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Review Paper

Quotient-4 Cordial Labeling Of Some Tricyclic Graphs

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Abstract
Let $G(V,E)$ be a simple graph of order p and size q. Let $\varphi: V(G) \to Z_5 - \{0\}$ be a function. For each edge set $E(G)$ define the labeling $\varphi^*: E(G) \to Z_4$ by $\varphi^*(uv) = \lceil(\frac{\varphi(u)}{\varphi(v)})\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1, 1 \leq i, j \leq 4, i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1, 0 \leq k, l \leq 3, k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y. Here some tricyclicgraphs such as Jelly fish graph and $C_n \circ v\ C_n'$ graph are quotient-4 cordial labeling.
Keywords: *Jelly fish graph, $C_n \circ v\ C_n'$graph, quotient-4 cordial labeling and quotient-4 cordial graph.*

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I. INTRODUCTION

Here the graphs considered are finite, simple, undirected and non-trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [4] for more information. The cordial labeling concept was first introduced by Cahit [2]. H- and H2 –cordial labeling was introduced by Freeda S and ChellathuraiR.S[3]. Mean Cordial Labeling was introduced by Albert William, Indira Rajasingh, and S Roy [1]. Quotient-4 cordiallabelingwas introduced byP.Sumathi and S.Kavitha [5]. A graph G is said to be quotient-4 cordial graph if it receives quotient-4 cordial labeling .Let $v_\varphi(i)$ denotes the number of vertices labeled with i and $e_\varphi(k)$ denotes the number of edges labeled with $k, 1 \leq i \leq 4, 0 \leq k \leq 3$.

II. DEFINITIONS

Definition 2.1. Let $G\ (V,E)$ be a simple graph of order p and size q.Let$\varphi: V(G) \to Z_5 - \{0\}$ be a function. For each edge set $E\ (G)$ define the labeling $\varphi^*: E(G) \to Z_4$ by $\varphi^*(uv) = \lceil(\frac{\varphi(u)}{\varphi(v)})\rceil$(mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1, 1 \leq i, j \leq 4, i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1, 0 \leq k, l \leq 3, k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y.

Definition 2.2. The *jelly fish graph* $J(m,n)$ is obtained from a cycle with vertices x, y, u, v of length 4 by joining x and y with a prime edge and appending m pendent edges to u and n pendent edges to v. The prime edge in jelly fish graph is defined to be the edge joining the vertices x and y.

Definition 2.3. A graph $C_n \circ v\ C_n'$obtained from two copies of the cycle C_n sharing$\frac{n}{2}$common vertex if n is even and $\frac{n-1}{2}$ common vertex if n is odd.

III. MAIN RESULT

Theorem3.1. Any Jelly fish graph$J(m,n)$is quotient-4 cordial if $m,n \geq 1$.

Proof. Let G be a Jelly fish graph.

$V\ (G) = \{u_i: 1 \leq i \leq 4\} \cup \{x_j\ : 1 \leq j \leq m\} \cup \{y_k\ : 1 \leq k \leq n\}$.

$E\ (G) = \{u_i u_{i+1}: 1 \leq i \leq 3\} \cup \{u_4 u_1\} \cup \{u_1 u_3\} \cup \left\{u_2 x_j: 1 \leq j \leq m\right\} \cup$

$\{u_4 y_k: 1 \leq k \leq n\}$.

Here $|V\ (G)| = m + n + 4, |E\ (G)| = m + n + 5$.

Define $\varphi\ : V\ (G) \rightarrow \{1, 2, 3, 4\}$.

The values of $u_i$are labeled as follows:

Case 1: $m \geq n$.

$\varphi\ (u_1) = \varphi\ (u_3) = 3, \varphi\ (u_2) = 1, \varphi\ (u_4) = 2$.

Case 2: $m < n$.

$\varphi\ (u_1) = \varphi\ (u_3) = 3, \varphi\ (u_2) = 2, \varphi\ (u_4) = 1$.

The values of $x_j$are labeled as follows:

When $m < n$.

For $1 \leq j \leq m$.

$\varphi(x_j) = 1 \quad if\ j \equiv 0$ (modulo 2).

$\varphi(x_j) = 2 \quad if\ j \equiv 1$ (modulo 2).

When $m \geq n$.

Case 1: When $m \equiv 0, 1$ (modulo 4) and $n \equiv 0$ (modulo 4).

For $1 \leq j \leq n$.

$\varphi(x_j) = 3 \quad if\ j \equiv 1$ (modulo 2)and $j \neq 1$.

$\varphi(x_j) = 4 \quad if\ j \equiv 0$(modulo 2)and $j = 1$.

For $n + 1 \leq j \leq m$.

$\varphi(x_j) = 1 \quad if\ j \equiv 1$ (modulo4).

$\varphi(x_j) = 2 \quad if\ j \equiv 2$(modulo 4).

$\varphi(x_j) = 3 \quad if\ j \equiv 3$ (modulo4).

$\varphi(x_j) = 4 \quad if\ j \equiv 0$(modulo 4).

Case 2: When $m \equiv 0$ (modulo 4) and $n \equiv 1$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 4$.

For $n + 2 \leq j\ \leq\ m$.

$\varphi(x_j) = 1 \quad if\ j \equiv 3$ (modulo4).

$\varphi(x_j) = 2 \quad if\ j \equiv 0$(modulo 4).

$\varphi(x_j) = 3 \quad if\ j \equiv 1$ (modulo4).

$\varphi(x_j) = 4 \quad if\ j \equiv 2$(modulo 4).

Case 3: When $m \equiv 0$ (modulo 4) and $n \equiv 2$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 3$.

For $n + 2 \leq j \leq m$.

$\varphi(x_j) = 1 \quad if\ j \equiv 0$ (modulo4).

$\varphi(x_j) = 2 \quad if\ j \equiv 1$(modulo 4).

$\varphi(x_j) = 3 \quad if\ j \equiv 2$ (modulo4).

$\varphi(x_j) = 4 \quad if\ j \equiv 3$(modulo 4).

Case 4: When $m \equiv 0$ (modulo 4) and $n \equiv 3$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 2.

$\varphi(x_{n+1}) = 4$.

For $n + 2 \leq j \leq m$, the labeling of x_j values are same as case 1.

Case 5: When $m \equiv 1$ (modulo 4) and $n \equiv 1$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 4, \varphi(x_{n+2}) = 3$.

For $n + 3 \leq j \leq m$, the labeling of $x_j$values are same as case 3.

Case 6: When $m \equiv 1$ (modulo 4) and $n \equiv 2$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 3, \varphi(x_{n+2}) = 4$.

For $n + 3 \leq j \leq m$, the labeling of $x_j$values are same as case 1.

Case 7: When $m \equiv 1$ (modulo 4) and $n \equiv 3$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 4$.

For $n + 2 \leq j \leq m$, the labeling of $x_j$values are same as case 1.

Case 8: When $m \equiv 2$ (modulo 4) and $n \equiv 0$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 3, \varphi(x_{n+2}) = 4$.

For $n + 3 \leq j \leq m$.

$\varphi(x_j) = 1 \quad if\ j \equiv 3$ (modulo4).

$\varphi(x_j) = 2 \quad if\ j \equiv 0$(modulo 4).

$\varphi(x_j) = 3 \quad if\ j \equiv 1$ (modulo4).

$\varphi(x_j) = 4 \quad if\ j \equiv 2$(modulo 4).

Case 9: When $m \equiv 2$ (modulo 4) and $n \equiv 1$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 4$.

For $n + 2 \leq j \ \leq \ m$,the labeling of $x_j$values are same as case 8.

Case 10: When $m \equiv 2$ (modulo 4) and $n \equiv 2$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

For $n + 1 \leq j \leq m$,the labeling of $x_j$values are same as case 8.

Case 11: When $m \equiv 2$ (modulo 4) and $n \equiv 3$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 4$.

For $n + 2 \leq j \ \leq \ m$,the labeling of $x_j$values are same as case 1.

Case 12: When $m \equiv 3$ (modulo 4) and $n \equiv 0$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+2}) = 4, \varphi(x_{n+1}) = 3$.

For $n + 3 \leq j \leq m$,the labeling of $x_j$values are same as case 8.

Case 13: When $m \equiv 3$ (modulo 4) and $n \equiv 1$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 4$.

For $n + 2 \leq j \leq m$,the labeling of $x_j$values are same as case 8.

Case 14: When $m \equiv 3$ (modulo 4) and $n \equiv 2$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

$\varphi(x_{n+1}) = 3$.

For $n + 2 \leq j \leq m$,the labeling of x_j values are same as case 3.

Case 15: When $m \equiv 3$ (modulo 4) and $n \equiv 3$ (modulo 4).

For $1 \leq j \leq n$,the labeling of $x_j$values are same as case 1.

For $n + 1 \leq j \leq m$,the labeling of $x_j$values are same as case 3.

The values of $y_k$are labeled as follows:

When $m \geq n$.

For $1 \leq k \leq n$.

$\varphi\ (y_k) = 1 \quad if\ k \equiv 0$ (modulo 2).

$\varphi\ (y_k) = 2 \quad if\ k \equiv 1$ (modulo 2).

When $m < n$.

Case 1: When $m \equiv 0$ (modulo 4) and $n \equiv 0, 1$ (modulo 4).

For $1 \leq k \leq m$.

$\varphi\ (y_k) = 3 \quad if\ k \equiv 1$ (modulo 2) and $k \neq 1$.

$\varphi\ (y_k) = 4 \quad if\ k \equiv 0$ (modulo 2) and $k = 1$.

For $m + 1 \leq k \leq n$.

$\varphi\ (y_k) = 1 \quad if\ k \equiv 1$ (modulo4).

$\varphi\ (y_k) = 2 \quad if\ k \equiv 2$ (modulo4).

$\varphi\ (y_k) = 3 \quad if\ k \equiv 3$ (modulo4).

$\varphi\ (y_k) = 4 \quad if\ k \equiv 0$ (modulo4).

Case 2: When $m \equiv 0$ (modulo 4) and $n \equiv 2$ (modulo 4).

For $1 \leq k \leq m$,the labeling of $y_k$values are same as case 1.

$\varphi\ (y_{m+1}) = 3$.

For $m + 2 \leq k \leq n$.

$\varphi\ (y_k) = 1 \quad if\ k \equiv 2$ (modulo4).

$\varphi\ (y_k) = 2 \quad if\ k \equiv 3$ (modulo4).

$\varphi\ (y_k) = 3 \quad if\ k \equiv 0$ (modulo4).

$\varphi\ (y_k) = 4 \quad if\ k \equiv 1$ (modulo4).

Case 3: When $m \equiv 0$ (modulo 4) and $n \equiv 3$ (modulo 4).

For $1 \leq k \leq m$,the labeling of $y_k$values are same as case 1.

$\varphi\ (y_{m+1}) = 3, \varphi\ (y_{m+2}) = 4$.

For $m + 3 \leq k \leq n$.

$\varphi\ (y_k) = 1 \quad if\ k \equiv 3$ (modulo4).

$\varphi\ (y_k) = 2 \quad if\ k \equiv 0$ (modulo4).

$\varphi\ (y_k) = 3 \quad if\ k \equiv 1$ (modulo4).

$\varphi\ (y_k) = 4 \quad if\ k \equiv 2$ (modulo4).

Case 4: When $m \equiv 1$ (modulo 4) and $n \equiv 0, 1, 2, 3$ (modulo 4).

For $1 \leq k \leq m$,the labeling of $y_k$values are same as case 1.

$\varphi\ (y_{m+1}) = 4, \varphi\ (y_{m+2}) = 1$.

For $m + 3 \leq k \leq n$,the labeling of $y_k$values are same as case 3.

Case 5: When $m \equiv 2$ (modulo 4) and $n \equiv 0, 1$ (modulo 4).

For $1 \leq k \leq m$,the labeling of $y_k$values are same as case 1.

$\varphi\ (y_{m+1}) = 3, \varphi\ (y_{m+2}) = 4$.

For $m + 3 \leq k \leq n$,the labeling of $y_k$values are same as case 1.

Case 6: When $m \equiv 2$ (modulo 4) and $n \equiv 2, 3$ (modulo 4).
For $1 \leq k \ \leq \ m$,the labeling of $y_k$values are same as case 1.
$\varphi\ (y_{m+1}) = 1, \varphi\ (y_{m+2}) = 2$.

For $m + 3 \leq k \leq n$, the labeling of $y_k$values are same as case 3.

Case 7: When $m \equiv 3$ (modulo 4) and $n \equiv 0, 1$ (modulo 4).
For $1 \leq k \leq m$,the labeling of $y_k$values are same as case 1.
$\varphi\ (y_{m+1}) = 4$.

For $m + 2 \leq k \leq n$, the labeling of $y_k$values are same as case 1.

Case 8: When $m \equiv 3$ (modulo 4) and $n \equiv 2$ (modulo 4).
For $1 \leq k \leq m$,the labeling of $y_k$values are same as case 1.
$\varphi\ (y_{m+1}) = 4, \varphi\ (y_{m+2}) = 3$.

For $m + 3 \leq k \leq n$, the labeling of $y_k$values are same as case 2.

Case 9: When $m \equiv 3$ (modulo 4) and $n \equiv 3$ (modulo 4).
For $1 \leq k \leq m$,the labeling of $y_k$values are same as case 1.
For $m + 1 \leq k \leq n$.

$\varphi\ (y_k) = 1 \quad if\ k \equiv 0$ (modulo4).

$\varphi\ (y_k) = 2 \quad if\ k \equiv 1$ (modulo4).

$\varphi\ (y_k) = 3 \quad if\ k \equiv 2$ (modulo4).

$\varphi\ (y_k) = 4 \quad if\ k \equiv 3$ (modulo4).

The following table shows that m & $n$concurrence is realized with modulo 4

Nature of m and n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$m \equiv 0$ $n \equiv 0$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$
$m \equiv 0$ $n \equiv 1$ $m \geq n$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4} + 1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$
$m \equiv 0$, $n \equiv 1$, $m < n$	$\frac{m+n+3}{4} + 1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$

$m \equiv 0$ $n \equiv 2$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}-1$
$m \equiv 0,$ $n \equiv 3,$ $m \geq n$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 0$ $n \equiv 3$ $m < n$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 1$ $n \equiv 0$ $m \geq n$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$
$m \equiv 1$ $n \equiv 0$ $m < n$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$
$m \equiv 1$ $n \equiv 1$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}-1$
$m \equiv 1$ $n \equiv 2$ $m \geq n$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 1$ $n \equiv 2$ $m < n$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 1$ $n \equiv 3$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$
$m \equiv 2$ $n \equiv 0$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$
$m \equiv 2$ $n \equiv 1$ $m \geq n$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 2$ $n \equiv 1$ $m < n$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$

$m \equiv 2$ $n \equiv 2$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$
$m \equiv 2$ $n \equiv 3$ $m \geq n$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$
$m \equiv 2$ $n \equiv 3$ $m < n$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$
$m \equiv 3$ $n \equiv 0$ $m \geq n$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 3$ $n \equiv 0$ $m < n$	$\frac{m+n+5}{4}-1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 3$ $n \equiv 1$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$
$m \equiv 3$ $n \equiv 2$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$
$m \equiv 3$ $n \equiv 3$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}-1$

Table 1: Vertex labeling of Jelly Fish graph

The following table shows that m & $n$concurrence is realized with modulo 4.

Nature of m and n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$m \equiv 0$ $n \equiv 0$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}+1$	$\frac{m+n+4}{4}$
$m \equiv 0$ $n \equiv 1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$
$m \equiv 0$ $n \equiv 2$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$
$m \equiv 0$ $n \equiv 3$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$

$m \equiv 1$ $n \equiv 0$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$
$m \equiv 1$ $n \equiv 1$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$
$m \equiv 1$ $n \equiv 2$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 1$ $n \equiv 3$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}+1$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$
$m \equiv 2$ $n \equiv 0$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$
$m \equiv 2$ $n \equiv 1$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 2$ $n \equiv 2$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}+1$	$\frac{m+n+4}{4}$
$m \equiv 2$ $n \equiv 3$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$
$m \equiv 3$ $n \equiv 0$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$	$\frac{m+n+5}{4}$
$m \equiv 3$ $n \equiv 1$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}+1$	$\frac{m+n+4}{4}$	$\frac{m+n+4}{4}$
$m \equiv 3$ $n \equiv 2$ $m \geq n$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}+1$
$m \equiv 3$ $n \equiv 2$ $m < n$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$	$\frac{m+n+3}{4}$	$\frac{m+n+3}{4}+1$
$m \equiv 3$ $n \equiv 3$	$\frac{m+n+6}{4}-1$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$	$\frac{m+n+6}{4}$

Table 2: Edge labeling of Jelly Fish graph

The above tables 1 and 2 show that $|v_\varphi(i) - v_\varphi(j)| \leq 1and|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the Jelly fish graphis quotient-4 cordial labeling.

Illustration3.2. Figure 1 gives the quotient-4 cordial labeling for the Jelly fish graph with $m = 6, n = 5$.

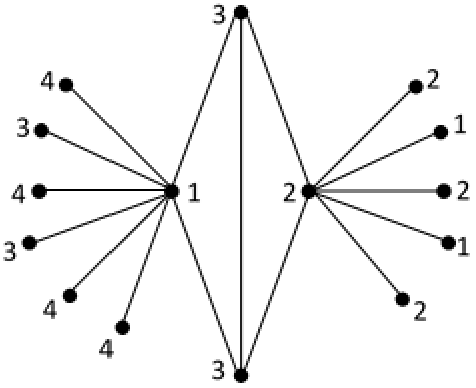

Figure 1.

Theorem 3.3.A graph $C_n \circ v\ C_n'$is quotient-4 cordial if $n \geq 3$.

Proof. Let G be a $C_n \circ v\ C_n'$graph.

If n is even,

V (G) = $\{x_i : 1 \leq i \leq n\} \cup \{y_j : \frac{n}{2} + 1 \leq j \leq n\}$.

$E\ (G)\ =\ \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{\ x_1 x_n\} \cup \{x_{\frac{n}{2}} y_{\frac{n}{2}+1}\} \cup \{y_j y_{j+1} : \frac{n}{2} + 1 \leq j \leq$

$n - 1\} \cup \{\ x_1 y_n\}$.

Here $|V\ (G)| = \frac{3n}{2}, |E\ (G)| = \frac{3n}{2} + 1$.

If n is odd,

$V\ (G) = \{x_i : 1 \leq i \leq n\} \cup \{y_j : \frac{n-1}{2} + 1 \leq j \leq n\}$.

$E\ (G) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{\ x_1 x_n\} \cup \{x_{\frac{n-1}{2}} y_{\frac{n-1}{2}+1}\} \cup \{y_j y_{j+1} : \frac{n-1}{2} + 1 \leq j$

$\leq n - 1\} \cup \{\ x_1 y_n\}$.

Here $|V\ (G)| = \frac{3n+1}{2}, |E\ (G)| = \frac{3n+1}{2} + 1$.

Define $\varphi : V\ (G) \rightarrow \{1, 2, 3, 4\}$.

The values of $x_i$are labeled as follows:

Case 1: When $n \equiv 0$ (modulo 16).

For $1 \leq i \leq\ n$.

$\varphi(x_i) = 1 \quad if\ i \equiv 1, 4$ (modulo 8).

$\varphi(x_i) = 2 \quad if\ i \equiv 6, 7$ (modulo 8).

$\varphi(x_i) = 3 \quad if\ i \equiv 0, 5$ (modulo 8).

$\varphi(x_i) = 4 \quad if\ i \equiv 2, 3$ (modulo 8).

Case 2: When $n \equiv 1$ (modulo 16).

For $1 \leq i \leq n - 1$, the labeling of $x_i$values are same as case 1.

$\varphi(x_n) = 4$.

Case 3: When $n \equiv 2$ (modulo 16).

For $1 \leq i \leq n-2$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-1}) = 4, \varphi(x_n) = 1$.

Case 4: When $n \equiv 3, 4, 12$ (modulo 16).

For $1 \leq i \leq n-2$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-1}) = 2, \varphi(x_n) = 3$.

Case 5: When $n \equiv 5, 13$ (modulo 16).

For $1 \leq i \leq n-3$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-2}) = \varphi(x_{n-1}) = 4, \varphi(x_n) = 1$.

Case 6: When $n \equiv 6$ (modulo 16).

For $1 \leq i \leq n-2$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-1}) = \varphi(x_n) = 2$.

Case 7: When $n \equiv 7$ (modulo 16).

For $1 \leq i \leq n-3$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-2}) = 4, \varphi(x_{n-1}) = \varphi(x_n) = 2$.

Case 8: When $n \equiv 8$ (modulo 16).

For $1 \leq i \leq n-3$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-2}) = 1, \varphi(x_{n-1}) = 4, \varphi(x_n) = 3$.

Case 9: When $n \equiv 9$ (modulo 16).

For $1 \leq i \leq n-3$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-2}) = 3, \varphi(x_{n-1}) = 1, \varphi(x_n) = 4$.

Case 10: When $n \equiv 10$ (modulo 16).

For $1 \leq i \leq n-3$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-2}) = 1, \varphi(x_{n-1}) = \varphi(x_n) = 3$.

Case 11: When $n \equiv 11$ (modulo 16).

For $1 \leq i \leq n-2$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-1}) = \varphi(x_n) = 3$.

Case 12: When $n \equiv 14$ (modulo 16).

For $1 \leq i \leq n-4$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-3}) = 1, \varphi(x_{n-2}) = \varphi(x_{n-1}) = \varphi(x_n) = 2$.

Case 13: When $n \equiv 15$ (modulo 16).

For $1 \leq i \leq n-5$, the labeling of $x_i$values are same as case 1.

$\varphi(x_{n-4}) = 1, \varphi(x_{n-3}) = 4, \varphi(x_{n-2}) = \varphi(x_{n-1}) = \varphi(x_n) = 2.$

The values of $y_j$are labeled as follows:

Case 1: When $n \equiv 0, 3, 4, 5, 9, 10, 11, 12, 13$ (modulo16).

For, if n is even$\frac{n}{2} + 1 \leq j \leq n$, if n is odd $\frac{n-1}{2} + 1 \leq j \leq n$.

$\varphi(y_j) = 1 \quad if\ j \equiv 1, 4$ (modulo 8).

$\varphi(y_j) = 2 \quad if\ j \equiv 6, 7$ (modulo 8).

$\varphi(y_j) = 3 \quad if\ j \equiv 0, 5$ (modulo 8).

$\varphi(y_j) = 4 \quad if\ j \equiv 2, 3$ (modulo 8).

Case 2: When $n \equiv 1$ (modulo 16).

For $1 \leq j \leq n - 2$, the labeling of $y_j$values are same as case 1.

$\varphi(y_{n-1}) = 1, \varphi(y_n) = 3.$

Case 3: When $n \equiv 2, 6, 14$ (modulo 16).

For $1 \leq j \leq n - 1$, the labeling of $y_j$values are same as case 1.

$\varphi(y_n) = 3.$

Case 4: When $n \equiv 7, 15$ (modulo 8).

For $1 \leq j \leq n - 4$, the labeling of $y_j$values are same as case 1.

$\varphi(y_{n-2}) = 1, \varphi(y_{n-3}) = \varphi(y_{n-1}) = \varphi(y_n) = 3.$

Case 3: When $n \equiv 8$ (modulo 8).

For $1 \leq j \leq n - 1$, the labeling of $y_j$values are same as case 1.

$\varphi(y_n) = 2.$

The following table shows that $n$concurrence is realized with modulo 16.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0, 8$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}$
$n \equiv 1$	$\frac{3n+5}{8}$	$\frac{3n+5}{8} - 1$	$\frac{3n+5}{8} - 1$	$\frac{3n+5}{8}$
$n \equiv 2$	$\frac{3n+2}{8}$	$\frac{3n+2}{8} - 1$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$
$n \equiv 3$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$	$\frac{3n-1}{8} + 1$

$n \equiv 4, 12$	$\frac{3n+4}{8}$	$\frac{3n+4}{8}-1$	$\frac{3n+4}{8}-1$	$\frac{3n+4}{8}$
$n \equiv 5, 13$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}$
$n \equiv 6, 14$	$\frac{3n-2}{8}+1$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$
$n \equiv 7, 15$	$\frac{3n+3}{8}$	$\frac{3n+3}{8}-1$	$\frac{3n+3}{8}$	$\frac{3n+3}{8}$
$n \equiv 9$	$\frac{3n+5}{8}$	$\frac{3n+5}{8}-1$	$\frac{3n+5}{8}$	$\frac{3n+5}{8}-1$
$n \equiv 10$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}-1$
$n \equiv 11$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}+1$	$\frac{3n-1}{8}$

Table 5.3.5: Vertex labeling of $C_n \circ v\ C_n'$ graph.

The following table shows that $n$concurrence is realized with modulo 16.

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 0, 8$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}+1$
$n \equiv 1, 9$	$\frac{3n+5}{8}$	$\frac{3n+5}{8}-1$	$\frac{3n+5}{8}$	$\frac{3n+5}{8}$
$n \equiv 2, 10$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$
$n \equiv 3$	$\frac{3n-1}{8}+1$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}+1$	$\frac{3n-1}{8}$
$n \equiv 4, 12$	$\frac{3n+4}{8}$	$\frac{3n+4}{8}$	$\frac{3n+4}{8}$	$\frac{3n+4}{8}-1$
$n \equiv 5, 13$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}+1$
$n \equiv 6, 14$	$\frac{3n-2}{8}+1$	$\frac{3n-2}{8}+1$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$
$n \equiv 7, 15$	$\frac{3n+3}{8}$	$\frac{3n+3}{8}$	$\frac{3n+3}{8}$	$\frac{3n+3}{8}$

$n \equiv 11$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}+1$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}+1$

Table 4: Edgelabeling of $C_n \circ v\ C_n'$graph.

The above tables 3 and 4show that $|v_\varphi(i) - v_\varphi(j)| \leq 1$ and$|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the graph $C_n \circ v\ C_n'$is quotient-4 cordial labeling.

Illustration 4. Figure 2 gives the quotient-4 cordial labeling for the graph$C_8 \circ v\ C_8'$.

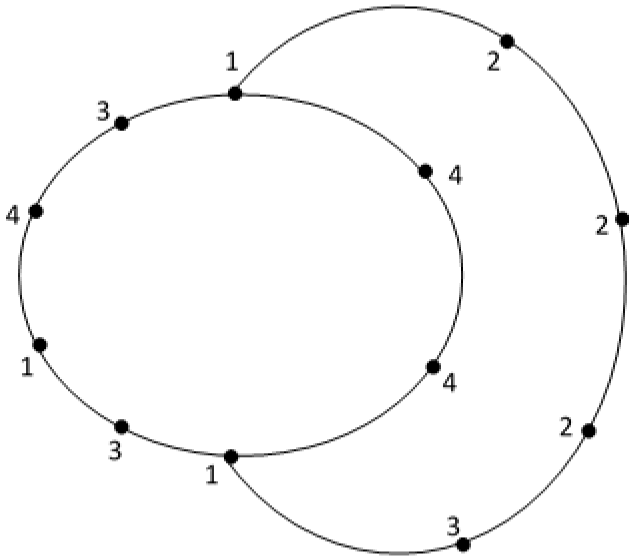

Figure 2.

IV. CONCLUSION

In this paper, it is proved that some tricyclic graphs which admits quotient-4 cordial. The existence of quotient-4 cordial labeling of different families of graphs will be the future work.

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