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Research Paper

Solving Fractional Differential Equations by Elzaki A domianDecoposition Method

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Abstract:*In this paper, a hybrid method called ElzakiAdomian decomposition method (EADM) has been implemented to solve fractional-order PDEs. Caputo-Fabrizio operator describes fractional-order derivatives. The solutions of three examples are presented to show the validity of the current method. The results of the proposed method are shown and analyzed with the help of figures. It is shown that the proposed method is found to be efficient, reliable, and easy to implement for various related problems of science and engineering.*

Keywords;*Korteweg-de Vries; Elzaki transform; Caputo-Fabrizio fractional derivative; Adomiandecomposition method.*

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I. Introduction

The subject of fractional calculus (FC) can be considered as an old yet novel topic. It has been an ongoing topic for more than300 years; however, since the 1970s, it has been gaining increasing attention [1]. Firstly,there were almost no practical applications of FC, and it was consideredby many as an abstract area containing only mathematical manipulations of little orno use [2]. Recently, FC has been widely used in various applications in almost everyfield of science, engineering, andmathematics, and it has gained considerable importancedue to its frequent applications in fluid flow, polymer rheology, economics, biophysics,control theory, psychology, and so on [3,4].

In recent years a lot of attention has been paid to finding effective numerical methodsto simulate the fractional PDEs [5-35]. The aim of this paper is to find an analytical method that provides us with an approximate solution that is very close to the exact solution with the easiest steps and to use it to solve PDEs with CFFD. This work is arranged as follows: in section ‘‘Preliminaries,’’ the preliminaries are introduced. In section ‘‘ElzakiAdomian decomposition method,’’ the description of the EADM is offered. In section ‘‘Applications,’’ the applications of EADM to the differential equations are illustrated and makes a comparison of ElzakiAdomiandecompositionmethod featuring Caputo–Fabrizio derivative. Eventually, in section ‘‘Conclusion,’’ some conclusions regarding the proposition method are pulled.

2. Preliminaries

Definition 1: Let $\boldsymbol{\varphi} \in \boldsymbol{H}(\boldsymbol{a}, \boldsymbol{b}), \boldsymbol{a} > b$, $a \in (-\infty, \boldsymbol{\tau}), \boldsymbol{0} \leq \boldsymbol{\alpha} \leq \boldsymbol{1}$, then The definition of the new Caputo fraction derivative is [36,37]:

$$ {}^{CF}_{a}\boldsymbol{D}^{\alpha}_{\tau}\boldsymbol{\varphi}(\boldsymbol{\tau}) = \frac{\boldsymbol{\beta}(\boldsymbol{\alpha})}{(\boldsymbol{1}-\boldsymbol{\alpha})}\int_{a}^{\tau} \acute{\boldsymbol{\varphi}}(\boldsymbol{s}) \exp\left(-\frac{\boldsymbol{\alpha}}{\boldsymbol{1}-\boldsymbol{\alpha}}(\boldsymbol{\tau}-\boldsymbol{\alpha})\right) \boldsymbol{ds}, \quad (\boldsymbol{1}) $$

where $\boldsymbol{\beta}(\boldsymbol{\alpha})$ is a normalization function satisfying $\boldsymbol{\beta}(\boldsymbol{0}) = \boldsymbol{\beta}(\boldsymbol{1}) = \boldsymbol{1}.$

Some properties fractional derivative

1. ${}^{CF}_{a}\boldsymbol{D}^{\alpha}_{\tau}[\boldsymbol{\lambda}\boldsymbol{\varphi}_{\boldsymbol{1}}(\boldsymbol{\tau}) + \boldsymbol{\varphi}_{\boldsymbol{2}}(\boldsymbol{\tau})] = \boldsymbol{\lambda}\, {}^{CF}_{a}\boldsymbol{D}^{\alpha}_{\tau}\boldsymbol{\varphi}_{\boldsymbol{1}}(\boldsymbol{\tau}) + {}^{CF}_{a}\boldsymbol{D}^{\alpha}_{\tau}\boldsymbol{\varphi}_{\boldsymbol{2}}(\boldsymbol{\tau})$

2. ${}^{CF}_{a}\boldsymbol{D}^{\alpha}_{\tau}(\boldsymbol{c}) = \boldsymbol{0}$,where c is constant

3. ${}^{CF}_{a}\boldsymbol{D}^{\alpha}_{\tau}\big(\boldsymbol{\varphi}\ \ (\boldsymbol{\tau})\big) = \boldsymbol{\varphi}\ \ (\boldsymbol{\tau})$, **where** $\boldsymbol{\alpha} = \boldsymbol{0}$.

Definition 2: Let $\boldsymbol{\varphi} \in \boldsymbol{H}(\boldsymbol{a}, \boldsymbol{b}), \boldsymbol{a} > b$, $a \in (-\infty, \boldsymbol{\tau}), \boldsymbol{0} \leq \boldsymbol{\alpha} \leq \boldsymbol{1}$,then The fractional integral of order $\boldsymbol{\alpha}$of a functional f is defined by [22-23]:

$$ {}^{CF}_{a}\boldsymbol{I}^{\alpha}_{\tau}\boldsymbol{\varphi}(\boldsymbol{\tau}) = \frac{\boldsymbol{1}-\boldsymbol{\alpha}}{\boldsymbol{\beta}(\boldsymbol{\alpha})}\boldsymbol{\varphi}(\boldsymbol{\tau}) + \frac{\boldsymbol{\alpha}}{\boldsymbol{\beta}(\boldsymbol{\alpha})}\int_{a}^{\tau}\boldsymbol{\varphi}(\boldsymbol{s})\boldsymbol{ds}. \quad (\boldsymbol{2}) $$

where $\boldsymbol{\beta}(\boldsymbol{\alpha})$ is a normalization function satisfying $\boldsymbol{\beta}(\boldsymbol{0}) = \boldsymbol{\beta}(\boldsymbol{1}) = \boldsymbol{1}.$

Some properties fractional integral

1. ${}^{\mathbf{CF}}_{\mathbf{a}}\mathbf{I}^{\alpha}_{\tau}[\boldsymbol{\lambda}\boldsymbol{\varphi}_{\mathbf{1}}(\boldsymbol{\tau}) + \boldsymbol{\varphi}_{\mathbf{2}}(\boldsymbol{\tau})] = \boldsymbol{\lambda}\, {}^{\mathbf{CF}}_{\mathbf{a}}\mathbf{I}^{\alpha}_{\tau}\boldsymbol{\varphi}_{\mathbf{1}}(\boldsymbol{\tau}) + {}^{\mathbf{CF}}_{\mathbf{a}}\mathbf{I}^{\alpha}_{\tau}\ \boldsymbol{\varphi}_{\mathbf{2}}(\boldsymbol{\tau}).$

2. ${}^{\mathbf{CF}}_{\mathbf{a}}\mathbf{I}^{\alpha}_{\tau}\,{}^{\mathbf{CF}}_{\mathbf{a}}\mathbf{D}^{\alpha}_{\tau}\boldsymbol{\varphi}(\boldsymbol{\tau}) = \boldsymbol{\varphi}(\boldsymbol{\tau}) - \boldsymbol{\varphi}(\mathbf{a}).$

Definition 3.1: The Elzaki transform (ET) is defined over the set of functions [24-25]:

$$ A = \left\{\varphi(\tau)/\exists M, k_1, k_2 > 0, |\varphi(\tau)| < Mexp\left(\frac{|\tau|}{k_j}\right), if\ \tau \in (-1)^j x[0,\infty)\right\}, $$

by the following integral

$$ E[\varphi(\tau)] = T(v) = v\int_{0}^{\infty}\varphi(\tau)\exp\left(\frac{-\tau}{v}\right)d\tau, \tau \geq 0, k_1 \leq v \leq k_2 (3) $$

where v is the factor of variable τ.

Some ET properties:

1. $E(k) = kv^2$, $k\ constant$

2. $E\left(\frac{t^n}{n!}\right) = v^{n+2}$, $n = 0,1,2, \dots$

3. $E(\tau^{\alpha}) = \Gamma(\alpha+1)v^{\alpha+2}$

4. $E(e^{a\tau}) = \frac{v^2}{1-av}$

5. $E[\lambda\varphi_1(\tau) \pm M\ \varphi_2(\tau)] = \lambda E[\varphi_1(\tau)] \pm ME[\varphi_2(\tau)],$

Theorem 3.2: The Elzaki transform of the Caputo-Fabrizio fractional derivative of the function $\varphi(\tau)$ of order $\alpha + n$, where $0 < \alpha \leq 1$ and $n \in \mathbb{N}U\{0\}$, is given by [24-25]

$$ E\left[D^{(\alpha+n)}\varphi(\tau)\right] = \frac{1}{1-\alpha(1-v)}\left[\frac{1}{v^n}E\big(\varphi(\tau)\big) - \sum_{k=0}^{n} v^{2-n+k}\ \varphi^{(k)}(0)\right] (4) $$

3. ElzakiAdomianDecomposition Method

We consider FPIDE

$$ {}^{CF}D^{\alpha}_{\tau}\varphi(\mu,\tau) + R\big(\varphi(\mu,\tau)\big) + N\big(\varphi(\mu,\tau)\big) = g(\mu,\tau) (5) $$

With the initial condition

$$ \varphi(\mu, 0) = \varphi_0(\mu) \quad (6\,) $$

where ${}^{CF}D_{\tau}^{\alpha}$ is Caputo-Fabrizio operator, R is a linear operator, N is an nonlinear operator, g is a source term and $0 < \alpha \leq 1$

Taking Elzaki transform to (5) we obtain

$$E\left[{}^{CF}D_{\tau}^{\alpha}\varphi(\mu,\tau)\right] + E\left[R\big(\varphi(\mu,\tau)\big)\right] + N\big(\varphi(\mu,\tau)\big) = E[g(\mu,\tau)] \tag{7}$$

Now by relation (4), we get

$$\frac{1}{\big(1-\alpha(1-v)\big)}\left[E\big(\varphi(\mu,\tau)\big) - v^2\varphi(\mu,0)\right] + E\left[R\big(\varphi(\mu,\tau)\big) + N\big(\varphi(\mu,\tau)\big)\right] = E[g(\mu,\tau)] \tag{8}$$

by substituting (6) in (8), we get

$$\begin{aligned} E\big(\varphi(\mu,\tau)\big) &= v^2\varphi_0(\mu) + \big(1-\alpha(1-v)\big)E[g(\mu,\tau)] \\ &\quad - \big(1-\alpha(1-v)\big)E\left[R\big(\varphi(\mu,\tau)\big) + N\big(\varphi(\mu,\tau)\big)\right] \end{aligned} \tag{9}$$

By using inverse Elzaki transform to both side of (9) we get

$$\begin{aligned} \varphi(\mu,\tau) &= E^{-1}\left[v^2\varphi_0(\mu) + \big(1-\alpha(1-v)\big)E[g(\mu,\tau)]\right] \\ &\quad - E^{-1}\left[\big(1-\alpha(1-v)\big)E\left[R\big(\varphi(\mu,\tau)\big) + N\big(\varphi(\mu,\tau)\big)\right]\right] \end{aligned} \tag{10}$$

let $f = E^{-1}\left[v^2\varphi_0(\mu) + \big(1-\alpha(1-v)\big)E[g(\mu,\tau)]\right]$
then

$$\begin{aligned} \varphi(\mu,\tau) &= f - E^{-1}\left[\big(1-\alpha(1-v)\big)\left(E\left(R\big(\varphi(\mu,\tau)\big)\right)\right)\right] \\ &\quad - E^{-1}\left[\big(1-\alpha(1-v)\big)\left(E\left(N\big(\varphi(\mu,\tau)\big)\right)\right)\right] \end{aligned} \tag{11}$$

Now, we represent solution as an infinite series given below

$$\varphi(\mu,\tau) = \sum_{n=0}^{\infty}\varphi_n(\mu,\tau) \tag{12}$$

and the nonlinear term can be decomposed as

$$N\big(\varphi(\mu,\tau)\big) = \sum_{n=0}^{\infty}A_n(\varphi_0,\varphi_1,\varphi_2) \tag{13}$$

where

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i\varphi_i\right)\right]_{\lambda=0}, n = 0,1,2,\ldots \tag{14}$$

By substituting (13)and (14)in (12), we have

$$\sum_{n=0}^{\infty}\varphi_n(\mu,\tau) = f - E^{-1}\left[\big(1-\alpha(1-v)\big)E\left(R\sum_{n=0}^{\infty}\varphi_n + \sum_{n=0}^{\infty}A_n\right)\right] \tag{15}$$

On comparing both sides of the Eq.(16) we get

$$\begin{aligned} \varphi_0(\mu,\tau) &= f \\ \varphi_1(\mu,\tau) &= -E^{-1}\left[\big(1-\alpha(1-v)\big)E\big(R(\varphi_0(\mu,\tau) + A_0)\big)\right] \\ \varphi_2(\mu,\tau) &= -E^{-1}\left[\big(1-\alpha(1-v)\big)E\big(R(\varphi_1(\mu,\tau) + A_1)\big)\right] \\ &\vdots \end{aligned} \tag{16}$$

In general, the recursive relation is given as

$$\varphi_{n+1}(\mu,\tau) = -E^{-1}\left[\big(1-\alpha(1-v)\big)\big(R(\varphi_n(\mu,\tau) + A_n)\big)\right] \tag{17}$$

Finally, we approximate the analytical solution (6) by truncated seriea:

$$\varphi(\mu,\tau) = \sum_{n=0}^{\infty} \varphi_n(\mu,\tau) \quad (18)$$

$\varphi($

4. Applications of (FEADM)

Example 4.1: We consider the following fractional KdV equation in Caputo-Fabrizio sense

$${}^{CF}D_\tau^\alpha\ \varphi(\mu,\tau) - 6\varphi\varphi_\mu + \varphi_{\mu\mu\mu}\ = 0 \quad (20)$$

Where $0 \le \alpha \le 1, \mu \in \mathbb{R}, \tau > 0$ and subject to the initial condition

$$\varphi(\mu,0) = 6\mu \quad (21)$$

Taking Elzaki transform to both side of (20), we get

$$E\left[{}^{CF}D_\tau^\alpha\ \varphi(\mu,\tau)\right] = E\left[6\varphi\varphi_\mu - \varphi_{\mu\mu\mu}\ \right] \quad (22)$$

By using the relation(9), we get

$$E[\varphi(\mu,\tau)] = 6\mu v^2 + \big(1-\alpha(1-v)\big)E\left[6\varphi\varphi_\mu - \varphi_{\mu\mu\mu}\ \right] \quad (23)$$

Taking the inverse Elzaki transform to both sides of (23) we get

$$\varphi(\mu,\tau) = 6\mu + E^{-1}\left[\big(1-\alpha(1-v)\big)E\left(6\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty}(\varphi_n)_{\mu\mu\mu}\right)\right] \quad (24)$$

Now

$$A_0 = \varphi_0\varphi_{0\mu}$$
$$A_1 = \varphi_0\varphi_{1\mu} + \varphi_1\varphi_{0\mu}$$
$$A_2 = \varphi_0\varphi_{2\mu} + \varphi_2\varphi_{0\mu} + \varphi_1\varphi_{1\mu}$$
$$\vdots$$

By relation (16), we get

$$\varphi_0(\mu,0) = 6\mu$$
$$\varphi_1(\mu,\tau) = E^{-1}\left[\big(1-\alpha(1-v)\big)E\big(6A_0 - \varphi_{0\mu\mu\mu}\ \big)\right]$$
$$= E^{-1}\left[\big(1-\alpha(1-v)\big)E\big(6_\mu^3 - 0\big)\right]$$
$$= 6^3\mu E^{-1}[v^2 - \alpha v^2 + \alpha v^3]$$
$$= 6^3\mu(1-\alpha+\alpha\tau)$$
$$\varphi_2(\mu,0) = E^{-1}\left[\big(1-\alpha(1-v)\big)E\big(6A_1 - \varphi_{1\mu\mu\mu}\ \big)\right]$$
$$= E^{-1}\left[\big(1-\alpha(1-v)\big)E\left(6\big(2(6^4\mu)\big)(1-\alpha+\alpha\tau)\right) - 0\right]$$
$$= 2(6^5\mu)E^{-1}\left[\big(1-\alpha(1-v)\big)E(1-\alpha+\alpha\tau)\right]$$
$$= 2(6^5\mu)E^{-1}[(1-\alpha+\alpha v)(v^2-\alpha v^2+\alpha v^3)]$$
$$= 2(6^5\mu)E^{-1}[v^2 - \alpha v^2 + \alpha v^3 - \alpha v^2 + \alpha^2 v^2 - \alpha^2 v^3 + \alpha v^3 - \alpha^2 v^3 + \alpha^2 v^4]$$
$$= 2(6^5\mu)E^{-1}[v^2 - 2\alpha v^2 + 2\alpha v^3 + \alpha^2 v^2 - 2\alpha^2 v^3 + \alpha^2 v^4]$$
$$= 2(6^5\mu)\left[1 - 2\alpha + 2\alpha\tau + \alpha^2 - 2\alpha^2\tau + \alpha^2\tfrac{\tau^2}{2!}\right]$$
$$= 2(6^5\mu)\left[(1-2\alpha+\alpha^2) + 2(\alpha-\alpha^2)\tau + \tfrac{1}{2}\alpha^2\tau^2\right]$$
$$\vdots$$

Then the approximate solution of (20) is

$$\varphi(\mu,\tau) = 6\mu + 6^3\mu(1-\alpha+\alpha\tau) + 2(6^5\mu)\left[(1-2\alpha+\alpha^2) + 2(\alpha-\alpha^2)\tau + \tfrac{1}{2}\alpha^2\tau^2\right] \quad (25)$$

The equation (25) is approximate solution to the form

$$\varphi(\mu,\tau) = \frac{6\mu}{1-6^2\tau}$$

for $\alpha = 1$, which is exact solution of (20) at $\alpha = 1$

which is the exact solution of (20). Figure 1 shows the graphs of the approximate and the exact solutions among different values of and when is fixed for the problem (20) in the Caputo-Fabrizio fractional operator. In Figures 2,3,4 we plotted the graphs of the approximate solutions among different values of μ and τ when α = 0.9, 0.95, 1. In Figure 5 we plotted the graphs of the exact solution among different values of μ and τ.

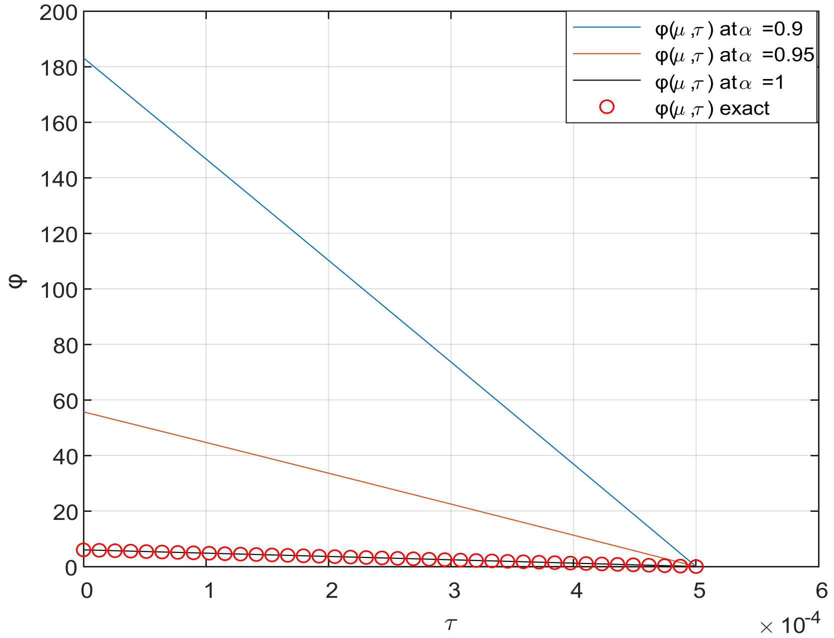

Figure 1: Plots of the exact and approximate solutions $\varphi(\mu , \tau)$ for different values of α with fixed value μ = 1

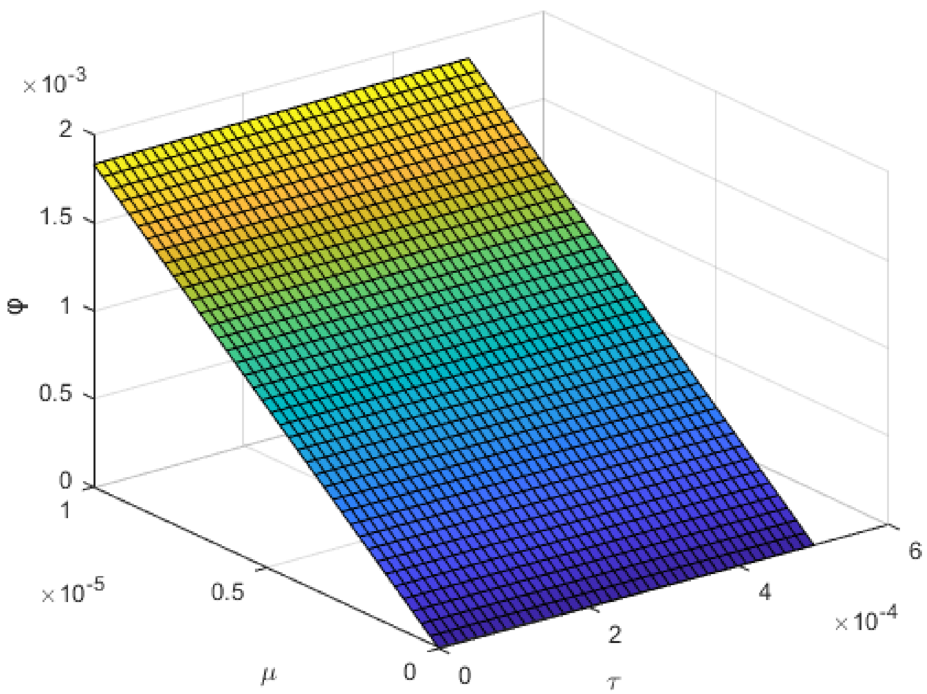

Figure 2. The surface graph of the approximate solution of fractional KdV equation when α =0.9.

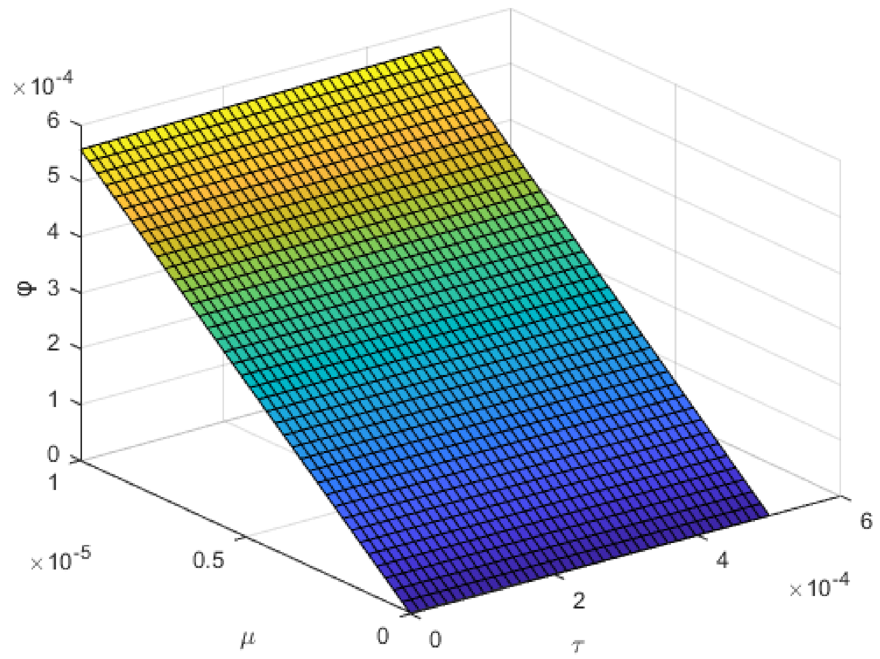

Figure 3. The surface graph of the approximate solution of fractional KdV equation when α =0.95.

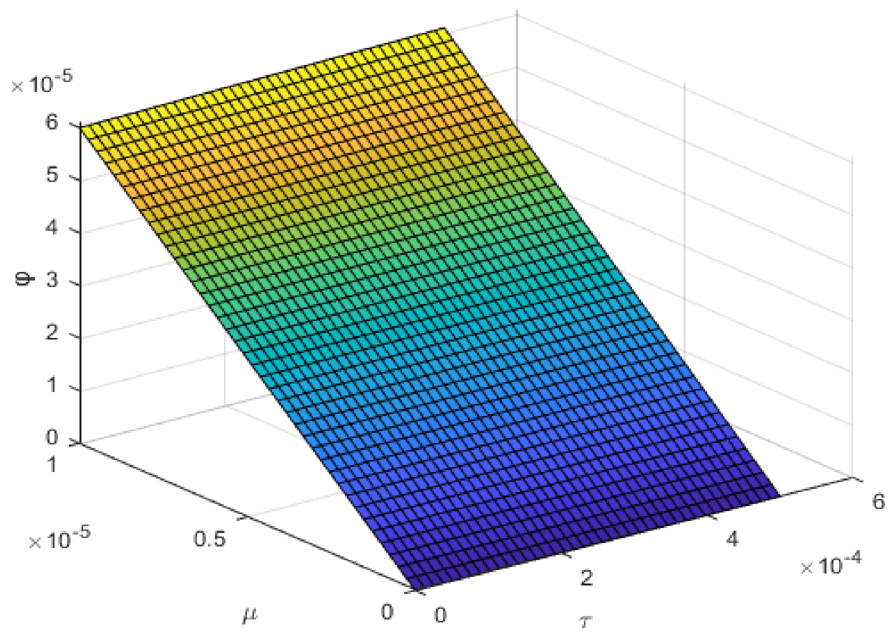

Figure 4.The surface graph of the approximate solution of fractional KdV equation when α =1.

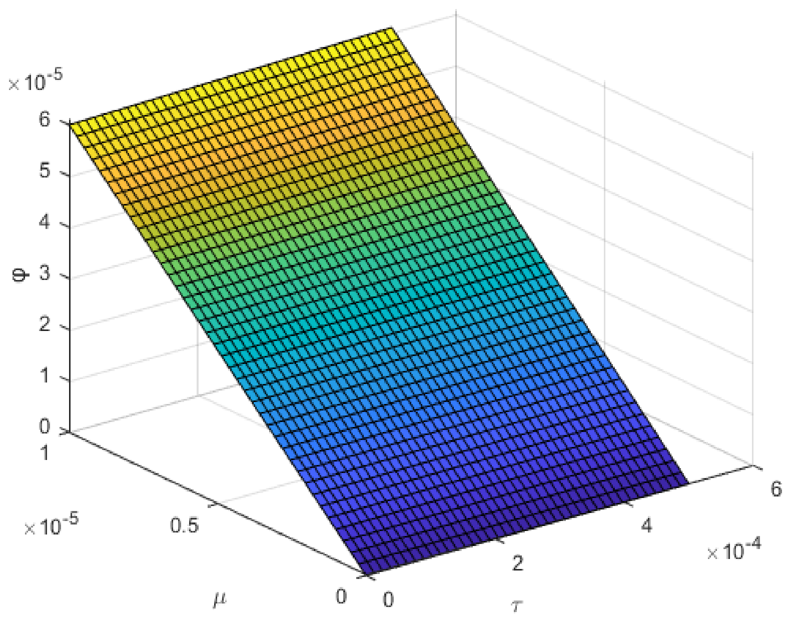

Figure 5. The surface graph of the exact solution of fractional KdV equation.

5. Conclusions

In this work, we have consideredPDEs with CFFD. The EADM has been successfully used to obtain the analytical approximate solutions . The obtained solutions were in the form of infinite power series which can be written in a closed form. The examples show that the results ofEADMare in excellent agreement with the exact solution when $\alpha = 1$. Because of the results, we can say that the proposed technique is a powerful mathematical tool for solving fractional PDEs. Also, we can use them to obtain approximate (or even analytical) solutions to other problems.

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