



A New Class of Sets Weaker Than Nano Regular*-Open Sets.

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Abstract: In this paper we define a new class of sets namely nano semi regular*-open sets in nano topological spaces. Also, we study in details the properties of nano semi regular*-open sets and its relation with other nano open sets. We also define the nano semi regular*-open -interior of a subset and study some of its basic properties.

Keywords: nano regular*-open, nano semi regular*-open, nano semi regular*-interior.

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I. INTRODUCTION

LellisThivagar [8] introduced nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X . He also introduced the weak forms of nano open sets namely nano α -open sets, nano semi-open sets, nano regular-open sets and nano pre-open sets. In 2014, K.Bhuvaneswari and K. Mythili Gnanapriya [3] introduced Nano Generalised Closed Sets in Nano Topological Space. Quite recently the authors C. Reena, B.Santhalakshmi and S.M.JanuPriyadharshini [6] have introduced the concept of nano regular*-open sets and investigated its properties. In this paper we introduce a new class of nano open sets namely nano semi regular*-open sets in nano topological spaces. We further show that the concept of nano semi regular*-open set is weaker than the concept of nano regular*-open sets but stronger than the concept of nano β -open sets. We also define the nano semi regular*-open -interior of a subset and study some of its basic properties.

II. PRELIMINARIES

Throughout this chapter $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U , U/R denotes the family of equivalence classes of U by R .

Definition 2.1[8]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be discernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

1. The **lower approximation** of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x \in U} \{R(x) / R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X .

2. The **upper approximation** of X with respect to R is the set of all objects which can be possibly defined as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x \in U} \{R(x) / R(x) \cap X \neq \phi\}$

3. The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2[8]: If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3[8]: Let U be the universe, R be an **equivalence relation** on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the proposition 2.2, $R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the **Nano topological space**. The elements of $\tau_R(X)$ are called as **Nano-open sets**.

Definition 2.4[8]: If $(U, \tau_R(X))$ is a Nano topological space with respect to X and if $A \subseteq U$, then the **Nano interior** of A is defined as the union of all Nano-open subsets of A and is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest Nano-open subset of A .

The **Nano closure** of A is defined as the intersection of all Nano-closed sets containing A and it is denoted by $Ncl(A)$. That is, $Ncl(A)$ is the smallest Nano-closed set containing A .

Definition 2.5[3]: Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called **Nano generalized-closed set** if $Ncl(A) \subseteq V$ where $A \subseteq V$ and V is Nano-open.

The complement of Nano generalized -closed set is called as **Nano generalized-open set**.

Definition 2.6[3]: For every set $A \subseteq U$, the **Nano generalized closure of A** is defined as the intersection of all Ng- closed sets containing A and is denoted by $Ncl^*(A)$.

Definition 2.7[3]: For every set $A \subseteq U$, the **Nano generalized interior of A** is defined as the union of all Ng-open sets contained in A and is denoted by $Nint^*(A)$.

Definition 2.8: Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

- (i) **nano pre-open**[8] if $A \subseteq Nint(Ncl(A))$
- (ii) **nano semi-open**[8] if $A \subseteq Ncl(Nint(A))$
- (iii) **nano α -open**[8] if $A \subseteq Nint(Ncl(Nint(A)))$
- (iv) **nano β -open**[7] if $A \subseteq Ncl(Nint(Ncl(A)))$
- (v) **nano regular-open** [8] if $A = Nint(Ncl(A))$
- (vi) **nano semi*- open** [1] if $A \subseteq Ncl^*(Nint(A))$
- (vii) **nanob β^* - open**[2] if $A \subseteq Ncl(Nint^*(Ncl(A)))$
- (viii) **nano regular*- open** if [6] $A = Nint(Ncl^*(A))$
- (ix) **nano pre*-open** if [5] $A \subseteq Nint^*(Ncl(A))$

Remark 2.9:[6] Every nano regular-open set is **nano regular*-open**.

III. NANO SEMI REGULAR*-OPEN SETS

Definition 3.1: A subset A of a nano topological space $(U, \tau_R(X))$ is called a nano semi regular*-open set if there is a nano regular*-open set V in U such that $V \subseteq A \subseteq Ncl(V)$

Notation 3.2 The class of all nano semi regular*-open sets in $(U, \tau_R(X))$ is denoted by $NSR^*O(U, \tau_R(X))$.

Theorem 3.3: A subset A of $(U, \tau_R(X))$ is nano semi regular*-open if $A \subseteq Ncl(Nint(Ncl^*(A)))$

Proof: Assume A is nano semi regular*-open set. Then there exist a nano regular*-open set V in U such that $V \subseteq A \subseteq Ncl(V)$. Now $V \subseteq A \Rightarrow V = Nint(Ncl^*(V)) \subseteq Nint(Ncl^*(A)) \Rightarrow Ncl(V) \subseteq Ncl(Nint(Ncl^*(A))) \Rightarrow A \subseteq Ncl(V) \subseteq Ncl(Nint(Ncl^*(A))) \Rightarrow A \subseteq Ncl(Nint(Ncl^*(A)))$.

Example 3.4: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c, d\}\}$. Let $X = \{a, c, d\}$. Then $\tau_R(X) = \{U, \phi, \{c, d\}, \{a, b\}\}$ and $\tau_{R^c} = \{U, \phi, \{a, b\}, \{c, d\}\}$. $NSR^*O(U, \tau_R(X)) = \{U, \phi, \{a, b\}, \{c, d\}\}$

Remark 3.5: In any nano topological space $(U, \tau_R(X))$, ϕ and X are nano semi regular*-open sets.

Theorem 3.6: If $\{A_\alpha\}$ is a collection of nano semi regular*-open sets in U , Then $\bigcup A_\alpha$ is also nano semi regular*-open set.

Proof: Let $\{A_\alpha\}$ be a collection of nano semi regular*-open sets in U . Since each A_α is nano semi regular*-open, $A_\alpha \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}^*(A_\alpha)))$. This implies $\cup A_\alpha \subseteq \cup (\text{Ncl}(\text{Nint}(\text{Ncl}^*(A_\alpha)))) \subseteq (\text{Ncl}(\cup \text{Nint}(\text{Ncl}^*(A_\alpha)))) \subseteq (\text{Ncl}(\cup \text{Nint}(\text{Ncl}^*(A_\alpha)))) \subseteq (\text{Ncl}(\text{Nint}(\cup \text{Ncl}^*(A_\alpha)))) \subseteq (\text{Ncl}(\text{Nint}(\text{Ncl}^*(\cup A_\alpha))))$. Hence $\cup A_\alpha$ is also nano semi regular*-open.

Remark 3.7: The intersection of two nano semi regular*-open sets need not be nano semi regular*-open as seen from the following example.

Example 3.8: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$. Let $X = \{a, c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\text{NSR}^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. The set $\{a, d\}$ and $\{b, c, d\}$ both are in nano semi regular*-open but their intersection $\{a, d\} \cap \{b, c, d\} = \{d\}$ is not in nano semi regular*-open.

Theorem 3.9: Every nano open set is nano semi regular*-open.

Proof: Let A be a nano open set. Then $A = \text{Nint}(A) \Rightarrow \text{Ncl}(A) = \text{Ncl}(\text{Nint}(A))$. Since $A \subseteq \text{Ncl}(A)$, we have $A \subseteq \text{Ncl}(\text{Nint}(A)) \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}^*(A)))$. Thus A is nano semi regular*-open.

Remark 3.10: The converse of the above theorem is not true as shown in the following example.

Example 3.11: Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b\}\}$. Let $X = \{a, c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}\}$ and $\text{NSR}^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Clearly the sets $\{a, b\}, \{a, c\}$ are nano semi regular*-open but not nano open.

Theorem 3.12: Every nano regular*-open set is nano semi regular*-open.

Proof: Let A be a nano regular*-open set. Then $A = \text{Nint}(\text{Ncl}^*(A)) \Rightarrow \text{Ncl}(A) = \text{Ncl}(\text{Nint}(\text{Ncl}^*(A)))$. Since $A \subseteq \text{Ncl}(A)$ we have $A \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}^*(A)))$. Hence A is nano semi regular*-open.

Remark 3.13: The converse of the above theorem is not true as shown in the following example.

Example 3.14: Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b\}\}$. Let $X = \{a, c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}\}$, $\text{NR}^*O(U, \tau_R(X)) = \{U, \phi, \{a\}\}$ and $\text{NSR}^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Clearly the sets $\{a, b\}, \{a, c\}$ are nano semi regular*-open but not nano regular*-open.

Theorem 3.15: Every nano regular open set is nano semi regular*-open.

Proof: Follows from remark 2.9 and theorem 3.12.

Remark 3.16: The converse of the above theorem is not true as shown in the following example.

Example 3.17: Let $U = \{a, b, c, d\}$, $U/R = \{\{b\}, \{c\}, \{d\}\}$. Let $X = \{a, b, c\}$. Then $\tau_R(X) = \{U, \phi, \{b, c\}\}$, $\text{NRO}(U, \tau_R(X)) = \{U, \phi\}$ and $\text{NSR}^*O(U, \tau_R(X)) = \{U, \phi, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Clearly the sets $\{b, c\}, \{a, b, c\}, \{b, c, d\}$ are nano semi regular*-open but not nano regular open.

Theorem 3.18: Every nano α -open set is nano semi regular*-open.

Proof: Let A be a nano α -open set. Then $A \subseteq \text{Nint}(\text{Ncl}(\text{Nint}(A))) \Rightarrow A \subseteq \text{Ncl}(\text{Nint}(A)) \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}^*(A)))$. Hence A is nano semi regular*-open.

Remark 3.19: The converse of the above theorem is not true as shown in the following example.

Example 3.20: Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$. Let $X = \{a, c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$, $\text{N}\alpha O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\text{NSR}^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Clearly the sets $\{a, d\}, \{b, c, d\}$ are nano semi regular*-open but not nano α -open.

Theorem 3.21: Every nano semi*-open set is nano semi regular*-open.

Proof: Let A be a nano semi*-open set. Then $A \subseteq \text{Ncl}^*(\text{Nint}(A)) \subseteq \text{Ncl}(\text{Nint}(A)) \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}^*(A)))$. Hence A is nano semi regular*-open.

Remark 3.22: The converse of the above theorem is not true as shown in the following example.

Example 3.23: Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b\}\}$. Let $X = \{a, c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}\}$, $\text{NS}^*O(U, \tau_R(X)) = \{U, \phi, \{a\}\}$ and $\text{NSR}^*O(U, \tau_R(x)) = \{U, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Clearly the sets $\{a, b\}, \{a, c\}$ are nano semi regular*-open but not nano semi*-open.

Theorem 3.24: Every nano semi regular*-open set is nano β -open.

Proof: Let A be a nano semi regular*-open set. Then $A \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}^*(A)))$. This implies $A \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}(A)))$. Hence A is nano β -open.

Remark 3.25: The converse of the above theorem is not true as shown in the following example.

Example 3.26: Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}\}$, $\text{NSR}^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{b, c\}\}$ and $\text{N}\beta O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Clearly the sets $\{b\}, \{c\}, \{a, b\}, \{a, c\}$ are nano β -open but not nano semi regular*open.

Theorem 3.27: Every nano semi regular*-open set is nano β^* -open.

Proof: Let A be a nano semi regular*-open set. Then $A \subseteq \text{Ncl}(\text{Nint}(\text{Ncl}^*(A)))$. This implies $A \subseteq \text{Ncl}(\text{Nint}^*(\text{Ncl}^*(A))) \subseteq \text{Ncl}(\text{Nint}^*(\text{Ncl}(A)))$. Hence A is nano β^* -open.

Remark 3.28: The converse of the above theorem is not true as shown in the following example.

Example 3.29: Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b\}, \{c\}\}$. Let $X = \{b, c\}$. Then $\tau_R(X) = \{U, \phi, \{b, c\}\}$, $\text{NSR}^*O(U, \tau_R(X)) = \{U, \phi, \{b, c\}\}$ and

$N\beta^*O(U, \tau_R(X)) = \{U, \phi, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$. Clearly the sets $\{b\}, \{c\}, \{a,b\}, \{a,c\}$ are nano β^* -open but not nano semi regular*-open.

Remark 3.30: The concept of nano g-open sets and nano semi regular*-open sets are independent as shown in the following example.

Example 3.31: Let $U = \{a,b,c,d\}$, $U/R = \{\{a\}, \{d\}, \{b,c\}\}$. Let $X = \{a,c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$, $NGO(U, \tau_R(X)) = \{U, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $NSR^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$. Here the subsets $\{b\}, \{c\}, \{b,c\}, \{a,b,c\}$ are nano g-open but not nano semi regular*-open and the subsets $\{a,d\}, \{b,c,d\}$ are nano semi regular*-open but not nano g-open.

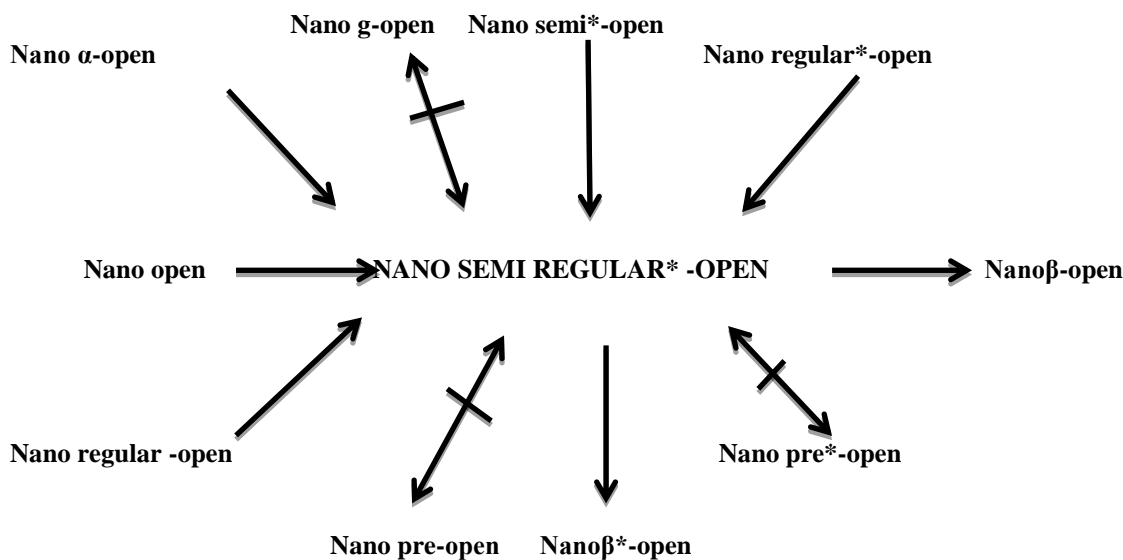
Remark 3.32: The concept of nano pre-open sets and nano semi regular*-open sets are independent as shown in the following example.

Example 3.33: Let $U = \{a,b,c,d\}$, $U/R = \{\{a\}, \{d\}, \{b,c\}\}$. Let $X = \{a,c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a,b,c\}, \{b,c\}\}$, $NPO(U, \tau_R(X)) = \{U, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$ and $NSR^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$. Here the subsets $\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,d\}, \{a,c,d\}$ are nano pre-open but not nano semi regular*-open and the subsets $\{a,d\}, \{b,c,d\}$ are nano semi regular*-open but not nano pre-open.

Remark 3.34: The concept of nano pre*-open sets and nano semi regular*-open sets are independent as shown in the following example.

Example 3.35: Let $U = \{a,b,c,d\}$, $U/R = \{\{a\}, \{d\}, \{b,c\}\}$. Let $X = \{a,c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{a,b,c\}, \{b,c\}\}$, $NP^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$ and $NSR^*O(U, \tau_R(X)) = \{U, \phi, \{a\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$. Here the subsets $\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,d\}, \{a,c,d\}$ are nano pre*-open but not nano semi regular*-open and the subsets $\{a,d\}, \{b,c,d\}$ are nano semi regular*-open but not nano pre*-open.

Diagram: The following diagram shows the relation between nano semi regular*-open sets and other nano open sets that are studied in this section



IV. NANO SEMI REGULAR*-INTERIOR

Definition 4.1: The nano semi regular*-interior of A is defined as the union of all nano semi regular*-open sets of U contained in A. It is denoted by $sr^*Nint(A)$.

Theorem 4.2: If A is any subset of a topological space $(U, \tau_R(X))$, then

- (i) $sr^*Nint(A)$ is the largest nano semi regular*-open set contained in A.
- (ii) A is nano semi regular*-open if and only if $sr^*Nint(A) = A$

Proof: (i) By theorem 3.6, $sr^*Nint(A)$ is nano semi regular*-open and contains every nano semi regular*-open subsets of A. This proves (i)

(ii) If A is nano semi regular*-open, then $sr^*Nint(A) = A$ is obvious from definition 4.1. Conversely, Suppose $sr^*Nint(A) = A$. By (i) $sr^*Nint(A)$ is nano semi regular*-open and hence A is nano semi regular*-open.

Theorem 4.3: (Properties of nano semi regular*-interior)

In any topological space $(U, \tau_R(X))$, the following statements hold:

(i) $sr^*Nint(\phi) = \phi$

(ii) $sr^*Nint(U) = U$

If A and B are subsets of U

(iii) $sr^*Nint(A) \subseteq A$

(iv) $A \subseteq B \Rightarrow sr^*Nint(A) \subseteq sr^*Nint(B)$

(v) $sr^*Nint(sr^*Nint(A)) = sr^*Nint(A)$

(vi) $r^*Nint(A) \subseteq sr^*Nint(A) \subseteq \beta Nint(A)$

(vii) $sr^*Nint(A \cup B) \supseteq sr^*Nint(A) \cup sr^*Nint(B)$

(viii) $sr^*Nint(A \cap B) \subseteq sr^*Nint(A) \cap sr^*Nint(B)$

Proof: (i),(ii),(iii) and (iv) follows from Definition 4.1. By theorem 4.2(i), $sr^*Nint(A)$ is semi regular*-open and by theorem 4.2(ii), $sr^*Nint(sr^*Nint(A)) = sr^*Nint(A)$. Thus (v) is proved. The statements (vi) follows from theorem 3.9 and theorem 3.21. Since $A \subseteq A \cup B$, from statement (iv) we have $sr^*(Nint(A)) \subseteq sr^*(Nint(A \cup B))$. Similarly, $sr^*(Nint(B)) \subseteq sr^*(Nint(A \cup B))$. This proves (vii). The proof for (viii) is similar.

Remark 4.4: In (vi) of theorem 4.3, each of the inclusions may be strict and equality may also hold. This can be seen from the following examples.

Example 4.5: In the space $(U, \tau_{R(X)})$ where $X = \{a, c\}$ and $\tau_{R(X)} = \{U, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Let $A = \{b, c\}$ Then $r^*(Nint(A)) = sr^*Nint(A) = \beta Nint(A) = \{b, c\} = A$

Let $B = \{a, b, c\}$. Then $r^*Nint(B) = \{b, c\}$

$sr^*Nint(B) = \beta Nint(B) = \{a, b, c\}$

Here $r^*Nint(B) \subsetneq sr^*Nint(B) = \beta Nint(B) = B$

Let $C = \{d\}$. Then $r^*Nint(A) = sr^*Nint(A) = \beta Nint(A) = \phi$

Here $r^*Nint(C) = sr^*Nint(C) = \beta Nint(C) \subsetneq C$

Remark 4.6: The inclusions (vii) and (viii) of theorem 4.3 May be strict and equality may also hold. This can be seen from the following example.

Example 4.7: Consider the space $(U, \tau_{R(X)})$ in example 4.5 Let $A = \{a, b\}$ and $B = \{b, c\}$ Then $A \cup B = \{a, b, c\}$; $sr^*Nint(A) = \{a\}$, $sr^*Nint(B) = \{b, c\}$.

Then $sr^*Nint(A \cup B) = \{a, b, c\}$

Here $sr^*Nint(A \cup B) = sr^*Nint(A) \cup sr^*Nint(B)$

Let $C = \{a, c\}$ and $D = \{a, d\}$ Then $C \cap D = \{a\}$

$sr^*Nint(C) = \{a\}$; $sr^*Nint(D) = \{a, d\}$

Then $sr^*Nint(C \cap D) = \{a\}$

Here $sr^*Nint(C \cap D) = sr^*Nint(C) \cap sr^*Nint(D)$

Let $E = \{a, d\}$ and $F = \{b, c, d\}$ Then $E \cap F = \{d\}$

$sr^*Nint(E) = \{a, d\}$; $sr^*Nint(F) = \{b, c, d\}$; $sr^*Nint(E \cap F) = \phi$

$sr^*Nint(E) \cap sr^*Nint(F) = \{d\}$

Here $sr^*Nint(E \cap F) \subsetneq sr^*Nint(E) \cap sr^*Nint(F)$

Let $G = \{a, b, c\}$ and $H = \{c, d\}$ Then $G \cup H = \{a, b, c, d\}$

$sr^*Nint(G) = \{a, b, c\}$; $sr^*Nint(H) = \phi$; $sr^*Nint(G \cup H) = \{a, b, c, d\}$

$sr^*Nint(G) \cup sr^*Nint(H) = \{a, b, c\}$

Here $sr^*Nint(G) \cup sr^*Nint(H) \subsetneq sr^*Nint(G \cup H)$

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