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Research Paper

Resource Allocation in Nigerian Banks Using Queue Theory

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ABSTRACT:*The development of a growing economy and the advancement in a developing economy holds its nodes on the banking sector and most especially commercial banks. Therefore, there is a need to focuson analyzing the queues in banks to optimize services at the teller counter in order to reduce customers waiting time and increase service satisfaction.Resource allocation is very essential in having and maintaining good flow of operation in any organisation. In this work, basic performance measures,like expected queue length, waiting times etc are computed for three Nigerian commercial Banks using queue theory.*
***KEYWORDS:* Resource allocation, Queues, waiting times, Poisson distribution**

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I. INTRODUCTION

Today, banks are one of the most important units of the public whose functions cannot be over-emphasized by either private or corporate sectors of the general economy.Daily transactions and exchange of goods and services require the endorsement of commercial banks for smooth running. Therefore, there is a need to focuson analyzing the queues in banks to optimize services at the teller counter in order to reduce customers waiting time and increase service satisfaction.Service delivery in banks is personal, customers are either served immediately or join a queue (waiting line) if the system is busy. A queue occurs where facilities are limited and cannot satisfy demands made against them at a particular period. However, most customers are not comfortable with waiting or queuing [1]. The danger of keeping customers in queues is that their waiting time may amount to or could become a cost to them (i.e Bank customers). Banks in the University community are generally characterized by congestion with students making deposits, withdrawal, payment of fees etc. Congestion in a bank often leads to a low level of customer satisfaction and encourages customer movements from one bank to another, in search of the services provided without much delay.

A queueing system consists of one or more servers that provide service of some sort to arriving customers. Customers who arrive to find all servers busy generally join one or more queues (waiting lines) in front of the servers, hence the name queuing systems. Waiting phenomenon is not an experience limited to human beings only: jobs wait to be processed in a machine, planes circles in a stack before given permission to land at an airport, trucks at central market wait for loading and offloading, in warehouse, items wait to be used, incoming calls wait to mature in the telephone exchange.In some queuing systems customers arrive according to some stochastic process (e.g., a Poisson process) and immediately upon arrival must join one of the queues, thereafter to be served on a first-come first-served basis, with no jockeying or defections allowed. The service times are independent and identically distributed with a known distribution. Moreover, the service times are independent of the arrival process and the customer decisions

Various studies have applied queueing theory to different aspect of lives. The studies in [3, 4, 5] evaluated the effectiveness of a queuing model in public teaching hospitals in Nigeria. Queue theory has been put to applications in various sectors like traffic, behavioural aspects, etc [6,7,8,9, 10 11].In this study, we consider three commercial nation's bank where the customers are coming from an infinite population and the system having c number of customers. We assume a first come first serve service discipline where the arrival rate has Poisson distribution and there is service rate is exponentially distributed. For the purpose of modeling, the arrivals (n) are the customers. As each customer reaches the bank, he/she books for service. If service is rendered immediately he/she leaves the bank or otherwise joins the queue. The teller counters are the servers (*c*).

The arrival rate, service time and number of servers were the data used for the study that have been collected using observation method. The data collection covered a period of five consecutive working days for each bank between the hours of 9:00 am to 1:00 pm which made it 3 weeks. The duration and time interval is to enable the researcher collect a representative data that considered traffic in each bank.

II. THE POISSON DISTRIBUTION

Consider the time interval t broken into small subintervals of length δt. If δt is sufficiently short then we can neglect the probability that two events will occur in it. We will find one event with probability $P(n;\delta t)$ for several values of the mean v

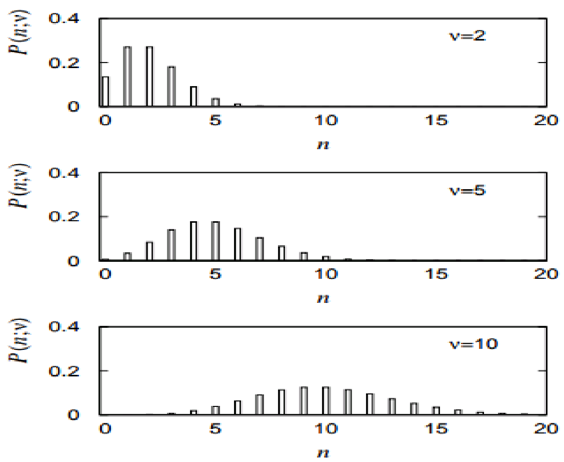

Fig1: The Poisson distribution

$$P(1;\delta t) = \lambda\delta t \quad (1)$$

And no events with probability

$$P(0;\delta t) = 1 - \lambda\delta t \quad (2)$$

What we want to find is the probability to find n events in t. We can start by finding the probability to find zero $P(0;t)$events in $t, P(0;t)$and then generalize this result by induction. Suppose we knew we could then ask what is the probability to find no events in the time $t+\delta t.$ since the events are independent, the probability for no events in both intervals, first none in t and then none in δt is given by the product of the two individual probabilities. That is,

$$P(0;t+\delta t) = P(0;t)(1-\lambda t) \quad (3)$$

This can be rewritten as

$$\frac{P(0;t+\delta t) - P(0;t)}{\delta t} = -\lambda P(0;t) \quad (4)$$

which in the limit of small δt becomes a differential equation

$$\frac{dP(0;t)}{dt} = -\lambda P(0;t) \quad (5)$$

Integrating to find the solution gives

$$P(0;t) = Ce^{-\lambda t} \quad (6)$$

For a length of time $t=0$ we must have zero events, i.e., we require the boundary condition $P(0;0)=1.$ The constant C must therefore be 1 and we obtain

$$P(0;t) = e^{-\lambda t} \quad (7)$$

Now consider the case where the number of events n is not zero. The probability of finding n events in a time $t+\delta t$ is given by the sum of two terms:

$$P(n;t+\delta t) = P(n;t)(1-\lambda\delta t) + P(n-1;t)\lambda\delta t \quad (8)$$

The first term gives the probability to have all n events in the first subinterval of time t and then no events in the final $\delta t.$ The second term corresponds to having n − 1 events in t followed by one event in the last δt. In the limit of small δt this gives differential equation for $P(n;t)$.

$$\frac{dP(n;t)}{dt} + \lambda P(n;t) = \lambda P(n;t) = \lambda P(n-1;t) \quad (9)$$

We can solve equation (9) by finding an integrating factor $\mu(t)$, i.e., a function which when multiplied by the left-hand side of the equation results in a total derivative with respect to t. That is, we want a function $\mu(t)$ such that

$$\mu(t)\left[\frac{dP(n;t)}{dt}+\lambda P(n;t)\right]=\frac{d}{dt}[\mu(t)P(n;t)] \qquad (10)$$

We can easily show that the function

$$\mu(t)=e^{\lambda t} \qquad (11)$$

has the desired property and therefore we find

$$\frac{d}{dt}\left[e^{\lambda t}P(n;t)\right]=e^{\lambda t}P(n-1;t) \qquad (12)$$

We can use this result, for example, with $n=1$ to find

$$\frac{d}{dt}\left[e^{\lambda t}P(1;t)\right]=\lambda e^{\lambda t}P(0;t)=\lambda e^{\lambda t}e^{-\lambda t}=\lambda \qquad (13)$$

where we substituted our previous result (7) for $P(0;t)$. Integrating equation (13) gives

$$e^{\lambda t}P(1;t)=\int \lambda dt=\lambda t+C \qquad (14)$$

Now the probability to find one event in zero time must be zero, i.e., $P(1;0)=0$ and therefore $C=0$, so we find

$$P(1;t)=\lambda te^{-\lambda t} \qquad (15)$$

We can generalize this result to arbitrary n by induction. We assert that the probability to find n events in a time t is

$$P(n;t)=\frac{(\lambda t)^n}{n!}e^{-\lambda t} \qquad (16)$$

We have already shown that this is true for $n=0$ as well as for $n=1$. Using the differential equation (12) with $n+1$ on the left-hand side and substituting (16) on the right, we find

$$\frac{d}{dt}\left[e^{\lambda t}P(n+1;t)\right]=e^{\lambda t}\lambda P(n;t)=e^{-\lambda t}\lambda\frac{(\lambda t)^n}{n!}e^{-\lambda t}=\lambda\frac{(\lambda t)^n}{n!} \qquad (17)$$

Integrating equation (17) gives

$$e^{\lambda t}P(n+1;t)=\int \lambda\frac{(\lambda t)^n}{n!}dt=\frac{(\lambda t)^{n+1}}{(n+1)!}+C \qquad (18)$$

Imposing the boundary condition $P(n+1;0)=0$ implies $C=0$ and therefore

$$P(n+1;t)=\frac{(\lambda t)^{n+1}}{(n+1)!}e^{-\lambda t} \qquad (19)$$

Thus the assertion (16) for n also hold for n+1 and the result is proved by induction

IV. THE EXPONENTIAL DISTRIBUTION

A continuous random variable X is said to have an Exponential distributionif it has probability density function

$$f_X(x|\lambda)=f(x)=\begin{cases}\lambda e^{-\lambda x}, & x>0\\ 0, & x\le 0\end{cases}, \qquad (20)$$

where $\lambda>0$ is called the rate of the distribution.

In the study of continuous-time stochastic processes, the exponential distribution is usually used to model the time until something happens in the process. The mean of the Exponential (λ) distribution is calculated using integration by parts as

$$E[X]=\int_0^\infty x\lambda e^{-\lambda t}\,dx=\frac{1}{\lambda} \qquad (21).$$

So one can see that as λ gets larger, the thing in the process we are waiting for to happen tends to happen more quickly, hence we think λ as a rate.

The system performance parameters used in this study were defined as follows:

λ: Arrival rate of customers at each selected bank per hour;

μ : Service rate (length of stay) of customers at each selected bank per hour.

C : Number of servers attending to customers in each selected bank.

ρ: Utilization factor for each selected bank $=\lambda/C\mu$.

Lq : Average number of customers in the queue.

L: Average number of customers in the system = $Lq+\frac{\lambda}{\mu}$

Wq: Waiting time of customers in the queue = Lq/λ

W : Waiting time of customers in the system = L/λ

$P_n=$ probability of n customers existing in the system.

$$P_n = \begin{cases} \left(\frac{\lambda}{\mu}\right)^n * p_0, & if\ 0 < n < c \\ \frac{(\lambda/\mu)^n}{c!\,c^{n-c}} p_0, & if\ c \le n \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!}\left(\frac{1}{1-\frac{\rho}{c}}\right)\right]^{-1}$$

PERFORMANCE MEASURES OF QUEUES IN THE BANKS

Three commercial banks (A, B and C) are considered.The main objective of this study is to analyze the congestion problem in the banks with a view to determining the optimal number of resources (tellers) necessary at points where bottlenecks existed in the banks. To perform any analysis, we first describe the data. To perform this we conduct the descriptive analysis as follows:

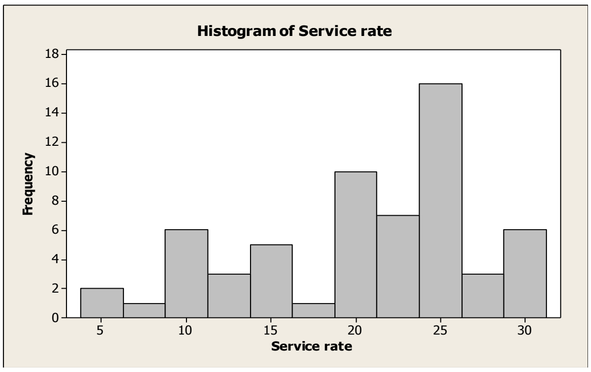

Fig.2: **Histogram of Service rate for Bank A**

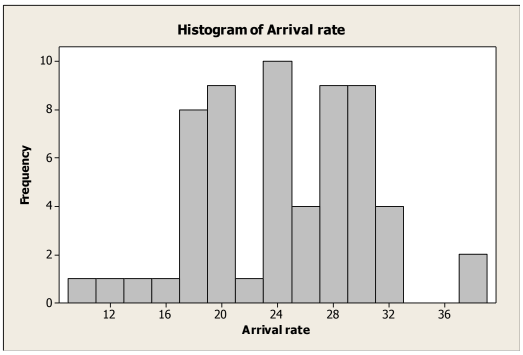

Fig.3: **Histogram of Arrival rate*for Bank A***

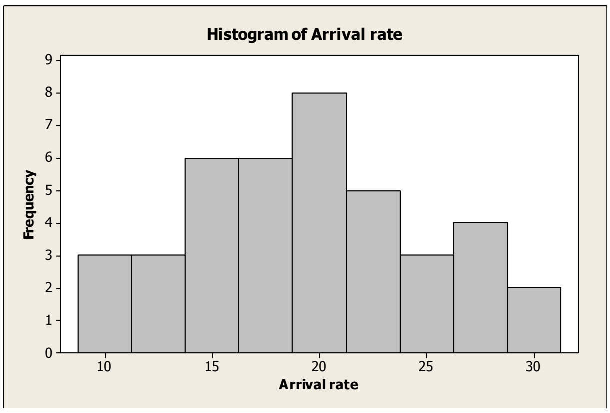

Fig 4. Histogram of Arrival rate for Bank B

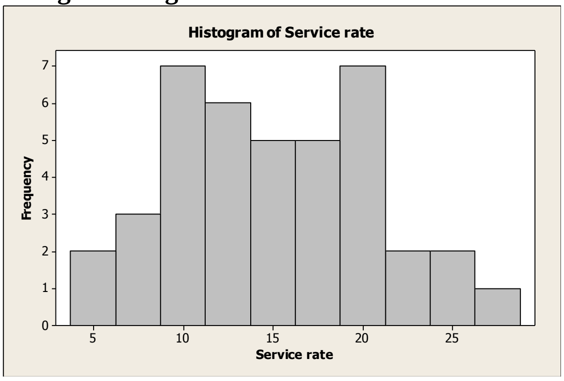

Fig.5: **Histogram of Service rate for Bank B**

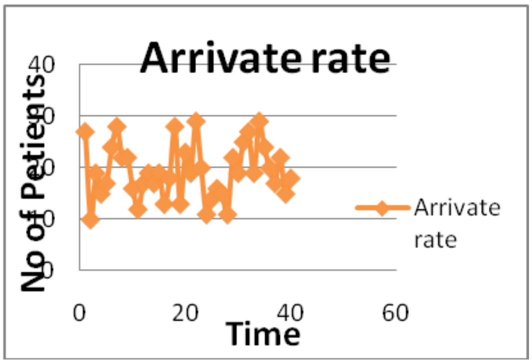

Fig.6: Arrival distribution graph for Bank B

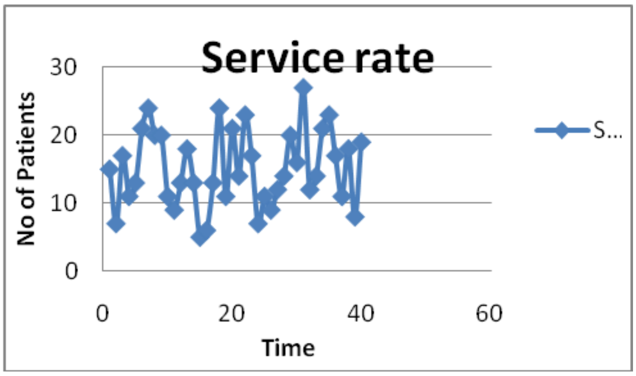

Fig.7: Service distribution graph for Bank B

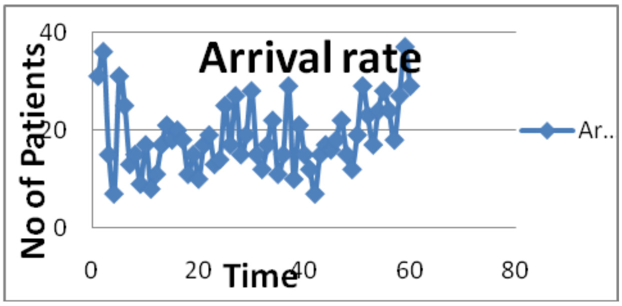

Fig. 8: **Histogram of Service rate for Bank C**

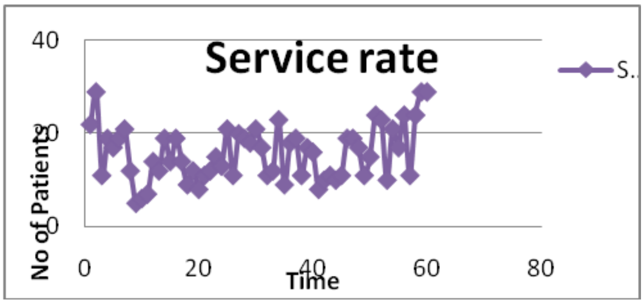

Fig 9. Service distribution graph for Bank C

Also, we perform a goodness of fit test to check if the arrival rate follows a Poisson distribution or not. The hypothesis is given as:

H_0: The arrival rate does not follow a Poisson distribution with mean rate λ.

H_1: The arrival rate follows the Poisson distribution with mean rate λ.

The result output (obtained by using Minitab 16) is then given as follows:

Table 1: Goodness of Fit for Poisson distribution for Bank B

Goodness-of-Fit Test for Poisson Distribution

Data column: Arrival rate

Poisson mean for Arrival rate = 19.275

Arrival rate	Observed	Poisson Probability	Expected
<=12	4	0.053793	2.15174
13 - 14	3	0.082340	3.29360
15 - 16	5	0.135125	5.40502
17 - 18	6	0.173362	6.93450
19 - 20	8	0.178591	7.14363
21 - 22	4	0.150930	6.03721
23 - 24	3	0.106507	4.26026
25 - 26	1	0.063690	2.54761
>=27	6	0.055661	2.22644

N	N*	DF	Chi-Sq	P-Value
40	0	7	10.2689	0.174

The table above shows that, at 5% level of significant, we reject the null hypothesis (since p-value$= 0.174 > 0.05$) and conclude that the arrival rate follows a Poisson distribution with mean rate λ.

For Bank A, the arrival rate$(\lambda) = \frac{Totalnumberofarrivals}{Totalarrivaltime} = 1.1925$ customers per minute.

Service rate$(\mu) = \frac{Totalnumberofcustomersserved}{timetakenforcustomerstobeserved} = 0.8283$ customers per minute.

Here arrival rate for Bank B, $\lambda = \frac{38.05}{60} = 0.6342$ customer per minute.

While the Service rate for Bank B, $\mu = \frac{30.25}{60} = 0.5042$ customer per minute.

For Bank C, $\lambda = \frac{55.9}{60} = 0.9317$ customer per minute.

And Service rate$(\mu) = \frac{47.3}{60} = 0.7883$ customers per minute.

Finally, we can obtain the performance measure for the queuing parameters (using MS-Excel) as follows:

Table 2: Queuing parameter for the Banks

Queue Station	Bank A	Bank B	Bank C
Arrival Rate	1.1925	0.634199977	0.9317
Service Rate/Channel	1.026667	0.504199982	0.7883
Number of Servers	3	2	3
Max. Number in System	***	***	***
Number in Population	***	***	***
Type	M/M/3	M/M/2	M/M/3
Mean Number at Station	1.244063	2.080910683	1.270434
Mean Time at Station	1.043239	3.281158924	1.363565
Mean Number in Queue	0.082537	0.823076613	0.088523
Mean Time in Queue	0.069213	1.297818737	0.095013
Mean Number in Service	1.161526	1.257834196	1.181911
Mean Time in Service	0.974026	1.983340025	1.268553
Throughput Rate	1.1925	0.634199977	0.9317
Efficiency	0.387175	0.628917098	0.39397
Probability All Servers Idle	0.306534	0.227809563	0.299902
Prob. All Servers Busy	0.13064	0.485643744	0.136172
Prob. System Full	0	0	0
Critical Wait Time	1	1	1
P(Wait >= Critical Wait)	0.019785	0.334044872	0.032483
P(0)	0.306534	0.227809563	0.299902
P(1)	0.356047	0.286546658	0.354457
P(2)	0.206779	0.180214092	0.209468

The fraction of time each server is busy (utilization factorρ)38.7%, 63.0% and 39.2% respectively.The probabilityP_0 that there are zero customers in the system for Banks A, B and C is given by 0.306, 0.2278, and 0.2999 respectively. The average number of customers in the system$L_s$for Banks A, B and C is given by 1.2441, 2.0809, 1.2704 customers respectively, The average time a customer spends in the waiting lineW_s and being served for Banks A, B and C is given by1.0432, 3.2812 and 1.3636 respectively. The average number of customers on the queue, waiting to be served at any time$L_q$for Banks A, B and C is given by 0.0825, 0.8231 and 0.0885 respectively. The average time a customer spends in the queue waiting for serviceW_q for Banks A, B and C is given by 0.0692, 0.2978 and 0.0950 respectively.

SENSITIVITY ANALYSIS

We next see how the systems perform if the number of tellers (resources) varies for each bank.

Table 3: Sensitivity Analysis for Bank A

c	ρ	P_0	L_s	W_s	L_q	W_q
2	58.1	0.2652	1.7527	1.4698	0.5912	0.4957
3	38.7	0.3065	1.2441	1.0432	0.0825	0.0692
4	29	0.3121	1.1752	0.9855	0.0136	0.0114
5	23.2	0.3129	1.1637	0.9758	0.0022	0.0018

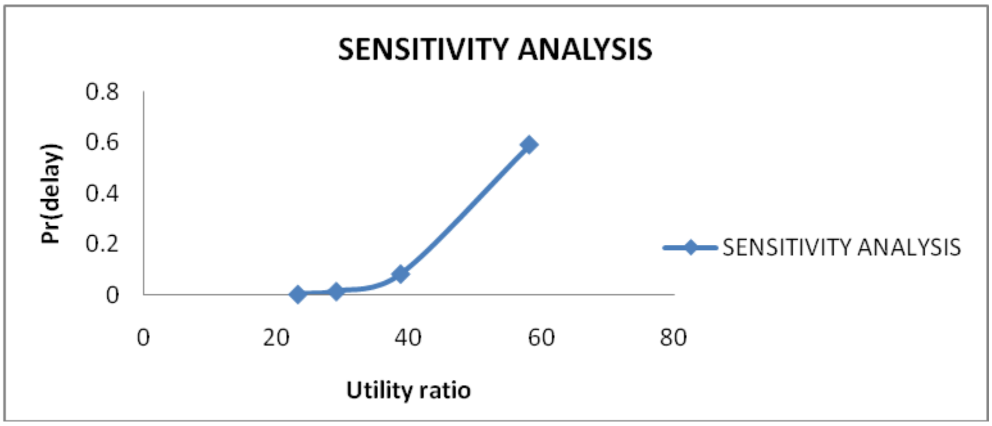

Fig.10: Analysis for Utility ratio Vs $\Pr\ (delay)$ For Bank A

Figure 10 shows that the average number of customers on the queue waiting to be served increases as the utility ratio increases.

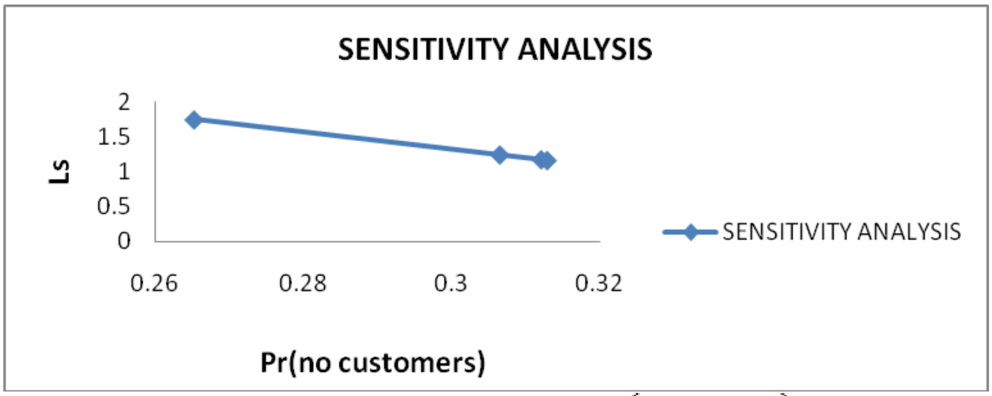

Fig.11: Average no of customers in the system Vs $\Pr(nocustomer)$ for Bank A

Figure 11 shows that the probability that there are zero customers in the system increases as the average number of customers in the system decreases.

Table 4: Sensitivity Analysis for Bank B

c	ρ	P_0	L_s	W_s	L_q	W_q
2	63	0.2278	2.0809	3.2812	0.8231	0.2978
3	42	0.2762	1.3717	2.1629	0.1139	0.1796
4	31.4	0.2831	1.2776	2.0145	0.09198	0.0311
5	25.2	0.2841	1.2612	1.9886	0.0033	0.0053

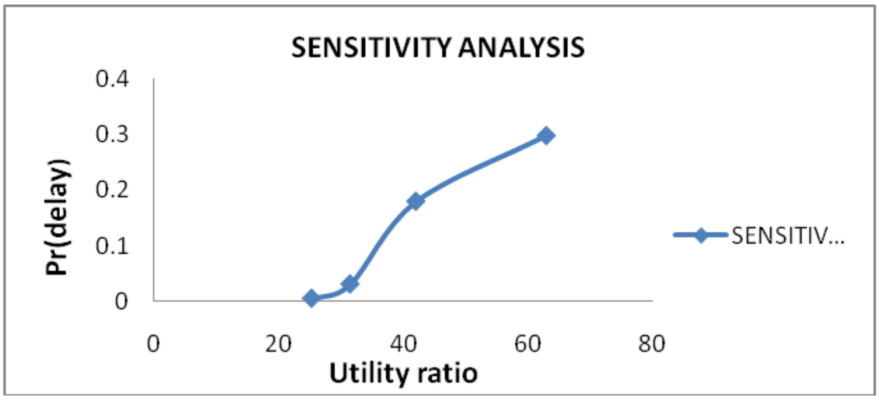

Fig.12: Analysis for Utility ratio and $\Pr\ (delay)$ for Bank B

Figure 12 shows that the average number of customers on the queue waiting to be served increases as the utility ratio increases.

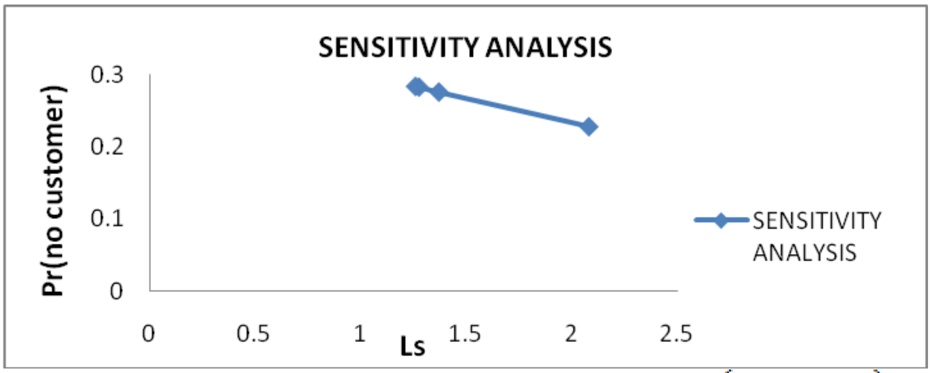

Fig.13: graph of Analysis showing average no of customers in the system and $\Pr(no customer)$ for Bank B

Figure 13 shows that the probability that there are zero customers in the system increases as the average number of customers in the system decreases.

Table 5: Sensitivity Analysis for Bank C

c	ρ	P_0	L_s	W_s	L_q	W_q
2	59.1	0.2571	1.8162	1.9493	0.6343	0.6808
3	39.2	0.2999	1.2704	1.3636	0.0885	0.905
4	29.5	0.3057	1.1967	1.2844	0.0148	0.0159
5	23.6	0.3066	1.1843	1.2711	0.0024	0.0026

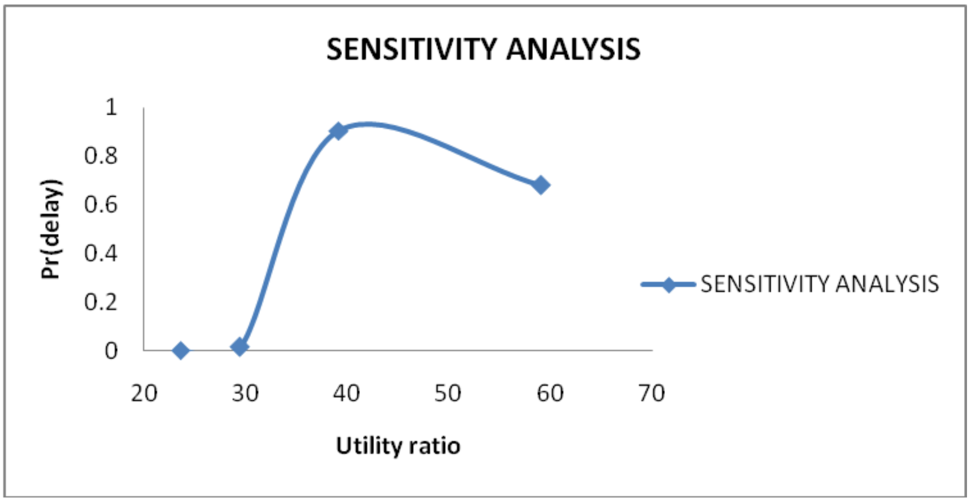

Fig.14: graph of Sensitivity Analysis for Utility ratio and $\Pr(delay)$ For Bank C

Figure 14 shows that the average number of customers on the queue waiting to be served increases as the utility ratio increases.

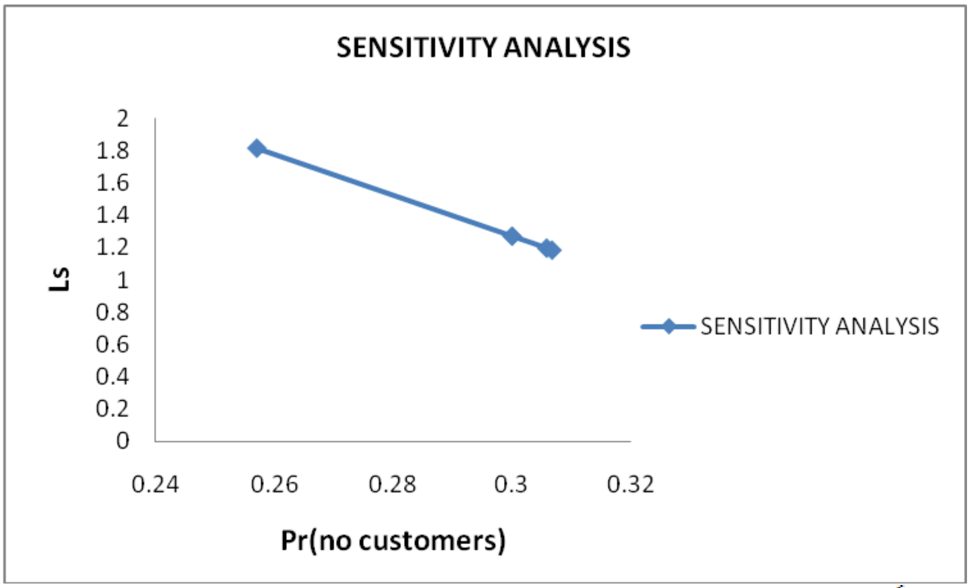

Fig.15: graph of Sensitivity Analysis showing average no of customers in the system and $\Pr(nocustomer)$.

Figure 15 shows that the probability that there are zero customers in the system increases as the average number of customers in the system decreases.

V. SUMMARY AND CONCLUSION

The arrival rate, service time and number of servers (tellers at the counter) were the data used for the study which have been collected using observation method. The data collection covered a period of five consecutive working days for each bank between the hours of 9:00 am to 1:00 pm which made it 3 weeks.It was analyzed using a multi-server queuing model, M/M/c. From the descriptive analysis conducted for each bank in the and the queueing analysis, it was discovered that the utility ratio ρ, is less than unity.Also, the results obtained by the queue analysis for each selected bank shows that, the arrival rates follows a Poisson distribution and the service rate follows an exponential distribution. This is in consonance with the assumption of M/M?c queueing model.

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