



Inverse Fair Restrained Domination in Graphs

Villa S. Verdad, Enrico C. Enriquez, Martin M. Bulay-og,

Enrico L. Enriquez

Department of Computer, Information Sciences and Mathematics,
School of Arts and Sciences
University of San Carlos 6000

ABSTRACT: Let G be a connected simple graph. A dominating subset S of $V(G)$ is a fair dominating set of G if all the vertices not in S are dominated by the same number of vertices from S . A subset $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to another vertex not in S . A fair dominating set $S \subseteq V(G)$ is a fair restrained dominating set if every vertex not in S is adjacent to a vertex in S and to another vertex not in S . The fair restrained domination number, $\gamma_{frd}(G)$ of G is the minimum cardinality of a fair restrained dominating set of G . A fair restrained dominating set of cardinality $\gamma_{frd}(G)$ is called a γ_{frd} -set. Let D be a minimum fair restrained dominating set of G . A fair restrained dominating set $S \subseteq V(G) \setminus D$ is called an inverse fair restrained dominating set of G with respect to D . The inverse fair restrained domination number of G denoted by $\gamma_{frd}^{-1}(G)$ is the minimum cardinality of an inverse fair restrained dominating set of G . In this paper, we investigate the concept and give the inverse fair restrained domination number of some special graphs.

KEYWORDS: dominating set, fair dominating set, restrained dominating set, inverse dominating set.

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I. INTRODUCTION

Domination in a graph has been a huge area of research in graph theory. It was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Accordingly, a subset S of a vertex set $V(G)$ is a dominating set of a graph G if, for every vertex $v \in V(G) \setminus S$, there exists a vertex $x \in S$ such that xv is an edge of G . The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set S of G . Some related studies on domination in graphs are found in [2-16].

One variant of domination in a graph is the fair domination in graphs [17]. A dominating subset S of $V(G)$ is a fair dominating set of G if all the vertices not in S are dominated by the same number of vertices from S , that is, $|N(u) \cap S| = |N(v) \cap S|$ for every two distinct vertices u and v from $V(G) \setminus S$ and a subset S of $V(G)$ is a k -fair dominating set in G if for every vertex $v \in V(G) \setminus S$, $|N(v) \cap S| = k$. The minimum cardinality of a fair dominating set of G , denoted by $\gamma_{fd}(G)$, is called the fair domination number of G . A fair dominating set of cardinality $\gamma_{fd}(G)$ is called γ_{fd} -set. Some related papers on fair domination in graphs is found in [18-25]. The restrained dominating set is another variant of domination in graphs [34,35]. A set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. Alternately, a subset S of $V(G)$ is a restrained dominating set if the close neighbor of S , $N[S] = V(G)$ and $\langle V(G) \setminus S \rangle$ is a subgraph without isolated vertices. The restrained domination number of G , denoted by $\gamma_r(G)$, is the minimum cardinality of a restrained dominating set of G .

Other variant of domination in a graph is the inverse domination in graphs [26]. Let D be a minimum dominating set in G . The dominating set $S \subseteq V(G) \setminus D$ is called an inverse dominating set with respect to D . The minimum cardinality of inverse dominating set is called an inverse domination number of G and is denoted by $\gamma^{-1}(G)$. An inverse dominating set of cardinality $\gamma^{-1}(G)$ is called γ^{-1} -set of G . Some variants of inverse domination in graphs are found [27-32].

The concepts of inverse, fair, and restrained dominating sets has motivated the researcher to define a new variant, the inverse fair restrained domination in graphs. Let D be a minimum fair restrained dominating set of a graph G . The fair restrained dominating set $S \subseteq V(G) \setminus D$ is called an inverse fair restrained dominating set

with respect to D . The minimum cardinality of an inverse fair restrained dominating set is called an inverse fair restrained domination number of G and is denoted by $\gamma_{frd}^{-1}(G)$. An inverse fair restrained dominating set of cardinality $\gamma_{frd}^{-1}(G)$ is called γ_{frd}^{-1} -set of G . In this paper, we investigate the concept and give inverse fair restrained domination number of some special graphs. For the general concepts, the readers may refer to [33].

II. REALIZATION PROBLEMS

Since the inverse fair dominating set does not always exist in a connected nontrivial graph G we denote $\mathcal{FD}(G)$, a family of all graphs with inverse fair dominating set. Thus, for the purpose of this study, it is assumed that all connected nontrivial graphs considered, belong to the family $\mathcal{FD}(G)$. The following remark is immediate.

Remark 2.1 Let G be any connected graph of order $n \geq 3$. Then $\gamma(G) \leq \gamma_{frd}(G) \leq \gamma_{frd}^{-1}(G)$.

Let a graph G be a cycle of order 5. Consider the vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and the edge set $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$ of G (see Figure 1).

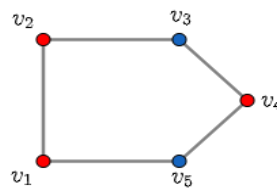


Figure 1: A graph C_5 with no inverse fair dominating set.

Then the set $D = \{v_1, v_2, v_4\}$ is a minimum fair dominating set of G and $S = V(G) \setminus D = \{v_3, v_5\}$ is not a fair dominating set of G . Hence, we cannot find an inverse fair restrained dominating set S of a graph $G = C_5$.

Since the inverse fair dominating set does not always exist in a connected nontrivial graph G , we denote $\mathcal{FRD}(G)$, a family of all graphs with inverse fair dominating set. Thus, for the purpose of this study, it is assumed that all connected nontrivial graphs considered, belong to the family $\mathcal{FRD}(G)$. The following remark is immediate.

Remark 2.1 Let G be any connected graph of order $n \geq 3$. Then $\gamma(G) \leq \gamma_{frd}(G) \leq \gamma_{frd}^{-1}(G)$.

The next result says that the value of the parameter $\gamma_{frd}^{-1}(G)$ ranges over all positive integers from $1, 2, \dots, n - 2$ where $n \leq 3$ is the order of G .

Theorem 2.2 Let k and n be positive integers such that $1 \leq k \leq n - 2$ and $n \geq 3$. Then there exists a connected nontrivial graph G such that $\gamma_{frd}^{-1}(G) = k$ and $|V(G)| = n$.

Proof: Consider the following cases:

Case 1. Suppose that $1 = k \leq n - 2$.

Let $G = K_n$ with $n \geq 3$. Clearly, $\gamma_{frd}^{-1}(G) = 1 = k$ and $|V(G)| = n$. (see Figure 2)

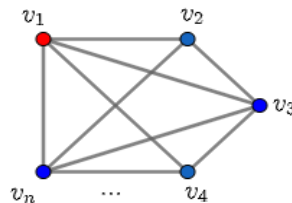


Figure 2: A graph G with $1 = k \leq n - 2$

Case 2. Suppose that $1 < k \leq n - 2$.

Subcase 1. Consider that $k = n - 2$. Let $G = C_4$. Then $S = \{x_1, x_2\}$ is a γ_{frd}^{-1} set of G . Thus, $\gamma_{frd}^{-1}(G) = |S| = 2 = k$ and $|V(G)| = 4 = n$ (see Figure 3)

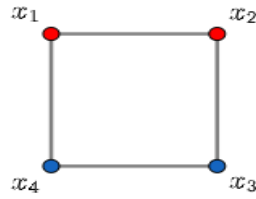


Figure 3: A graph G with $k = n - 2$

Subcase 2. Consider that $k < n - 2$. Let $G = P_2 \square P_r$ (the Cartesian product of two paths P_2 and P_r). Without loss of generality assume that $r \neq 1$ is an odd integer. Then $n \geq 6$. Let $r = 2k - 1$. If k is an even integer, then the set $S = \{v_{4i-3} : i = 1, 2, \dots, \frac{r+1}{4}\} \cup \{u_{4j-1} : j = 1, 2, \dots, \frac{r+1}{4}\}$ is a γ_{frd}^{-1} -set of G (see Figure 4). Thus, $\gamma_{frd}^{-1}(G) = |S| = \frac{r+1}{4} + \frac{r+1}{4} = \frac{r+1}{2} = k$.

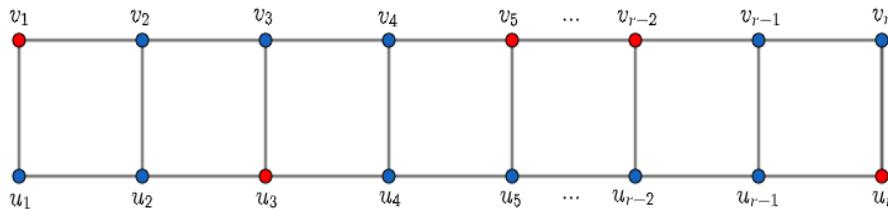


Figure 4: A graph G with $1 < k < n - 2$.

Similarly, if k is an odd integer, then the set $S = \{v_{4i-1} : i = 1, 2, \dots, \frac{r+1}{4}\} \cup \{u_{4j-3} : j = 1, 2, \dots, \frac{r+1}{4}\}$ is a γ_{frd}^{-1} -set of G and $\gamma_{frd}^{-1}(G) = |S| = \frac{r+1}{4} + \frac{r+1}{4} = \frac{r+1}{2} = k$. This proves the assertion. ■

Theorem 2.3 Let k, m , and $n \geq 3$ be positive integers such that $1 \leq k \leq m \leq n - 2$. Then there exists a connected nontrivial graph G such that $\gamma_{frd}(G) = k$, $\gamma_{frd}^{-1}(G) = m$, and $|V(G)| = n$.

Proof: Consider the following cases:

Case 1. Suppose that $1 = k = m \leq n - 2$ with $n \geq 3$
 Let $G = K_n$. Then, $\gamma_{frd}(G) = 1 = \gamma_{frd}^{-1}(G)$ and $|V(G)| = n$.

Case 2. Suppose that $1 = k < m \leq n - 3$ with $n \geq 5$.
 Subcase 1. If $m = 2$, then let $G = P_1 + P_4$, that is $n = 5$ (see Figure 5).

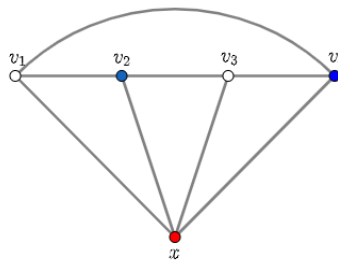


Figure 5: A graph G with $1 = k < m = n - 3$.

The set $D = \{x\}$ is a γ_{frd} -set of G and the set $S = \{v_2, v_4\}$ a γ_{frd}^{-1} -set of G . Thus, $\gamma_{frd}(G) = 1 = k$ and $\gamma_{frd}^{-1}(G) = 2 = m$. Further, $|V(G)| = 5 = n$.

Subcase 2. If $m > 2$, then let G be a graph (see Figure 6) with an integer $r \geq 6$ and $n = r + 1$. Let $r = 2m$.

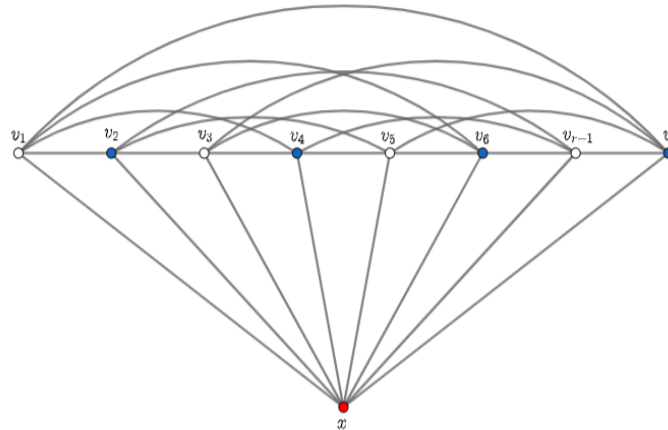


Figure 6: A graph G with $1 = k < m < n - 3$.

Now, $D = \{x\}$ is a γ_{frd} -set of G and $S = \{v_2, v_4, v_6, \dots, v_r\}$ a γ_{frd}^{-1} -set of G . Thus, $\gamma_{frd}(G) = |D| = 1 = k$. Thus, $\gamma_{frd}^{-1}(G) = |S| = \frac{r}{2} = m$. Further, $|V(G)| = r + 1 = n$.

Case 3. Suppose that $1 < k = m < n - 4$. Let $G = P_2 \square P_r$ and $n = 2r$ (see Figure 7). Without loss of generality assume that an odd integer $r \neq 1$. Then $r \geq 3$ and $\frac{r+1}{2}$ is a positive integer. Let $k = \frac{r+1}{2}$.

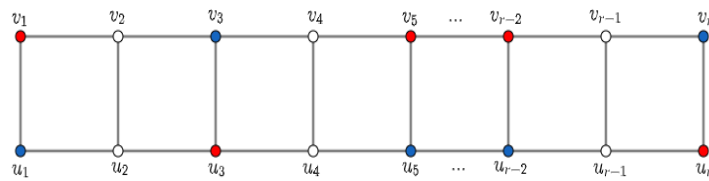


Figure 7: A graph G with $1 < k = m < n - 4$.

The set $D = \{v_{4i-3} : i = 1, 2, \dots, \frac{r+1}{4}\} \cup \{u_{4j-1} : j = 1, 2, \dots, \frac{r+1}{4}\}$ is a γ_{frd} -set of G . Thus,

$$\gamma_{frd}(G) = |D| = \frac{r+1}{4} + \frac{r+1}{4} = \frac{r+1}{2} = k.$$

Further, the set $S = \{v_{4i-1} : i = 1, 2, \dots, \frac{r+1}{4}\} \cup \{u_{4j-3} : j = 1, 2, \dots, \frac{r+1}{4}\}$ is a γ_{frd}^{-1} -set of G . Thus,

$$\gamma_{frd}^{-1}(G) = |S| = r + \frac{r+1}{4} + \frac{r+1}{4} = \frac{r+1}{2} = k = m \text{ and } |V(G)| = 2r = n.$$

Case 4. Suppose that $1 < k < m < n - 1$. Let H be a nontrivial connected graph of order k such that $G = H \circ P_4$ (see Figure 8).

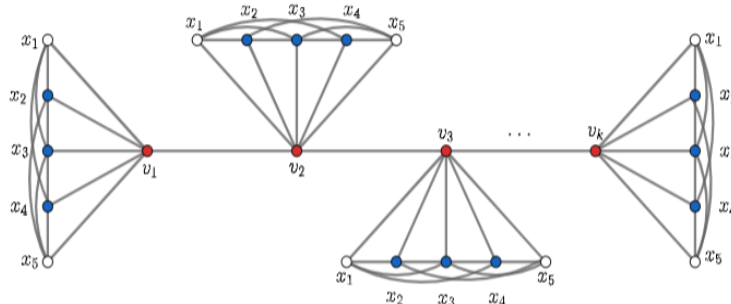


Figure 8: A graph G with $1 < k < m < n - 1$.

Let $n = 6k$ and $m = 3k$. The set $D = V(H)$ is a γ_{frd} -set and $S = \bigcup_{v \in V(H)} S_v$ is a γ_{frd}^{-1} -set of G where $S_v = \{x_2, x_3, x_4\}$. Thus, $\gamma_{frd}(G) = |D| = |V(H)| = k$ and

$$\gamma_{frd}^{-1}(G) = |S| = \left| \bigcup_{v \in V(H)} S_v \right| = \sum_{v \in V(H)} |S_v| = |V(H)| \cdot |S_v| = k \cdot 3 = m$$

Further, $|V(G)| = |V(H)| \cdot (|V(P_5)| + 1) = k \cdot (5 + 1) = k \cdot 6 = n$.

This proves the assertion. ■

The following result is an immediate consequence of Theorem 2.3.

Corollary 2.4 The difference $\gamma_{frd}^{-1}(G) - \gamma_{frd}(G)$ can be made arbitrarily large.

Proof: Let r be any positive integer such that $n = 2k$ for any integer k and let G be a graph as shown in Figure 8. Then $\gamma_{frd}^{-1}(G) - \gamma_{frd}(G) = m - k = 3k - k = 2k = r$. ■

III. SOME SPECIAL GRAPHS

The cycle $C_n = a_1 a_2 \dots a_n a_1$ is the graph with $V(C_n) = \{a_1, a_2, \dots, a_n\}$ and $E(C_n) = \{a_1 a_2, a_2 a_3, \dots, a_{n-1} a_n, a_n a_1\}$ where $n \geq 3$.

Proposition 3.1 Let $G = C_n$ of order $n \geq 3$. Then

$$\gamma_{frd}^{-1}(G) = \begin{cases} \frac{n}{3}, & \text{if } n \equiv 0 \pmod{3} \\ \frac{n+2}{3}, & \text{if } n \equiv 1 \pmod{3} \\ \frac{n+4}{3}, & \text{if } n \equiv 2 \pmod{3}, n \neq 5 \end{cases}$$

Proof: Suppose that $G = \{a_1, a_2, \dots, a_{n-1}, a_n, a_1\}$ is a cycle of order $n \geq 3$. Consider the following,

Case 1. If $n \equiv 0 \pmod{3}$, then $D = \{a_2, a_5, a_8, \dots, a_{n-1}\}$ is a γ_{frd} -set of G and $S = \{a_1, a_4, a_7, \dots, a_{n-2}\}$ is a γ_{frd}^{-1} -set of G (see Figure 4).

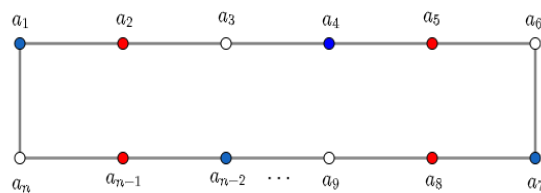


Figure 4. A cycle of order $n \equiv 0 \pmod{3}$

Thus, by routine computations,

$$|D| = |\{a_2, a_5, a_8, \dots, a_{n-1}\}| = \frac{(n-1) + 1}{3} = \frac{n}{3}$$

and

$$|S| = |\{a_1, a_4, a_7, \dots, a_{n-2}\}| = \frac{(n-2) + 2}{3} = \frac{n}{3}$$

Hence, $\gamma_{frd}^{-1}(G) = |S| = \frac{n}{3}$.

Case 2. If $n \equiv 1 \pmod{3}$, then $D = \{a_2, a_5, a_8, \dots, a_{n-2}, a_{n-1}\}$ is a γ_{frd} -set of G and $S = \{a_1, a_4, a_7, \dots, a_n\}$ is a γ_{frd}^{-1} -set of G (see Figure 5).

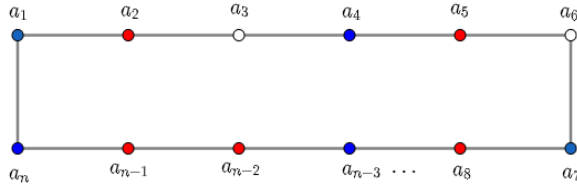


Figure 5. A cycle of order $n \equiv 1(\text{mod } 3)$

Thus, by routine computations,

$$|D| = |\{a_2, a_5, a_8, \dots, a_{n-2}, a_{n-1}\}| = \frac{n-1}{3} + 1 = \frac{n+2}{3}$$

and

$$|S| = |\{a_1, a_4, a_7, \dots, a_{n-3}, a_n\}| = \frac{n+2}{3}.$$

Hence,

$$\gamma_{frd}^{-1}(G) = |S| = \frac{n+2}{3}.$$

Case 3. If $n \equiv 2(\text{mod } 3)$, then $D = \{a_2, a_5, a_8, \dots, a_{n-6}, a_{n-5}, a_{n-2}, a_{n-1}\}$ is a γ_{frd} -set of G and $S = \{a_1, a_4, a_7, \dots, a_{n-6}, a_{n-5}, a_{n-2}, a_{n-1}\}$ is a γ_{frd}^{-1} -set of G (see Figure 6).

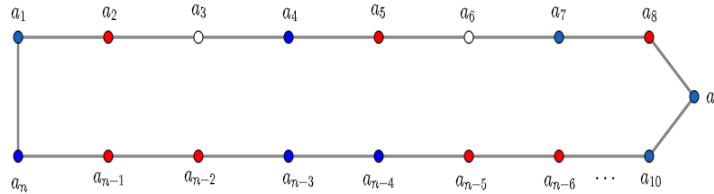


Figure 6. A path of order $n \equiv 2(\text{mod } 3)$

Thus, by routine computations,

$$|D| = |\{a_2, a_5, a_8, \dots, a_{n-6}, a_{n-5}, a_{n-2}, a_{n-1}\}| = \frac{n-2}{3} + 2 = \frac{n+4}{3}$$

and

$$|S| = |\{a_1, a_4, a_7, \dots, a_{n-8}, a_{n-7}, a_{n-4}, a_{n-3}, a_n\}| = \frac{n-5}{3} + 3 = \frac{n+4}{3}.$$

Hence,

$$\gamma_{frd}^{-1}(G) = |S| = \frac{n+4}{3}.$$

and the proof is completed. ■

A complete bipartite graph is a graph whose vertex set can be partitioned into V_1 and V_2 such that every edge joins a vertex in V_1 with a vertex in V_2 ; and every vertex in V_1 is adjacent with every vertex in V_2 .

Proposition 3.2 Let $G = K_{m,n}$ be a complete bipartite. Then $\gamma_{frd}^{-1}(G) = 2$ for all $m \geq 2$ and $n \geq 2$.

Proof. Suppose that G is a complete bipartite. Then $G = K_{m,n} = \bar{K}_m + \bar{K}_n$. Consider the following,

Case 1. If $m = 2$, then $D = \{x_1, v_1\}$ is a γ_{frd} -set of G and $S = \{x_n, v_2\}$ is a γ_{frd}^{-1} -set of G for all $n \geq 2$ (see Figure 7).

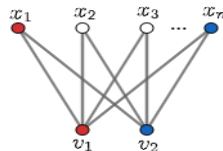


Figure 7. A complete bipartite $K_{2,n}$ for all $n \geq 2$.

Thus, $\gamma_{frd}^{-1}(G) = |S| = 2$.

Case 2. If $n = 2$, then $D = \{x_1, v_1\}$ is a γ_{frd} -set of G and $S = \{x_m, v_2\}$ is a γ_{frd}^{-1} -set of G for all $m \geq 2$ (see Figure 8).

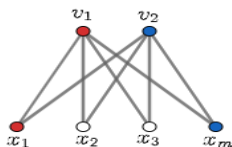


Figure 8. A complete bipartite $K_{m,2}$ for all $m \geq 2$.

Thus, $\gamma_{frd}^{-1}(G) = |S| = 2$. ■

The complete graph K_n is the graph of order n where every pair of vertices is adjacent.

Proposition 3.3. Let $G = K_n$ be a complete graph. Then $\gamma_{frd}^{-1}(G) = 1$ for all $n \geq 2$.

IV. CONCLUSION

In this paper, we introduced a new parameter of domination in graphs G - the inverse fair restrained domination number of G . The realization problems of the inverse fair restrained dominating set were investigated and the inverse fair domination number is computed. Further, we give the inverse fair restrained domination number of some special graphs. This study will motivate new research such as bounds and some binary operations of two graphs such as the join, corona, lexicographic, and Cartesian product of two graphs. Other parameters involving inverse fair restrained domination in graphs may also be explored. Finally, the characterization of an inverse fair restrained domination in graphs and its bounds is a promising extension of this study.

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