



HQ-normal Spaces in General Topology

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Abstract. In this paper, we introduced the notion of HQ-normal space which is a simultaneous generalization of quasi normal and meekly π -normal spaces and properties of HQ-normal spaces are investigated. Further we obtained some characterizations and properties of HQ-normal spaces with other normal and regular spaces and several decomposition of normality are obtained. We studied the behavior of HQ-normality with respect to continuous functions.

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I. Introduction

Normality plays a prominent role in general topology. Normality behaves differently from the other separation axioms in the terms of subspaces. Several generalized notions of normal spaces studied in literature such as almost normal [22], π -normal [12], Δ -normal [5], weakly Δ -normal [5], quasi-normal [30], nearly normal [21], softly normal [26], densely normal [25], κ -normal [27] (or mildly normal [24]). Some of these forms are used to serve as a necessary ingredient towards a decomposition of normality. Das et al. [8] introduced almost β -normal spaces as a generalizations of normal, β -normal [1] and almost normal [22] spaces. In 2015, Sharma and Kumar [26] introduced a new class of normal spaces called softly normal and obtained a characterization of softly normal space. In 2018, Kumar and Sharma [16] introduced the concepts of softly regular and partly regular spaces and obtained some characterizations of softly regular spaces. In 2023, Kumar [17] introduced the concepts of epi π -normal spaces, which lies between epi-normal and epi-almost normal spaces, and epi-normal and epi-quasi normal spaces. Interrelation among some existing variants of normal spaces is discussed. Recently, Kumar and Sharma [18] introduced the concepts of meekly π -normal spaces and obtained some characterizations of meekly π -normal spaces with other generalized normal spaces. In this paper, we introduced the notion HQ-normal spaces which is a simultaneous generalization of quasi normal and meekly π -normal spaces. This paper is organized as follows: Section-2 develops the necessary preliminaries. In Section-3, the concept of HQ-normal spaces is introduced and properties of HQ-normality are investigated. In Section-4, some characterizations and properties of HQ-normal spaces with other normal and regular spaces have been obtained. In Section-5, several decomposition of normality are obtained. Further we studied the behavior of HQ-normality with respect to continuous functions.

II. Preliminaries

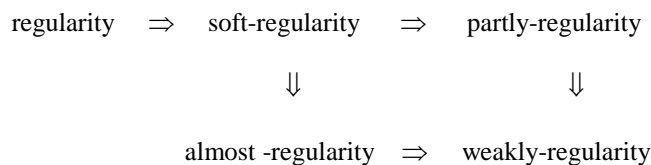
Let (X, \mathfrak{T}) or X be a topological space or space and let A is a subset of X . We will denote by $\text{cl}(A)$ and $\text{int}(A)$ the closure of A and the interior of A , respectively. A subset A of a space X is said to be **regularly open** or **open domain** [19] if it is the interior of its own closure or, equivalently, if it is the interior of some closed set. A complement of an open domain subset of X is called **closed domain** (or a subset A is said to be **regular open**

(resp. **regular closed**) if $A \subset \text{int}(\text{cl}(A))$ (resp. $A \subset \text{cl}(\text{int}(A))$). The finite union of regular open sets is said to be π -**open** [30]. The complement of a π -open set is said to be π -**closed**. Two sets A and B of X are said to be **separated** [3, 10] if there exist two disjoint-open subsets U and V of X such that $A \subset U$ and $B \subset V$.

Definition 2.1. A topological space is said to be

1. **normal** [3, 10] if any pair of disjoint closed subsets A and B of X can be separated.
2. **k-normal** [27] (**mildly normal** [24]) if for every pair of disjoint regularly closed sets H and K of X , there exists disjoint open subsets U and V of X such that $H \subset U$ and $K \subset V$.
3. **almost normal** [22] if for every pair of disjoint closed sets H and K one of which is regularly closed, there exist disjoint open sets U and V of X such that $H \subset U$ and $K \subset V$.
4. **α -normal** [1] if for any two disjoint closed subsets H and K of X , there exist disjoint open sets U and V of X such that $\text{cl}(H \cap U) = H$ and $\text{cl}(K \cap V) = K$.
5. **β -normal** [1] if for any two disjoint closed subsets H and K of X , there exist disjoint open sets U and V of X such that $\text{cl}(H \cap U) = H$, $\text{cl}(K \cap V) = K$ and $\text{cl}(U) \cap \text{cl}(V) = \phi$.
6. **almost β -normal** [8] if for every pair of disjoint closed sets H and K , one of which is regularly closed, there exist disjoint open sets U and V of X such that $\text{cl}(H \cap U) = H$, $\text{cl}(K \cap V) = K$ and $\text{cl}(U) \cap \text{cl}(V) = \phi$.
7. **π -normal** [12] if for any two disjoint closed subsets H and K of X , one of which is π -closed, there exist disjoint open sets U and V of X such that $H \subset U$ and $K \subset V$.
8. **quasi normal** [30] if for every pair of disjoint π -closed subsets H and K of X , there exist disjoint open sets U and V of X such that $H \subset U$ and $K \subset V$.
9. **softly normal** [26] if for any two disjoint closed subsets H and K of X , one of which is π -closed and other is regularly closed, there exist disjoint open sets U and V of X such that $H \subset U$ and $K \subset V$.
10. **almost regular** [22] if for every regularly closed set F and a point $x \notin F$, there exist disjoint open sets U and V of X such that $F \subset U$ and $x \in V$.
11. **softly regular** [16] if for every π -closed set A and a point $x \notin A$, there exist two open sets U and V such that $x \in U$, $A \subset V$, and $U \cap V = \phi$.
12. **partly regular** [16] if for every point x and every π -open set U containing x , there is an open set V such that $x \in V \subset \text{cl}(V) \subset U$.
13. **weakly regular** [22] if for every point x and every regularly open set U containing x , there is an open set V such that $x \in V \subset \text{cl}(V) \subset U$.

2.2 Remark. By the definitions stated above, we have the following diagram [16]:



Where none of the implications is reversible.

III. H-Quasi Normal Spaces

Definition 3.1. A topological space is called H-quasi normal (briefly HQ-normal) if for every pair of disjoint π -closed sets H and K , there exist disjoint open sets U and V of X such that $\text{cl}(H \cap U) = H$, $\text{cl}(K \cap V) = K$ and $\text{cl}(U) \cap \text{cl}(V) = \phi$.

Example 3.2. Let X be the union of any infinite set Y and two distinct one point sets p and q . The modified Fort [28] space on X as defined in is T_1 -space. The regularly closed sets of this space are finite subsets of Y and sets of the form $A \cup \{p, q\}$, where $A \subset Y$ is infinite. Thus the space is $\beta\kappa$ -normal.

Example 3.3. Let R be a commutative ring with unity. Let $X = \text{Spec}(R)$ be the set of all prime ideals of R . For each ideal I of R , let $V(I) = \{P \in \text{Spec}(R) : I \subset P\}$. The collections $\{\text{Spec}(R) \setminus V(I) : I \text{ is an ideal of } R\}$ is a topology on X and the collection of all sets $X_f = \{P \in \text{Spec}(R) : f \notin P\}$, $f \in R$ constitutes a base for this topology. The topology on $\text{Spec}(R)$ described above is called **Zariski topology**. $\text{Spec}(R)$ endowed with Zariski topology is called **prime spectrum** of the ring R and is always compact T_0 -space but not a βk -normal space.

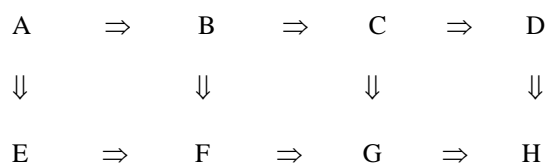
Theorem 3.4. Every quasi normal space is HQ-normal.

Proof. The proof follows from the definitions.

Theorem 3.5. Every quasi normal space is HQ-normal.

Proof. Let X be a quasi normal space. Let H and K be two disjoint π -closed sets in X . Since X is quasi normal, there exist disjoint open sets W and V containing H and K respectively. Since $W \cap V = \emptyset$, $W \cap \text{cl}(V) = \emptyset$. Let $U = \text{int}(A)$. Then $\text{cl}(U) \cap \text{cl}(V) = \emptyset$, $\text{cl}(H \cap U) = H$ and $\text{cl}(K \cap V) = K$. Thus, the space X is HQ-normal.

By the definitions stated above, we have the following diagram:



Where none of the implications is reversible.

Where

A : normal, B : π - normal, C : quasi normal, D : k -normal, E : β -normal, F : meekly π -normal, G : HQ-normal, H : βk -normal.

Now it is clear that every normal space is HQ-normal normal. Arhangel'skii et. al [1] have shown that a space is normal if and only if it is k -normal and β -normal. Therefore, every non-normal space which is β -normal is an example of a βk -normal space which is not κ -normal.

Example 3.6. Let X be the set of positive integers. Define a topology on X by taking every odd integer to be open and a set $U \subset X$ is open if for every even integer $p \in U$, the predecessor and successor of p also belongs to U . The space X is βk -normal. But the space X is not k -normal since for disjoint regular closed subsets $A = \{2, 3, 4\}$ and $B = \{6, 7, 8\}$ of X there does not exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

Theorem 3.7. For any topological space X , the following statements are equivalent:

- (1). X is HQ-normal;
- (2). whenever $E, F \subset X$ are disjoint π -closed, there is an open set V such that $F = \text{cl}(V \cap F)$ and $E \cap \text{cl}(V) = \emptyset$;
- (3). whenever $E \subset X$ is π -closed, $U \subset X$ is π -open and $E \subset U$, there is an open set V such that $E = \text{cl}(E \cap V)$ and $\text{cl}(V) \subset U$.

Proof. (1) \Rightarrow (2). Suppose that $E, F \subset X$ are disjoint π -closed. Since X is HQ-normal, there exist open sets U and V such that $E = \text{cl}(U \cap E) \subset \text{cl}(U)$, $F = \text{cl}(V \cap F) \subset \text{cl}(V)$ and $\text{cl}(U) \cap \text{cl}(V) = \emptyset$. Then $E \cap \text{cl}(V) = \emptyset$.

(2) \Rightarrow (1). Suppose that $E, F \subset X$ are disjoint π -closed. By the hypothesis, there exist an open set V such that $F = \text{cl}(V \cap F)$ and $E \cap \text{cl}(V) = \emptyset$. Let $U = \text{int}(E)$. Then $E = \text{cl}(U \cap E)$ and $\text{cl}(U) \cap \text{cl}(V) = E \cap \text{cl}(V) = \emptyset$.

(1) \Rightarrow (3). Suppose that $E \subset X$ is π -closed, $U \subset X$ is π -open and $E \subset U$. Since U is π -open, $X - U$ is π -closed. Since X is HQ-normal, there exist open sets W and V such that $(X - U) = \text{cl}(W \cap (X - U)) \subset \text{cl}(W)$, $E = \text{cl}(V \cap E) \subset \text{cl}(V)$ and $\text{cl}(W) \cap \text{cl}(V) = \emptyset$. Then $(X - U) \cap \text{cl}(V) = \emptyset$, which implies that $\text{cl}(V) \subset U$.

(3) \Rightarrow (2). Suppose that $E, F \subset X$ are disjoint π -closed. Then $F \subset X - E$ and $X - E$ is π -open. By the hypothesis, there is an open set V such that $F = \text{cl}(V \cap F) \subset \text{cl}(V) \subset (X - E)$. Then $\text{cl}(V) \cap E = \emptyset$.

IV. HQ-normal Spaces with Other Separation Axioms

Recall that a Hausdorff space X is said to be **extremally disconnected** if the closure of every open set in X is open.

Theorem 4.1. Every extremally disconnected HQ-normal space is quasi normal.

Proof. Let X be an extremally disconnected HQ-normal space and let H and K be two disjoint π -closed sets in X . By HQ-normality of X , there exist disjoint open sets U and V such that $\text{cl}(H \cap U) = H$, $\text{cl}(K \cap V) = K$ and $\text{cl}(U) \cap \text{cl}(V) = \emptyset$. Thus $H \subset \text{cl}(U)$ and $K \subset \text{cl}(V)$. By the extremally disconnectedness of X , $\text{cl}(U)$ and $\text{cl}(V)$ are disjoint open sets containing H and K , respectively.

Recall that a topological space X is called a **locally indiscrete space** [9] if every closed set in X is open in X .

Theorem 4.2. Every locally indiscrete, T_1 and HQ-normal space is softly regular.

Proof. Let A be a π -closed set in X and x be a point outside A . Since X is T_1 , so $\{x\}$ is closed. Also since X is locally indiscrete, $\{x\}$ is π -closed. By HQ-normality there exist disjoint open sets U and V such that $x \in U$, $\text{cl}(A \cap V) = A$ and $\text{cl}(U) \cap \text{cl}(V) = \emptyset$. Since $A \subset \text{cl}(V)$, U and $X - \text{cl}(U)$ are disjoint open sets containing $\{x\}$ and A , respectively. Thus, the space is softly regular.

Recall that a space X is said to be **almost compact** [4] if every open cover of X has a finite subcollection, the closure of whose members covers X .

Theorem 4.3. If X is an almost regular space in which every π -closed set is almost compact, then X is quasi normal [20].

Corollary 4.4. If X is an almost regular space in which every π -closed set is almost compact, then X is HQ-normal.

Proof. Since every quasi normal space is HQ-normal.

Corollary 4.5. If X is a softly regular space in which every π -closed set is almost compact, then X is HQ-normal.

Proof. Since every softly regular is almost regular.

Theorem 4.6. Every almost regular Lindelof space is quasi normal [20].

Corollary 4.7. Every almost regular Lindelof space is HQ-normal.

Proof. Since every quasi normal space is HQ-normal.

Corollary 4.8. Every softly regular Lindelof space is HQ-normal.

Proof. Since every softly regular is almost regular.

Theorem 4.9. An almost regular space with a σ -locally finite base is quasi normal [20].

Corollary 4.10. An almost regular space with a σ -locally finite base is HQ-normal.

Proof. Since every quasi normal space is HQ-normal.

Corollary 4.11. A softly regular space with a σ -locally finite base is HQ-normal.

Proof. Since every softly regular is almost regular.

Theorem 4.12. An almost regular nearly paracompact space is quasi normal [20].

Corollary 4.13. An almost regular nearly paracompact space is HQ-normal.

Proof. Since every quasi normal space is HQ-normal.

Corollary 4.14. A softly regular nearly paracompact space is HQ-normal.

Proof. Since every softly regular is almost regular.

Theorem 4.15. A nearly paracompact Hausdorff space is quasi normal [20].

Corollary 4.16. A nearly paracompact Hausdorff space is HQ-normal.

Proof. Since every quasi normal space is HQ-normal.

Theorem 4.17. Every weakly regular paracompact space is π -normal [29].

Corollary 4.18. Every weakly regular paracompact space is HQ-normal.

Proof. Since every π -normal space is HQ-normal.

Corollary 4.19. Every almost regular paracompact space is HQ-normal.

Proof. Since every almost regular is weakly regular.

Corollary 4.20. Every softly regular paracompact space is HQ-normal.

Proof. Since every softly regular is weakly regular.

Theorem 4.21. Every weakly regular compact space is π -normal [29].

Corollary 4.22. Every weakly regular compact space is HQ-normal.

Proof. Since every π -normal space is HQ-normal.

Corollary 4.23. Every almost regular compact space is HQ-normal.

Proof. Since every almost regular is weakly regular.

Corollary 4.24. Every softly regular compact space is HQ-normal.

Proof. Since every softly regular is weakly regular.

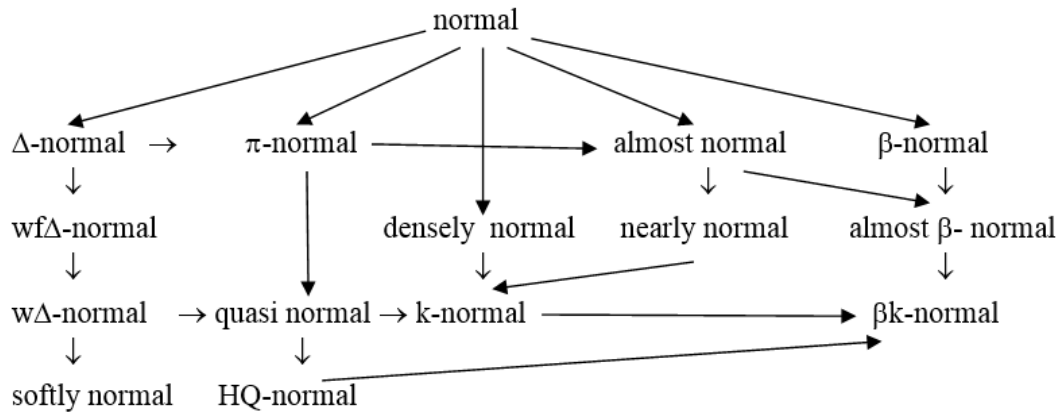
V. Decomposition of Normality

Recall a topological space is said to be **semi-normal** if for every closed set A contained in an open set U , there exists a regularly open set V such that $A \subset V \subset U$.

Definition 5.1. A topological space X is said to be

1. **Δ -normal [5]** if every pair of disjoint closed sets one of which is δ -closed are contained in disjoint open sets.
2. **weakly Δ -normal [5] ($w\Delta$ -normal)** if every pair of disjoint δ -closed sets are contained in disjoint open sets.
3. **weakly functionally Δ -normal [5] ($wf\Delta$ -normal)** if for every pair of disjoint δ -closed sets A and B there exists a continuous function $f : X \rightarrow [0, 1]$ such that $f(A) = 0$ and $f(B) = 1$.
4. **nearly normal [21]** if for any two disjoint sets one of which is δ -closed and the other is regularly closed sets are contained in disjoint open sets.
5. **densely normal [25]** if there exists a dense subspace Y of X such that X is normal on Y .

The following implications hold but none are reversible.



Definition 5.2. [] A space X is called **weakly β -normal** [7] if for any two disjoint closed subsets A and B of X there exist open subsets U and V of X such that $A \subset \text{cl}(U)$, $B \subset \text{cl}(V)$ and $\text{cl}(U) \cap \text{cl}(V) = \phi$.

It is clear from definitions that every normal space is β -normal and hence, weakly β -normal.

Theorem 5.3. [1] A topological space X is normal if and only if it is k -normal and β -normal.

Theorem 5.4. [25] A topological space X is normal if and only if it is κ -normal and weakly β -normal.

Definition 5.5. A space X is called **meekly weakly β -normal** if for every pair of disjoint closed sets A and B one of which is π -closed, there exist open subsets U and V of X such that $A \subset \text{cl}(U)$, $B \subset \text{cl}(V)$ and $\text{cl}(U) \cap \text{cl}(V) = \phi$.

The following implications are follows by the definitions:

normality \Rightarrow π -normality \Rightarrow meekly π -normality \Rightarrow meekly weakly β -normal.

Theorem 5.6. [18] A topological space is π -normal if and only if it is meekly π -normal and quasi normal.

The following theorem is an improvement of Theorem 5.6.

Theorem 5.7. A topological space is π -normal if and only if it is meekly weakly β -normal and quasi normal.

Proof. Let X be a meekly weakly β -normal and quasi normal space. Let Let A and B be two disjoint closed sets in X in which A is π -closed. By meekly weakly β -normality of X , there exist disjoint open subsets U and V such that $A \subset \text{cl}(U)$, $B \subset \text{cl}(V)$ and $\text{cl}(U) \cap \text{cl}(V) = \phi$. Now, $\text{cl}(U)$ and $\text{cl}(V)$ are disjoint π -closed sets. By quasi normality, there exist disjoint open sets V_1 and V_2 such that $\text{cl}(U) \subset V_1$ and $\text{cl}(V) \subset V_2$. Thus X is π -normal.

The following corollary is an improvement of [Corollary 2.20, [8]].

Corollary 5.8. In a semi-normal and quasi normal space the following statements are equivalent:

- (1). X is normal;
- (2). X is π -normal;
- (3). X is β -normal;
- (4). X is meekly π -normal;
- (5). X is meekly weakly β -normal.

Proof. $(1) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5)$ and $(1) \Rightarrow (2) \Rightarrow (4) \Rightarrow (5)$ are obvious. We have to show that $(5) \Rightarrow (1)$. Let X be semi-normal, quasi normal and meekly weakly β -normal space. Thus by Theorem 5.7, X is π -normal. Since every π -normal, semi-normal space is normal. Thus X is normal.

Theorem 5.9. In a extremally disconnected space X the following statements hold [25]:

1. The following statements are equivalent:

- (a). X is normal;
- (b). X is β -normal;
- (c). X is weakly β -normal.

Corollary 5.10. In a extremally disconnected space X the following statements hold:

- (a). X is π -normal;
- (b). X is meekly π -normal;
- (c). X is meekly weakly β -normal.

Theorem 5.11 [25]. If X is a compact space, then X is normal if and only if X is weakly β -normal and almost regular.

Corollary 5.12 [25]. In a compact, almost regular space the following statements are equivalent:

- (a). X is normal;
- (b). X is β -normal;
- (c). X is weakly β -normal.

Theorem 5.13 [25]. A Hausdorff space is normal if and only if it is nearly normal, $w\theta$ -normal and weakly β -normal.

Corollary 5.14. In a extremally disconnected semi normal space the following statements are equivalent:

- 1. X is normal;
- 2. X is Δ -normal;
- 3. X is weakly functionally Δ -normal;
- 4. X is weakly Δ -normal;
- 5. X is π -normal;
- 6. X is meekly π -normal;
- 7. X is almost normal;
- 8. X is quasi normal;
- 9. X is βQ -normal;
- 10. X is densely normal;
- 11. X is k -normal;
- 12. X is βk -normal.

The following corollary follows from Theorem 3.9 and [Theorem 2.13, [6]].

Corollary 5.15. In a extremally disconnected β -normal space the following statements are equivalent:

- 1. X is normal;
- 2. X is Δ -normal;
- 3. X is weakly functionally Δ -normal;
- 4. X is weakly Δ -normal;
- 5. X is π -normal;
- 6. X is meekly π -normal;
- 7. X is almost normal;
- 8. X is quasi normal;

9. X is βQ -normal;
10. X is densely normal;
11. X is k -normal;
12. X is βk -normal.

Theorem 5.16. βQ -normality is preserved under regular closed subspace.

Proof. Easy to verify.

Theorem 5.17. Suppose that X and Y are topological spaces, X is βQ -normal, and $f : X \rightarrow Y$ is onto, continuous, open, and closed. Then Y is βQ -normal.

Proof. Easy to verify.

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