



Important Identities in Numerical Analysis

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Abstract

This paper contains some important identities of mathematics which are commonly used in regular practice. These novel statements/equations along with the proofs are discussed in this article. These Identities will help a person to find the square root of a number which is not a perfect square and/but it's a perfect cube number. These identities will also be helpful to understand how to find a particular number with the help of its square root in a different way and how can we reduce calculation with the use of these novel identities which uses a system of naming the numbers like x , y and z and then solving the equation.

Keywords: Square root, Perfect cube numbers, Simplified equations.

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Identity:

An identity is an equation which is true for all values of the variable/ variables.

Square root of a perfect cube number:

We can easily find the square root of a perfect square number like 9, 16 and 25 etc.

But to find the square root of a perfect cube number instantly following noel statement can be of great help.

Statement: If $x = y^3$, then $\sqrt{x} = y\sqrt{y}$

Suppose we have to find the square root of a perfect cube number, then you can use the above statement.

For example:

(1) $\sqrt{125} = ?$

We know that $5^3 = 125$,

Thus,

Let 125 be x and 5 be y

So,

If $x = y^3$, then $\sqrt{x} = y\sqrt{y}$.

Ans- If $125 = 5^3$, then $\sqrt{125} = 5\sqrt{5}$

(2) $\sqrt{27} = ?$

We know that 3^3 is 27, thus,

Let 27 be x and 3 be y .

So, If $x = y^3$, then $\sqrt{x} = y\sqrt{y}$

Ans- If $27 = 3^3$, then $\sqrt{27} = 3\sqrt{3}$

(3) $\sqrt[3]{343} = ?$

We know that 7^3 is 343, thus,

Let 343 be x and 7 be y

So, If $x = y^3$, then $\sqrt{x} = y\sqrt{y}$.

Ans- If $343 = 7^3$, then $\sqrt[3]{343} = 7\sqrt[3]{7}$.

Similarly square root of 5832 will be $18\sqrt{18}$ and square root of 1728 will be $12\sqrt{12}$.

Proof of the statement:

If $x = y^3$ then $\sqrt{x} = y\sqrt{y}$

Let's take L.H.S.

$$x = y^3$$

$$x = y \times y \times y$$

$$\sqrt{x} = \sqrt{y \times y \times y}$$

$$\sqrt{x} = \sqrt{y^2 \times y}$$

$$\sqrt{x} = y\sqrt{y} \text{ (As square will cut square root)}$$

$$\sqrt{x} = y\sqrt{y}$$

Hence Proved

Identity to reduce calculation

We should try to reduce the calculation whenever we solve any mathematical problem. The best way to reduce the calculation is to name the numbers x, y and z and then simplify the equation formed. The following statement/equation can be a better choice to simplify the equation with reduced calculation load.

Statement:

$$\frac{x \frac{y}{z}}{y} - \frac{y}{z} = \frac{x-y}{z}$$

For example:

Let x be 5, y be 3 and z be 9

$$\begin{aligned}
 &x \times \frac{y}{z} - \frac{y}{z} = \frac{x-y}{z} \\
 &5 \times \frac{3}{9} - \frac{3}{9} = \frac{5-3}{9} \\
 &= \frac{2}{9}
 \end{aligned}$$

Let x be 4, y be 10 and z be 7

$$x \times \frac{y}{z} - \frac{y}{y} = \frac{x-y}{z}$$

$$4 \times \frac{10}{7} - \frac{10}{7}$$

$$\frac{40}{7} - \frac{10}{7} = \frac{4-10}{7}$$

$$= \frac{6}{7}$$

Let x be 1, y be 2 and z be 3

$$x \times \frac{y}{z} - \frac{y}{y} = \frac{x-y}{z}$$

$$1 \times \frac{2}{3} - \frac{2}{3}$$

$$\frac{2}{3} - \frac{2}{3} = \frac{1-2}{3}$$

$$= \frac{-1}{3}$$

Proof of the statement:

Let's take the L.H.S.

$$x \times \frac{y}{z} - \frac{y}{y}$$

$$\frac{x \times y}{yz} - \frac{y}{z}$$

$$\frac{x}{z} - \frac{y}{z} = \frac{x-y}{z} \quad \text{Simplified}$$

Squaring Differently:

Suppose, if the question is $3\sqrt{5}$ is the square root of which of these 58,45,20,25 numbers?

We can find the answer of this question by simply squaring $3\sqrt{5}$. However, there is another method to solve this question.

According to my observation,

$$x\sqrt{y} = \sqrt{(x \times y)x}$$

So, if $x\sqrt{y} = \sqrt{(x \times y)x}$, then

$$(x\sqrt{y})^2 = (x \times y)x$$

Thus, $(x\sqrt{y})^2 = (x \times y)x$

Statement: $(x\sqrt{y})^2 = (x \times y)x$

For example:

Let's take $3\sqrt{5}$

Let x be 3 and y be 5 in $x\sqrt{y}$, So,

$$(x\sqrt{y})^2 = (x \times y)x$$

$$(3\sqrt{5})^2 = (3 \times 5)3$$

$$(3\sqrt{5})^2 = 15 \times 3$$

$$(3\sqrt{5})^2 = 45$$

$3\sqrt{5}$ is the square root of 45.

Let's take $4\sqrt{7}$

Let x be 4 and y be 7 in $x\sqrt{y}$

Now, $(x\sqrt{y})^2 = (x \times y)x$

$$(4\sqrt{7})^2 = (4 \times 7)4$$

$$(4\sqrt{7})^2 = 28 \times 4$$

$$(4\sqrt{7})^2 = 112$$

$4\sqrt{7}$ is the square root of 112

Let's take $4\sqrt{13}$

Let x be 4 and y be 13 in $x\sqrt{y}$

Now, $(x\sqrt{y})^2 = (x \times y)x$

$$(4\sqrt{13})^2 = (4 \times 13)4$$

$$(4\sqrt{13})^2 = 52 \times 4$$

$$(4\sqrt{13})^2 = 208$$

$4\sqrt{13}$ is the square root of 208.

Similarly, $9\sqrt{8}$ is the square root of 648 and $2\sqrt{10}$ is the square root of 40.

Proof of the statement:

$$(x\sqrt{y})^2 = (x \times y)x$$

Let's take L.H.S.

$$x\sqrt{y} = \sqrt{(x\sqrt{y})^2}$$

$$x\sqrt{y} = \sqrt{x \times x \times y \times y}$$

$$x\sqrt{y} = \sqrt{x \times x \times y} \quad (\text{As } \sqrt{y \times y} = \sqrt{y^2} = y)$$

$$x\sqrt{y} = \sqrt{(x \times y)x}$$

$$(x\sqrt{y})^2 = (x \times y)x.$$

Hence proved

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