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Research Paper

A New Class of Micro Open Set In Micro Topology

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Abstract: *In this article, we first establish a new type of open sets in micro topology, namely micro α*- open sets and analyze the various properties of micro α*- open sets and compare it with other existing micro open sets.*
Keywords: *$\mathfrak{M}\alpha$*- open, $\mathfrak{M}\alpha$*- closed, $\mathfrak{M}\alpha$*- interior, $\mathfrak{M}\alpha$*- closure.*

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I. Introduction:

Nano topology was introduced by Lellis Thivagar[5] in the year 2013. This topology is based on the concept of lower approximation, upper approximation and boundary region. It has maximum five open sets and minimum three open sets including universal & empty set. Every nano topology is micro topology. Micro topology was introduced by sakkraiveeranan chandrasekar[2] in 2019. Micro topology was the extension of nano topology. And it has maximum nine open sets and minimum four open sets. In this paper, we introduced micro α* open set in micro topological space.

II. Preliminaries:

Definition: 2.1[2]

Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let X ⊆ U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L_R(X). That is L_R(X) = $\cup_{x\in U}\{R(x): R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by x∈ U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U_R(X). That is U_R(X) = $\cup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by B_R(X). That is B_R(X) = U_R(X)- L_R(X).

Definition: 2.2[2]

Let U be an universe, R be an equivalence relation on U and τ_R(X) = $\{U, \emptyset, L_R(\text{X}), U_R(\text{X}), B_R(\text{X})\}$ where X$\subseteq U$ satisfies the following axioms.

1. U,$\emptyset \in \tau_R$(X)
2. The union of the elements of any sub collection of τ_R(X) is in τ_R(X).
3. The intersection of the elements of any finite sub collection of τ_R(X) is in τ_R(X).Then τ_R(X) is called the nano topology on U with respect to X. The space $(U, \tau_R(\text{X}))$ is the nano topological space. The elements are called nano open sets.

Definition: 2.3[2]

$(U, \tau_R(\text{X}))$ is a nano topological space here μ_R(X) = $\{N \cup (N' \cap \mu)\}$ / N,N' $\in \tau_R$(X) and it is called Micro topology of τ_R(X) by μ where $\mu \notin \tau_R$(X). The micro topology μ_R(X) satisfies the following axioms.

1. U,$\emptyset \in \mu_R$(X)
2. The union of the elements of any subcollection of μ_R(X) is in μ_R(X).
3. The intersection of the elements of any finite subcollection of μ_R(X) is in μ_R(X).

Then μ_R(X) is called the micro topology on U with respect to X. The triplet $(U, \tau_R(\text{X}), \mu_R(\text{X}))$ is called micro topological spaces and the elements of μ_R(X) are called micro open sets and the complement of a micro open set is called a micro closed set.

Example: 1

Let U = $\{p, q, r, s, t\}$ and U/R = $\{\{p\}, \{q, r, s\}, \{t\}\}$ and let X = $\{q, r\} \subseteq U$. Then Nτ_R(X) = $\{U, \phi, \{q, r, s\}\}$ and $\mu = \{p\}$. $\mathfrak{M}$ –open, μ_R(X) = $\{u, \phi, \{p\}, \{p, q, r, s\}, \{q, r, s\}\}$.

Definition: 2.4[2]

The micro closure of a set A is denoted by $\mathfrak{M}$-cl(A) and is defined as $\mathfrak{M}$-cl(A) = $\cap \{B : B \text{ is micro closed and } A \subseteq B\}$.

Definition: 2.5[2]

The micro interior of a set A is denoted by $\mathfrak{M}$-int(A) and is defined as $\mathfrak{M}$-int(A) = $\cup \{B : B \text{ is micro open and } A \supseteq B\}$.

Definition: 2.6[3]

Let $(U, \tau_R(\text{X}), \mu_R(\text{X}))$ be a micro topological space and A$\subseteq U$. Then A is said to be $\mathfrak{M}$ pre- open if A $\subseteq$ $\mathfrak{M}$ -int ($\mathfrak{M}$ -cl(A)) and $\mathfrak{M}$ pre closed set if A$\subseteq$ $\mathfrak{M}$ -cl ($\mathfrak{M}$ -int(A)).

Definition: 2.7[3]

Let $(U, \tau_R(\text{X}), \mu_R(\text{X}))$ be a micro topological space and A$\subseteq U$. Then A is said to be $\mathfrak{M}$ semi- open if A$\subseteq$ $\mathfrak{M}$ -cl ($\mathfrak{M}$ -int(A)) and $\mathfrak{M}$ semi closed if A $\subseteq$ $\mathfrak{M}$ -int ($\mathfrak{M}$ -cl(A)).

Definition: 2.8[3]

Let $(U, \tau_R(\text{X}), \mu_R(\text{X}))$ be a micro topological space. An set A is called an $\mathfrak{M}\alpha -$ open set if A$\subseteq$ $\mathfrak{M}$ -int ($\mathfrak{M}$ -cl($\mathfrak{M}$ -int(A))), The complement of a $\mathfrak{M}\alpha -$ open set is called an $\mathfrak{M}\alpha -$ closed set.

Definition: 2.9[4]

Let $(U, \tau_R(\text{X}), \mu_R(\text{X}))$ be a micro topological space. The set A is called $\mathfrak{M}\beta -$ open set if A $\subseteq$ $\mathfrak{M}$ -cl($\mathfrak{M}$ -int($\mathfrak{M}$ -cl(A))), The complement of a $\mathfrak{M}\beta -$ open set is called an $\mathfrak{M}\beta -$ closed set.

Definition: 2.10[1]

A subset A of a micro topology$(U, \tau_R(\text{X}), \mu_R(\text{X}))$ is called $\mathfrak{M}$ gs- closed set if $\mathfrak{M}$ - $scl(A)$ $\subseteq$ U whenever A$\subseteq U$ and U is micro open in U, the complement of $\mathfrak{M}$ gs-closed set is called $\mathfrak{M}gs$-open set.

Definition: 2.11[1]

A subset A of a micro topology$(U, \tau_R(\text{X}), \mu_R(\text{X}))$ is called $\mathfrak{M}$ αg- closed set if $\mathfrak{M}$ -$\alpha cl(A)$ $\subseteq$ U whenever A$\subseteq U$ and U is micro open in U, the complement of $\mathfrak{M}$ αg-closed set is called $\mathfrak{M}$ αg-open set.

Definition: 2.12

Let $(U, \tau_R(\text{X}), \mu_R(\text{X}))$ be a micro topological space. The set A is called $\mathfrak{M}b -$ open set if A $\subseteq$ $\mathfrak{M}$ -int($\mathfrak{M}$ -cl(A)$\cup$($\mathfrak{M}$ -cl($\mathfrak{M}$ -int(A)).

III. Micro α*- open set

In this section, we introduce the concept of $\mathfrak{M}$ α*- open sets and some of their properties are discussed below. Hereafter, $\mathfrak{M}g -$ open set is denoted by $\mathfrak{M} -$ int***.**

Definition: 3.1.1

Let $(U, \tau_R(\text{X}), \mu_R(\text{X}))$ be a micro topological space. A set A is called an $\mathfrak{M}\alpha$*-open set if A $\subseteq$ $\mathfrak{M}$ -int*($\mathfrak{M}$ -cl ($\mathfrak{M}$ -int*(A))).

Example: 3.1.2

Let U = {a,b,c,d} with U/R = {{a,b},{c,d}} and X = {a,b}⊆ U. Then τ_R(X) = { U,ϕ, {a,b}} and then μ = {a,b,c}, 𝔐 topology, μ_R(X) = {U, ϕ, {a,b},{a,b,c}}, 𝔐 α*- open set = {U, ϕ, {a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

Theorem: 3.1.3

Every micro open set is micro α*- open.

Proof:

Let A be an micro open set in $(U, \tau_R(X), \mu_R(X))$. Since A is micro open, we have A = 𝔐 -int(A). Also, we have A ⊆ 𝔐 -cl(A) ⊆ 𝔐 -cl (𝔐 -int(A))

𝔐 -int(A) ⊆ 𝔐-int(𝔐 -cl(𝔐 -int(A)))

A ⊆ 𝔐 -int*(𝔐 -cl(𝔐 -int(A)))

⊆ 𝔐 -int*(𝔐 -cl (𝔐 -int*(A))).

Hence A is micro α*- open.

The converse of the above theorem need not be true as shown in the following example

Example:

Let U = {a,b,c,d} with U/R = {{a},{b},{c,d}} and X = {b,d}⊆ U,Then τ_R(X) = { U,ϕ, {b},{c,d},{b,c,d}} and then μ = {a,b}, 𝔐 topology, μ_R(X) = {U, ϕ,{b},{c,d}, {b,c,d}, {a,b}}, 𝔐 open, μ_R(X) = {U, ϕ, {b},{a,b},{c,d},{b,c,d}}, 𝔐α*-open={U,ϕ,{b},{c},{d},{a,b},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{b,c,d}}

Here, {{c},{d},{b,c},{b,d},{a,b,c},{a,b,d}} are 𝔐α*- open but not in 𝔐 open.

Theorem: 3.1.4

Every micro $\alpha -$ open set is micro α*- open.

Proof:

Let A be micro $\alpha - open$. Then we have A ⊆ 𝔐-int(𝔐 -cl(𝔐 -int(A))) ⊆ 𝔐 -int*(𝔐 -cl (𝔐 -int*(A))). Hence, A is micro α*- open.

The converse of the above theorem need not be true as shown in the following example

Example:

Let U = {a,b,c} with U/R = {{a,c},{b}} and X = {a,c}⊆ U, Then τ_R(X) = { U,ϕ, {a,c}} and then μ = {b}, 𝔐 topology, μ_R(X) = {U, ϕ,{a,c},{b}}, 𝔐 $\alpha - open$ = {U, ϕ, {a},{b,c}}, 𝔐 α*- open = {U,ϕ,{a},{b},{c},{a,b},{a,c},{b,c}}

Here, {{b},{c},{a,b},{a,c}} are in 𝔐 α*- open but not in 𝔐 α -open.

Theorem: 3.1.5

If A is micro- open, then every micro pre- open set is micro α*- open.

Proof:

Since A is micro open, we have A = 𝔐 -int(A)

A ⊆ 𝔐-int(𝔐 -cl(A)).

A ⊆ 𝔐-int(𝔐 -cl(𝔐 -int(A)))

A ⊆ 𝔐 -int*(𝔐 -cl (𝔐 -int*(A))).

Hence, A is micro α*- open.

The converse of the above theorem need not be true is shown in the following example.

Example:

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}} and X = {a,c}⊆ U **.** Then τ_R(X) = { U,ϕ, {a,c}} and then μ = {a,b}**.** 𝔐 topology, μ_R(X) = {U, ϕ,{a,c},{a,b},{a},{a,b,c}}**,** 𝔐 pre- open = {U, ϕ, {a},{a,b},{a,c},{a,d},{a,b,c},{a,c,d},{a,b,d}}, 𝔐 α*- open = {U, ϕ, {a}, {b}, {c}, {a,b},{a,c},{a,d},{b,c},{a,b,c},{a,b,d},{a,c,d}}. Here, {{b},{c},{b,c}} are in 𝔐 α*-open but not in 𝔐 pre-open.

Theorem: 3.1.6

Every micro g-open set is micro α*- open.

Proof:

Let A be a micro g- open set, implies A=$𝔐 - int$*(A). Then we have 𝔐 -cl(A) ⊆ U whenever A ⊆ U, U is micro open.

A ⊆ 𝔐 -cl(A) ⊆ U

A ⊆ 𝔐 -cl(A)

𝔐 -int*(A) ⊆ 𝔐 - int*(𝔐 -cl(A))

A ⊆ 𝔐 -int*(𝔐 -cl(𝔐 -int*(A)))

Hence, A is micro α*- open.

The converse of the above theorem need not be true as shown in the following example.

Example:

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}} and X = {a,c}⊆ U, Then τ_R(X) = { U,ϕ, {a,c}} and then μ = {a,b}, 𝔐 topology, μ_R(X) = {U, ϕ,{a,c},{a,b},{a},{a,b,c}},𝔐 g- open = {U, ϕ,

{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}, $\mathfrak{M}\alpha$*- open = {U, ϕ,{a}, {b},{c},{a,b},{a,c},{a,d},{b,c},{a,b,c},{a,b,d},{a,c,d}}
Here, {{a,d}} are in $\mathfrak{M}$ α*-open but not in $\mathfrak{M}$ g-open.

Theorem:3.1.7

Every micro $\alpha g -$ open set is micro α*-open.

Proof:

Let A be a micro αg- open set. Then we have $\mathfrak{M} - \alpha$cl(A) ⊆ U whenever A⊆ U, U is micro open.

$$A \subseteq \mathfrak{M} - \alpha cl(A) \subseteq \mathfrak{M} - cl(A) \subseteq U$$
$$A \subseteq \mathfrak{M} - cl(A)$$
$$\mathfrak{M} - int(A) \subseteq \mathfrak{M} - int(\mathfrak{M} - cl(A))$$
$$\mathfrak{M} - int(A) \subseteq A \subseteq \mathfrak{M} - int(\mathfrak{M} - cl(\mathfrak{M} - int(A)))$$
$$A \subseteq \mathfrak{M} -int^*(\mathfrak{M} -cl(\mathfrak{M} -int^*(A)))$$

Hence A is micro α*- open.

The converse of the above theorem need not be as shown in the following example.

Example:

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}} and X = {b,d}⊆ U, Then τ_R(X) = { U,ϕ, {b,d}} and then μ = {a}, $\mathfrak{M}$ topology, μ_R(X) = {U, ϕ,{a},{b,d},{a,b,d}}, $\mathfrak{M}$ $\alpha g - open$ = {U,ϕ, {a},{b},{d},{a,b},{a,d},{b,d},{a,b,d}}, $\mathfrak{M}$ α*- open = {U,ϕ, {a}, {b}, {d}, {a,b}, {a,d}, {b,d}, {a,b,c}, {a,b,d}, {a,c,d}}
Here, {{b,d},{a,b,c},{a,c,d}} are in $\mathfrak{M}$ α*- open but not in $\mathfrak{M}$ αg -open.

Theorem: 3.1.8

An arbitrary union of micro α*- open sets is micro α*- open.

Proof:

Let $\{A_i : i\epsilon I\}$ be a family of micro α*- open sets. Then for each i , $A_i \subseteq \mathfrak{M} -$ int*($\mathfrak{M}$ -cl ($\mathfrak{M} -$ int*(A_i)))

$$\cup_{i\epsilon I} A_i \subseteq \cup_{i\epsilon I} [\mathfrak{M}\text{- int*}(\mathfrak{M}\text{- cl } (\mathfrak{M}\text{- int*}(A_i)))]$$
$$\subseteq [\cup_{i\epsilon I} \mathfrak{M}\text{- int*}(\mathfrak{M}\text{- cl } (\mathfrak{M} - \text{int*}(A_i)))]$$
$$\subseteq [\mathfrak{M} - \text{int*}(\cup_{i\epsilon I} \mathfrak{M}\text{- cl } (\mathfrak{M}\text{- int*}(A_i)))]$$
$$\subseteq [\mathfrak{M}\text{- int*}(\mathfrak{M}\text{- cl } (\cup_{i\epsilon I} \mathfrak{M}\text{- int*}(A_i)))]$$
$$\subseteq [\mathfrak{M} - \text{int*}(\mathfrak{M}\text{- cl } (\mathfrak{M}\text{- int*}(\cup_{i\epsilon I} A_i)))].$$

Then $\cup_{i\epsilon I} A_i$ is a micro α*- open set.

Remark: 3.1.9

The intersection of two micro α*- open sets need not be micro α*- open.

Example: 3.1.10

Let U = {a,b,c,d} with U/R = {{a,b},{c,d}} and X = {a,b}⊆ U, Then τ_R(X) = { U,ϕ, {a,b}} and then μ = {a,b,c}, $\mathfrak{M}$ topology, μ_R(X) = {U, ϕ, {a,b},{a,b,c}}, $\mathfrak{M}$ α*- open set = {U, ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.
The intersection of {b,d} and {a,c,d} is {d} which is not $\mathfrak{M}$ α*- open set.

Remark: 3.1.11

The concept of $\mathfrak{M}$ α*- open set and $\mathfrak{M}$ b-open set are independent as seen in the following example

Example: 3.1.12

Let U = {a,b,c} with U/R = {{a,b},{c}} and X = {c}⊆ U , Then τ_R(X) = { U,ϕ, {c}} and then μ = {a}, $\mathfrak{M}$ topology, μ_R(X) = {U, ϕ, {a},{c},{a,c}}, $\mathfrak{M}$ b-open = {U,ϕ,{a},{c},{a,b},{a,c},{b,c}}, $\mathfrak{M}$ α*- open set = {U, ϕ,{a},{c},{a,c}}. The set {a,b} and {b,c} is $\mathfrak{M}$ b-open but not $\mathfrak{M}$ α*- open.

Example: 3.1.13

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}} and X = {a,c}⊆ U , Then τ_R(X) = { U,ϕ, {a,c}} and then μ = {a,b}, $\mathfrak{M}$ topology, μ_R(X) = {U, ϕ, {a},{a,b},{a,c}, {a,b,c}}, $\mathfrak{M}$ b-open = {U,ϕ,{a},{a,c},{a,b},{a,d},{a,b,c}, {a,c,d},{a,b,d}}, $\mathfrak{M}$ α*- open set = {U, ϕ,{a},{b},{c}, {a,b},{a,c},{a,d},{b,c}, {a,b,c}, {a,c,d}, {a,b,d}}. The set {b}, {c} and {b,c} is $\mathfrak{M}$ α*-open but not $\mathfrak{M}$ b- open.

Remark: 3.1.14

The concept of $\mathfrak{M}$ α*- open set and $\mathfrak{M}$ β-open are independent as shown in the following example

Example: 3.1.15

Let U = {a,b,c,d} with U/R = {{a,b},{c,d}} and X = {a,b}⊆ U. Then τ_R(X) = { U,ϕ, {a,b}} and then μ = {a,b,c}, $\mathfrak{M}$ topology, μ_R(X) = {U, ϕ, {a,b},{a,b,c}}, $\mathfrak{M}$ β- open = { {U, ϕ, {a},{b},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}, $\mathfrak{M}\alpha$*- open set = {U, ϕ, {a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. The set {c} is $\mathfrak{M}$ α*- open but not $\mathfrak{M}$ $\beta -$ open.

Example: 3.1.16

Let U = {a,b,c} with U/R = {{a,b},{c}} and X = {c}⊆ U , Then τ_R(X) = { U,ϕ, {c}} and then μ = {a}, 𝔐 topology, μ_R(X) = {U, ϕ, {a},{c},{a,c}}, 𝔐β-open = {U,ϕ,{a},{c},{a,b},{a,c},{b,c}}, 𝔐 α*- open set = {U, ϕ,{a},{c},{a,c}}. The set {a,b} and {b,c} is 𝔐 β-open but not 𝔐 α*- open.

Remark: 3.1.17

The concept of 𝔐 α*- open set and 𝔐 semi- open are independent as shown in the given example

Example: 3.1.18

Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}} and X = {a,c}⊆ U. Then τ_R(X) = { U,ϕ,{a,c}} and then μ = {a,b}, 𝔐 topology, μ_R(X) = {U, ϕ, {a,c},{a,b},{a},{a,b,c}}, 𝔐$semi$- open = {U, ϕ,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}, 𝔐 α*- open set = {U, ϕ, {a},{b},{c},{a,b},{a,c},{a,d},{b,c},{a,b,c},{a,b,d},{a,c,d}}. The set {b}, {c}, {b,c} is 𝔐 α*- open but not 𝔐 semi- open.

Example: 3.1.19

Let U = {a,b,c} with U/R = {{a,b},{c}} and X = {c}⊆ U , Then τ_R(X) = { U,ϕ, {c}} and then μ = {a}, 𝔐 topology, μ_R(X) = {U, ϕ, {a},{c},{a,c}}, 𝔐 $semi$-open = {U,ϕ,{a},{c},{a,b},{a,c},{b,c}}, 𝔐 α*- open set = {U, ϕ,{a},{c},{a,c}}. The set {a,b} and {b,c} is 𝔐 $semi$-open but not 𝔐 α*- open.

Remark: 3.1.20

The concept of 𝔐 α* -open set and 𝔐 gs- open set are independent as shown in the example

Example: 3.1.21

Let U = {a,b,c,d} with U/R = {{a,b},{c,d}} and X = {a,b}⊆ U. Then τ_R(X) = {U ,ϕ ,{a,b}} and then μ = {a,b,c}, 𝔐 topology, μ_R(X) = {U, ϕ, {a,b},{a,b,c}}, 𝔐 gs-open = {U, ϕ,{b},{c},{d},{b,c},{b,d},{c,d},{a,b,d},{b,c,d}}, 𝔐 α*- open set = {U, ϕ,{a}, {b},{c}, {a,b},{a,c},{b,c},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. The set {a} is 𝔐α*-open but not in 𝔐 $gs - open$. And the set {d} is 𝔐 gs- open but not in 𝔐 α*- open.

3.2MICRO α* CLOSED SET

In this section, we discuss about 𝔐 α*- closed sets and its properties are verified.

Definition: 3.2.1

Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then the complement of a 𝔐 α*- open set is called an 𝔐 α*- closed set.

Example: 3.2.2

Let U = {a,b,c} with U/R = {{a,b},{c}} and X = {c}⊆ U. Then τ_R(X) = { U,ϕ, {c}} and then μ = {a}, 𝔐 topology, μ_R(X) = {U, ϕ, {a},{c},{a,c}}, 𝔐 α*- open set = {U, ϕ, {a}, {c},{a,c}}, 𝔐 α*- closed set = {U, ϕ, {b,c}, {a,b}, {b}}.

Lemma: 3.2.3

1. Every 𝔐 closed set is 𝔐 α*- closed.
2. Every 𝔐 α closed set is 𝔐 α*- closed.
3. Every 𝔐 pre closed set is 𝔐 α*- closed.
4. Every 𝔐 g-closed set is 𝔐 α* - closed.
5. Every micro $\alpha g -$ closed set is micro α*- closed.

Remark: 3.2.4

1. The concept of 𝔐 α*- closed set and 𝔐 b-closed set are independent.
2. The concept of 𝔐 α*- closed set and 𝔐 β-closed set are independent.
3. The concept of 𝔐 α*- closed set and 𝔐 semi- open set are independent.
4. The concept of 𝔐 α*- closed set and 𝔐 gs-closed set are independent.

3.3 MICRO α* INTERIOR AND MICRO α* CLOSURE

Definition: 3.3.1

Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. Then

1. The union of all 𝔐 α*- open sets contained in A is called the 𝔐 α*- interior of A and denoted by 𝔐 α*- int(A).
2. The intersection of all 𝔐 α*- open sets containing A is called the 𝔐 α*- closure of A and denoted by 𝔐 α*- cl(A).

Theorem: 3.3.2

Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space. For any subsets A,B of U, we have the following

1. A is 𝔐 α*-open if and only if A = 𝔐 α*- int(A).
2. A is 𝔐 α*-closed if and only if A = 𝔐α*- cl(A).
3. If A⊆B, then 𝔐α*- int(A) ⊆ 𝔐α*- int(B) and 𝔐α*- cl(A) ⊆ 𝔐α*- cl(B).
4. 𝔐α*- int(A) ∪ 𝔐α*- int(B) ⊆ 𝔐α*- int(A∪ B).
5. 𝔐α*- int(A∩ B) ⊆ 𝔐α*- int(A) ∩ 𝔐α*- int(B).

6. $\mathfrak{M}\alpha$*- cl(A) ∪ $\mathfrak{M}\alpha$*- cl(B) ⊆ $\mathfrak{M}\alpha$*- cl(A∪ B).
7. $\mathfrak{M}\alpha$*- cl(A∩ B) ⊆ $\mathfrak{M}\alpha$*- cl(A) ∩ $\mathfrak{M}\alpha$*- cl(B).
8. $\mathfrak{M}\alpha$*- int(U\A) = U\ $\mathfrak{M}\alpha$*- cl(A).
9. $\mathfrak{M}\alpha$*- cl(U\A) = U\ $\mathfrak{M}\alpha$*- int(A).
10. U\ $\mathfrak{M}\alpha$*- cl(U\A) = $\mathfrak{M}\alpha$*- int(A).
11. U\ $\mathfrak{M}\alpha$*- int(U\A) = $\mathfrak{M}\alpha$*- cl(A).
12. X ∈ $\mathfrak{M}\alpha$*- int(A) if and only if there exist a $\mathfrak{M}\alpha$*- open set L such that x ∈ L ⊆A.

Proposition: 3.3.3
Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and B and C are subset of X such that B ⊆ C, then

i. If B is $\mathfrak{M}$ α*- open set then it is not necessary that C is $\mathfrak{M}$ α*- open set.
ii. If C is $\mathfrak{M}$ α*- open set then it is not necessary that B is $\mathfrak{M}$ α*- open set.

The proposition above needs the following example:

Example: 3.3.4
Let U = {a,b,c,d} with U\R = {{a},{b},{c,d}} and X = {b,d}. Then τ_R(X) = { U,ϕ, {b},{c,d},{b,c,d}}. Let μ = {a,b}, Then μ_R(X) = {U, ϕ, {b},{c,d},{a,b},{b,c,d}}, $\mathfrak{M}$ α*- open set = {U, ϕ,{b},{c},{d},{a,b},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{b,c,d}}. We have:

i. The set B = {c,d} is $\mathfrak{M}$ α*- open but the set C = {a,c,d} is not $\mathfrak{M}$ α* - open.
ii. The set C = {a,b,d} is $\mathfrak{M}$ α*- open but the set B = {a,d} is not $\mathfrak{M}$ α*- open.

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