

Step 5: An optimal path is the shortest one that minimizes the idle time for job 1 in horizontal movement or the shortest one that minimizes the idle time for job 2in vertical movement consisting of horizontal, vertical and $45°$ lines from the origin to the end. Obviously we must choose such a combination in which the diagonal movement is as much as possible.

Step 6: The required minimum total elapsed time is calculated by as follows

Processing time of job 1 $+$ idle time of job 1

Or

Processing time of job 2$+$ idle time of job 2

The optimal sequence of processing is found from the graph drawn.

IV. NUMERICAL EXAMPLES

Example 4.1:

Using graphical method, determine the optimal sequence needed to process job1 and 2 on five machines, *A, B, C, D,* and *E*. For each machine find the job which should be done first. Also, calculate the total time needed to complete both the jobs.

Job 1	Sequence	A	B	C	D	E
	Time	(1, 1, 1)	(3, 2, 1)	(2, 3, 4)	(2, 6, 4)	(4, 0, 2)
Job 2	Sequence	C	A	D	E	B
	Time	(4, 3, 2)	(5, 4, 3)	(5, 1, 3)	(2, 1, 0)	(4, 6, 2)

Solution:

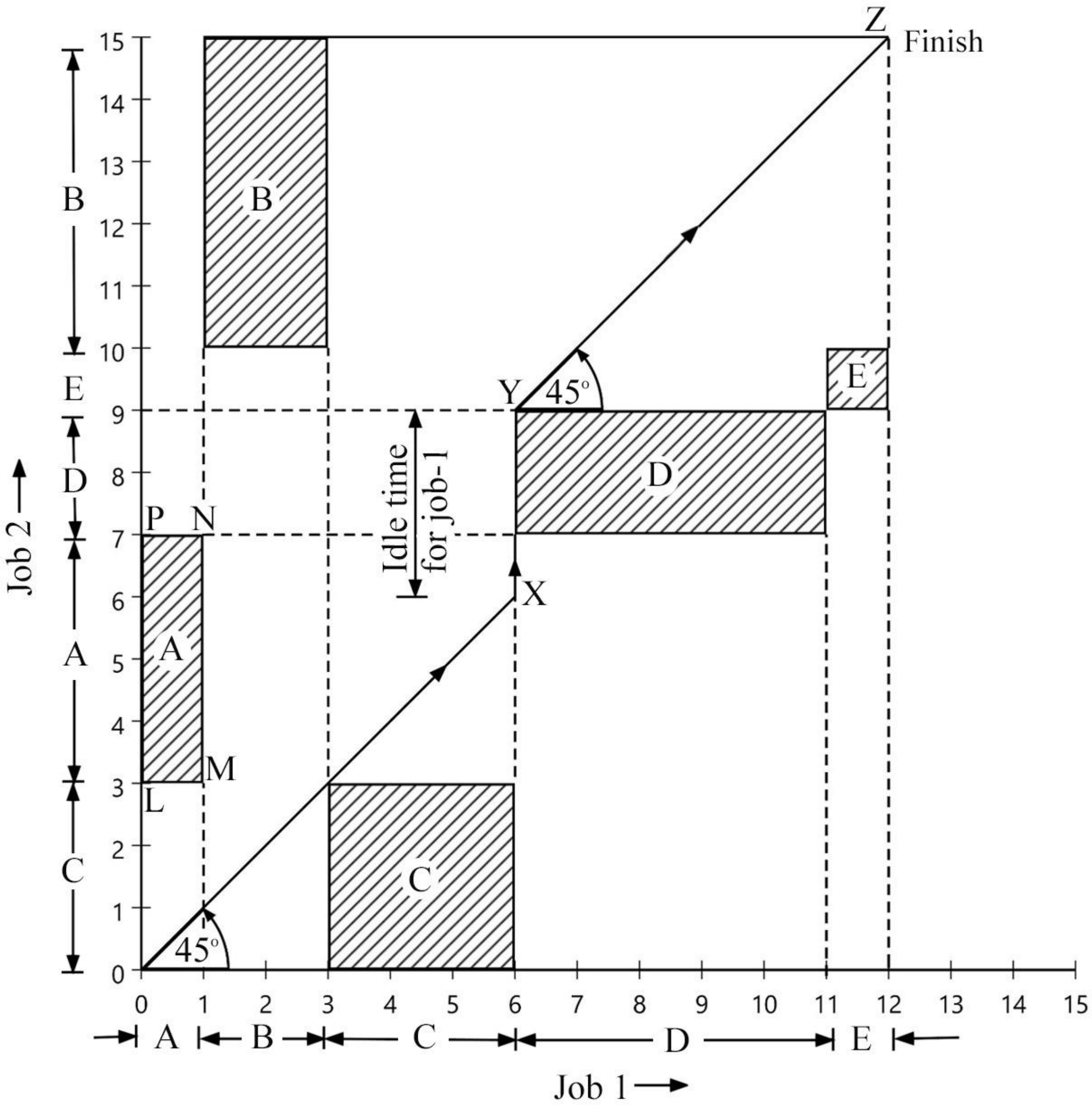
Using ranking function $\Re\,(a_1, a_2, a_3) = \frac{a_1+4a_2+a_3}{6}$ for the triangular fuzzy number, the fuzzy times can be converted to crisp times as follows

$$\Re(1, 1, 1) = 1;\ \Re\,(3, 2, 1) = 2;\ \Re\,(2, 3, 4) = 3;\ \Re\,(2, 6, 4) = 5;\ \Re\,(4, 0, 2) = 1$$
$$\Re\,(4, 3, 2) = 3;\ \Re\,(5, 4, 3) = 4;\ \Re\,(5, 1, 3) = 2;\ \Re\,(2, 1, 0) = 1;\ \Re\,(4, 6, 2) = 5$$

Now, we set the following Crisp Problem.

Job 1	Sequence	A	B	C	D	E
	Time (hrs.)	1	2	3	5	1
Job 2	Sequence	C	A	D	E	B
	Time (hrs.)	3	4	2	1	5

Graphical solution of above 2 job 5 machine problem is mentioned in the following graph.



A good path, accordingly, is chosen by eye and drawn on the graph (path OXYZ).

∴ Total elapsed time = 12 + 3 = 15 hours (considering job 1) (or)

= 15 + 0 = 15 hours (considering job 2).

The optimal schedule corresponding to the chosen path is shown in the following figure

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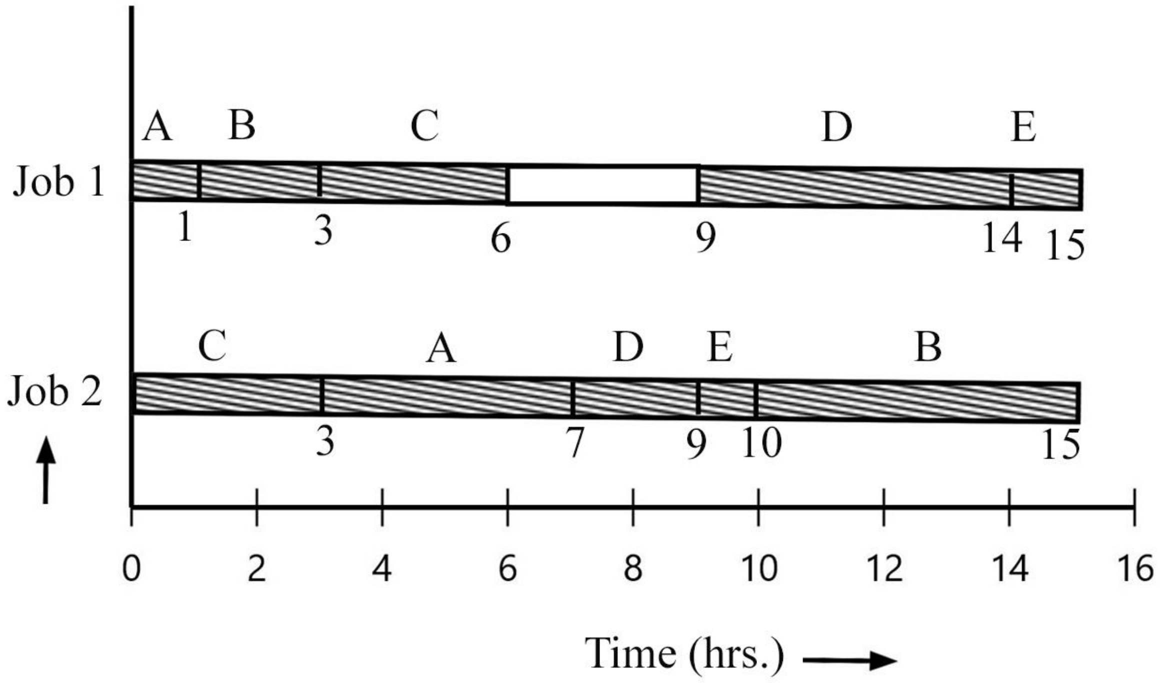


The optimal sequence or schedule on various machines for the two jobs as evident from the above figure is:

Machine A: job 1 precedes job 2,
Machine B: job 1 precedes job 2,
Machine C: job 2 precedes job 1,
Machine D: job2 precedes job 1,
Machine E: job 2 precedes job 1.

Example 4.2:

Use graphical method to minimize the time required to process the following jobs on the machines. For each machine specify the job which should be done first. Also calculate the total elapsed time to complete both jobs.

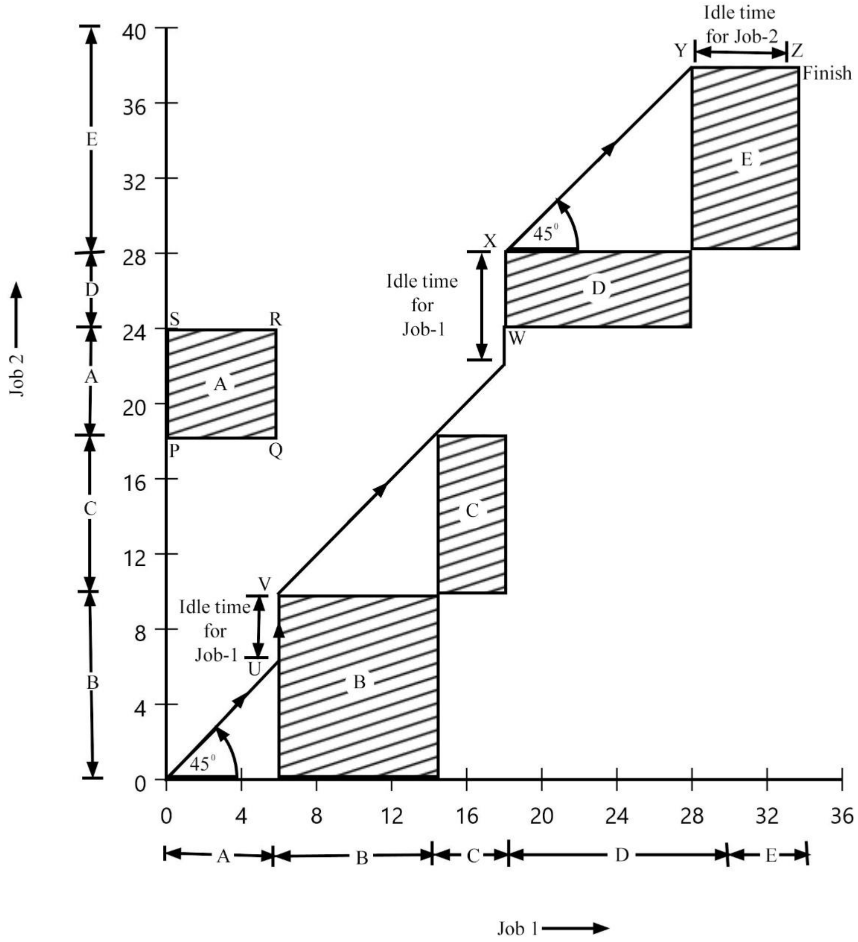
Job 1	Sequence	A	B	C	D	E
	Time	(9, 5, 4, 6)	(10, 9, 7, 6)	(5, 6, 3, 2)	(10, 13, 14, 11)	(4, 5, 6, 1)
Job 2	Sequence	B	C	A	D	E
	Time	(11, 12, 9, 8)	(7, 6, 9, 11)	(9, 7, 5, 3)	(6, 4, 5, 1)	(10, 11, 13, 14)

Solution: Using Yager's ranking function $Y(\tilde{A}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$ for Trapezoidal Fuzzy Number the fuzzy time can be converted in to cribs time, as follows

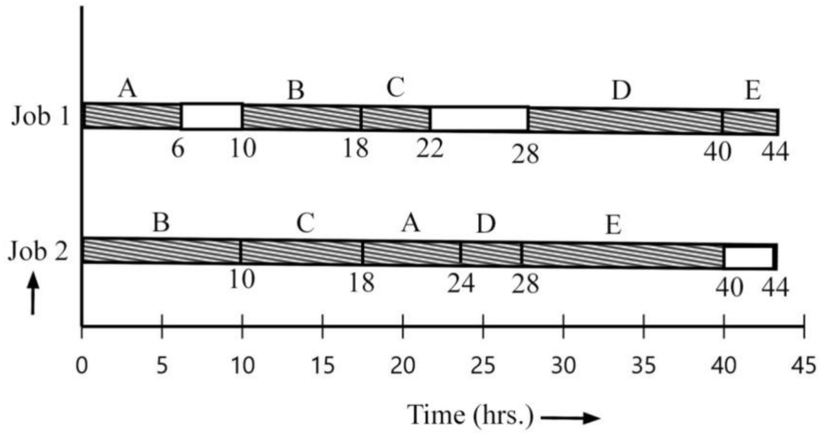
Y (9, 5, 4, 6) =6; Y (10, 9, 7, 6)= 8; Y (5, 6, 3, 2)= 4; Y (10, 13, 14, 11)= 12, Y (4, 5, 6, 1) = 4
Y (11, 12, 9, 8) =10; Y (7, 6, 8, 11)= 8; Y (9, 7, 5, 3)= 6; Y (6, 4, 5, 1)= 4; Y (10, 11, 13, 14) = 12
Now, we set the following Crisp Problem.

Jobs *1*	Sequence	A	B	C	D	E
	Time (hours)	6	8	4	12	4
Jobs *2*	Sequence	B	C	A	D	E
	Time (hours)	10	8	6	4	12

Graphical solution of above 2 job 5 machine problem is mentioned in the following graph.



. A good path, accordingly, is chosen by eye and drawn on the graph (path OUVWXYZ).
Total elapsed time = 34 + 10 = 44 hours. (Considering job 1) (or)
= 40 + 4 = 44 hours. (Considering job 2)
The optimal schedule corresponding to the chosen path is shown in the following figure



The optimal sequence or schedule on various machines for the two jobs as evident from the above figure is:
Machine A: job 1 precedes job 2,
Machine B: job 2 precedes job 1,
Machine C: job 2 precedes job 1,

Machine D: job 2 precedes job 1,
Machine E: job 2 precedes job 1,

V. CONCLUSION

In this paper, the duration hours are treated as imprecise numbers described by triangular and trapezoidal fuzzy numbers which are more realistic and general in nature. Processing ‘2’jobs through ‘m’ machines for total elapsed time and optimal sequence of graphical method to solve a fuzzy sequencing problem. Finally numerical examples discussed by the fuzzy sequencing problem converted into classical sequencing problem using ranking technique to solve the total elapsed time

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