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**Research Paper**

# An Alternative Proposed System for Solving Assignment Problems

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***ABSTRACT:*** *In this paper, we proposed a new technique for solving balanced assignment problems. At first, we examine some examples by the new method and then compared the obtained results with the Hungarian method. The newlyproposed method is very logical and different from the existing method.*
***KEYWORDS:*** *Assignment problems, Hungarian method, proposed technique, optimization.*

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## I. INTRODUCTION

An assignment problem is a special type of linear programming problem. The aim of the assignment problem is to assign a number of origins to an equal number of destinations at a minimum cost (or maximum profit) which is to be made on a one-to-one basis.

There are two types of assignment problems namely balanced and unbalanced assignment problems. We discussed the balanced assignment problem in this paper. In the practical field, we are faced with types of problems that consists of jobs to machines, drivers to trucks, men to offices, etc. in which the assigned possess a varying degree of efficiency, called cost or effectiveness.

The Hungarian method is the most popular method for solving assignment problems. The method was developed and published by H.W.Kuhn [1]. He gave the name Hungarian method. Because the algorithm was largely based on the earlier works of two Hungarian mathematicians: D. Konig and J. Egervary. James Munkres [2] reviewed the algorithm and observed that it is strongly polynomial. Since then the algorithm has been known as Kuhn-Munkres algorithm. Besides the Hungarian method, the simplex method for linear programming was modified to solve the assignment problem [3], [4], [5]. Also the signature method for the assignment problem was presented by Balinski [6]. Kore [7] proposed a new approach to solve an unbalanced assignment problem. Basirzadeh [8] developed a Hungarian-like method, called Ones Assignment Method which can be applied for assignment problem.We attempt to propose a new approach for solving assignment problems that is completely different from the preceding method and an easy system to solve assignment problems.

## II. MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEMS

Here, we have a cost matrix with $n$ origins (rows) and $n$ destinations (columns). Its represented below,

| Origin *Destinations* ↓ | $A_1$ | $A_2$ | … | | $A_n$ | *Variables* ↓ |
|---|---|---|---|---|---|---|
| $R_1$ | $C_{11}$ | $C_{12}$ | … | | $C_{1n}$ | 1 |
| $R_2$ | $C_{21}$ | $C_{22}$ | … | | $C_{2n}$ | 1 |
| … | … | … | … | … | | … |
| $R_n$ | $C_{n1}$ | $C_{n2}$ | … | | $C_{nn}$ | 1 |
| *Required* → | 1 | 1 | … | 1 | | |

Here $C_{ij}$ represents the cost of assignments of $i-th$ origin to $j-th$ destinations.
Let $x_{ij}$ denotes the assignment of origin $i$ to destination $j$ such that,

$$x_{ij} = \begin{cases} 1 & if\ origin\ i\ is\ assign\ to\ j \\ 0 & otherwise \end{cases}$$

Then the mathematical formulation of the assignment problem is,

Minimize, $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}\, x_{ij}$
Subject to the constraints,

$\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1$ ; $x_{ij} = 0\ or\ 1$ for all $i = 1,2,\dots\dots,n$ and $j = 1,2,\dots\dots,n$.

## III. PROPOSED ALGORITHM

The proposed algorithm of the new technique is discussed as follows:
Step 1:- Find smallest number from the all numbers of the given cost matrix. Then subtract this number from every number of the matrix.
Step 2:- Now add 1 to all numbers of the cost matrix.
Step 3:- Find smallest number from every row and then divide every number of the row by its smallest number.
Step 4:- Find smallest number from every column and then divide every number of the column by its smallest number.
Step 5:- Make assignment in term of ones. If there are some rows and columns without assignment, then we cannot get the optimum solution. Then go to the next steps.
Step 6:- Draw the minimum number of lines passing through all ones by using the following procedure:

a. Mark (√) rows that do not have assignments.
b. Mark (√) columns that have crossed ones in that marked rows.
c. Mark (√) rows that have assignment in marked columns.
d. Repeat (b) and (c) till no more rows or column can be marked.
e. Draw straight lines through all unmarked rows and marked columns. If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal. Otherwise go to next step.

Step 7:- Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered numbers by this smallest number. Other numbers covered by lines remain unchanged. Then we get some new ones in row and column. Again make assignments in terms of ones.

Step 8:- If we cannot get the optimal assignment in each row and column, then repeat steps (6) and (7) successively till an optimum solution is obtained.

## IV. NUMERICAL EXAMPLES

In this section, we analyze two problems of assignment problem by the new proposed system as follows
**Example 1:**There are four jobs to be assigned to be four machines. Only one job can be assigned to one machine. The amount of time in hours required for the jobs per machine are given in the following matrix:

Machines

| | | *A* | *B* | *C* | *D* |
|---|---|---|---|---|---|
| *1* | | 12 | 30 | 21 | 15 |
| Jobs | *2* | 18 | 33 | 9 | 31 |
| *3* | | 44 | 25 | 24 | 21 |
| *4* | | 23 | 30 | 28 | 14 |

Find an optimum assignment of jobs to the machines to minimize the total processing time.

**Solution:**
**Step-1:** Find the smallest number from the given cost matrix. Then subtract the smallest element from every number of the matrix.
Machines

| | | A | B | C | D |
|---|---|---|---|---|---|
| *1* | | 3 | 21 | 12 | 6 |
| Jobs | *2* | 9 | 24 | 0 | 22 |
| *3* | | 35 | 16 | 15 | 12 |
| *4* | | 14 | 21 | 19 | 5 |

**Step-2:** Now add 1 to all numbers of the matrix.
Machines

| | | A | B | C | D |
|---|---|---|---|---|---|
| *1* | | 4 | 22 | 13 | 7 |
| Jobs | *2* | 10 | 25 | 1 | 23 |
| *3* | | 36 | 17 | 16 | 13 |
| *4* | | 15 | 22 | 20 | 6 |

**Step-3:**Find the smallest number from each row. Then divide every numberof each row by its smallest number.
Machines

| | | A | B | C | D |
|---|---|---|---|---|---|
| *1* | | 1 | 5.5 | 3.25 | 1.75 |
| Jobs | *2* | 10 | 25 | 1 | 23 |
| *3* | | 2.77 | 1.31 | 1.23 | 1 |
| *4* | | 2.5 | 3.64 | 3.33 | 1 |

**Step-4:**Find the smallest number from each column. Then divide every number of each column by its smallest number.

Machines

| | | A | B | C | D |
|---|---|---|---|---|---|
| *1* | | 1 | 4.2 | 3.25 | 1.75 |
| Jobs | *2* | 10 | 19.08 | 1 | 23 |
| *3* | | 2.77 | 1 | 1.23 | 1 |
| *4* | | 2.5 | 2.8 | 3.33 | 1 |

**Step-5:**Make initial assignments.
Machines

| | | A | B | C | D |
|---|---|---|---|---|---|
| *1* | | 1 | 4.2 | 3.25 | 1.75 |
| Jobs | *2* | 10 | 19.08 | 1 | 23 |
| *3* | | 2.77 | 1 | 1.23 | 1 |
| *4* | | 2.5 | 2.8 | 3.33 | 1 |

Hence the optimum assignment schedule is,

$$1 \rightarrow A, \quad 2 \rightarrow C, \ 3 \rightarrow B, \ 4 \rightarrow D$$

And the minimum time is,

$$(12 + 9 + 25 + 14) = 60 \ hours.$$

**Example 2:** A work manager has to assign four different drivers to four schools to supply lunch for students. Depending on the efficiency of the time taken by the individual differ by the capacity as shown in the table:
Schools

| | | *A* | *B* | *C* | *D* |
|---|---|---|---|---|---|
| *1* | | 8 | 26 | 17 | 11 |
| Drivers | *2* | 13 | 28 | 4 | 26 |
| *3* | | 38 | 19 | 18 | 15 |
| *4* | | 19 | 26 | 24 | 10 |

How should the drivers to be assigned to school so as to minimize the total man-hours?

**Solution:**
**Step-1:**Find the smallest number from the given cost matrix. Then subtract the smallest element from every number of the matrix.
Schools

| | | *A* | *B* | *C* | *D* |
|---|---|---|---|---|---|
| *1* | | 4 | 22 | 13 | 7 |
| Drivers | *2* | 9 | 24 | 0 | 22 |
| *3* | | 34 | 15 | 14 | 11 |
| *4* | | 15 | 22 | 20 | 6 |

**Step-2:** Now add 1 to all numbers of the matrix.

Schools

| | | *A* | *B* | *C* | *D* |
|---|---|---|---|---|---|
| *1* | | 5 | 23 | 14 | 8 |
| Drivers | *2* | 10 | 25 | 1 | 23 |
| *3* | | 35 | 16 | 15 | 12 |
| *4* | | 16 | 23 | 21 | 7 |

**Step-3:**Find the smallest number from each row. Then divide every number of each row by its smallest number.
Schools

| | | *A* | *B* | *C* | *D* |
|---|---|---|---|---|---|
| *1* | | 1 | 4.6 | 2.8 | 1.6 |
| Drivers | *2* | 10 | 25 | 1 | 23 |
| *3* | | 2.92 | 1.33 | 1.25 | 1 |
| *4* | | 2.29 | 3.29 | 3 | 1 |

**Step-4:**Find the smallest number from each column. Then divide every number of each column by its smallest number.

Schools

| | | *A* | *B* | *C* | *D* |
|---|---|---|---|---|---|
| | *1* | 1 | 3.46 | 2.8 | 1.6 |
| Drivers | *2* | 10 | 18.8 | 1 | 23 |
| | *3* | 2.92 | 1 | 1.25 | 1 |
| | *4* | 2.29 | 2.47 | 3 | 1 |

**Step-5:**Make4 initial assignments.

Schools

| | | *A* | *B* | *C* | *D* |
|---|---|---|---|---|---|
| | *1* | 1 | 3.46 | 2.8 | 1.6 |
| Drivers | *2* | 10 | 18.8 | 1 | 23 |
| | *3* | 2.92 | 1 | 1.25 | 1 |
| | *4* | 2.29 | 2.47 | 3 | 1 |

Here, the optimum assignment schedule is,

$$1 \rightarrow A, \quad 2 \rightarrow C, \quad 3 \rightarrow B, \quad 4 \rightarrow D$$

And the minimum time is,

$$(8 + 4 + 19 + 10) = 41\ hours.$$

## V. COMPARISON OF RESULTS

Here, we compare the obtained result of the proposed method with the Hungarian method. The comparison is given in the following table:

| Example | Proposed Method | Hungarian Method | Optimum |
|---|---|---|---|
| 01 | 60 | 60 | 60 |
| 02 | 41 | 41 | 41 |

**Table 1**

The above table shows that the new proposed technique is very logical, efficient and easy procedure to solve assignment problem.

## VI. CONCLUSION

In this paper, we have provided a different approach for finding optimal solution of assignment problem which is very easy to understand and consumes less time. Also we have compared the new approach with Hungarian method and we get the same results. This new method will be easy to apply in the practical field and helpful decision makers.

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