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Research Paper

On Intuitionistic L-Fuzzy Bi-ideals of Magnified Translation in Ring

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ABSTRACT:
In this paper we introduce and study the concept of intuitionistic L-fuzzy bi-ideals of magnified translation in rings.
KEYWORDS:
Intuitionistic fuzzy subring, Intuitionistic L-fuzzy ideal, Intuitionistic fuzzy magnified translation, Intuitionistic L-fuzzy bi-ideal.

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I. INTRODUCTION

The idea of L-fuzzy set was introduced by J.A Goguen[2] as a generalization of zadeh's[6] fuzzy sets.The concept of intuitionistic fuzzy set was introduced by Atanassov [1].Lajos and szasz[3] initiated the idea of bi-ideals in a ring.The concept of fuzzy magnified translation in ring was introduced by Sanjib kumar Datta[4]P.K. Sharma [5] introduced the notion of an intuitionistic fuzzy magnified translation in ring.In this paper we introduce the concept of intuitionistic L- fuzzy bi-ideals of magnified translation in rings and established some of its properties in detail.

II. PRELIMINARIES

2.1 Definition:
Let X is a fixed non-empty set. An intuitionistic fuzzy set (IFS) A of X is an object of the following form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ where the functions $\mu_A : X\to[0,1]\,\&\,\nu_A : X\to[0,1]$ denote the degree of membership and the degree of non membership respectively, and $0\le\mu_A(x)+\nu_A(x)\le 1 \;\; for \;\; all \; x\in X$.

2.2Definition:
Let R be a ring. An intuitionistic fuzzy set (IFS) A= $\{< x,\mu_A(x),\nu_A(x) >: x\in R\}$ of R is said to be intuitionistic L-fuzzy subring of R (ILFSR) of R if

$(i) \quad \mu_A(x-y)\ge\mu_A(x)\wedge\mu_A(y)$
$(ii) \quad \mu_A(xy) \ge\mu_A(x)\wedge\mu_A(y)$
$(iii) \quad \nu_A(x-y)\le\nu_A(x)\vee\nu_A(y)$
$(iv) \quad \nu_A(xy) \le\nu_A(x)\vee\nu_A(y) \;\; for\, all\; x,y\in R.$

2.3 Definition:
Let R be a ring. An Intuitionistic fuzzy subring A of R is said to be intuitionistic fuzzy normal subring (IFNSR) of R if,

$$(i)\ \mu_A(xy) = \mu_A(yx)$$
$$(ii)\ \nu_A(xy) = \nu_A(yx)$$

2.4 Definition:

An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in R\}$ of a ring R said to be

(a) Intuitionistic L-fuzzy left ideal of R(ILFLI) of R if

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\mu_A(xy) \geq \mu_A(y)$$
$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$$
$$\nu_A(xy) \leq \nu_A(y)\ \ \textit{for all } x, y \in R.$$

(b) Intuitionistic L-fuzzy Right ideal of R (ILFRI) of R if

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\mu_A(xy) \geq \mu_A(x)$$
$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$$
$$\nu_A(xy) \leq \nu_A(x)\ \ \textit{for all } x, y \in R.$$

(c) Intuitionistic L-fuzzy ideal of R(ILFI) 0f R if,

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$$
$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$$
$$\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)\ \ \textit{for all } x, y \in R.$$

2.5 Definition:
Let R be a ring and L be a lattice. A fuzzy set μ of R is said to be L-fuzzy bi-ideal of R if

$$(i)\quad \mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$(ii)\quad \mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$$
$$(iii)\quad \mu_A(xyz) \geq \mu_A(x) \wedge \mu_A(z)\ \ \textit{for all } x, y, z \in R$$

Example:
Consider the fuzzy set μ of R by

$$\mu_A(x) = \begin{Bmatrix} 0.9\, \textit{if}\ \ x\, \textit{is rational} \\ 0.5\, \textit{if}\ \ x\, \textit{is irrational} \end{Bmatrix}$$

Then μ_A is an L-fuzzy bi-ideal of R.

III. INTUITIONISTIC L-FUZZY BI-IDEAL OF MAGNIFIED TRANSLATION

3.1 Definition:
Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X and $\beta \in [0,1]$ & $\alpha \in [0, 1 - \sup\{\mu_A(x) + \nu_A(x) : x \in X, 0 < \mu_A(x) + \nu_A(x) < 1\}]$. Then the intuitionistic fuzzy magnified

translation (IFMT) T of A is an object of the form : $T = \left\{\left\langle x, \mu^A_{(\beta,\alpha)}(x), \nu^A_{(\beta,\alpha)}(x)\right\rangle : x \in X\right\}$ or briefly as $\left\{\left\langle x, \mu_T(x), \nu_T(x)\right\rangle : x \in X\right\}$ where the functions,

$\mu^A_{(\beta,\alpha)}(x) = \mu_T : X \to [0,1]$ & $\nu^A_{(\beta,\alpha)}(x) = \nu_T : X \to [0,1]$ are defined as

$$\mu_T(x) = \mu^A_{(\beta,\alpha)}(x) = \beta\mu_A(x) + \alpha,$$

$$\nu_T(x) = \nu^A_{(\beta,\alpha)}(x) = \beta\nu_A(x) + \alpha, \text{ for all } x \in X.$$

Example:

Let $X = \{1, \omega, \omega^2\}$

Let $A = \left\{\langle 1, 0.2, 0.7\rangle, \langle \omega, 0.3, 0.45\rangle, \langle \omega^2, 0.6, 0.1\rangle\right\}$ be an IFS of X. Then ,

$\left[0, 1 - \sup\{\mu_A(x) + \nu_A(x) : x \in X, 0 < \mu_A(x) + \nu_A(x) < 1\}\right] = [0, 0.1]$ Take, $\alpha = 0.3, \beta = 0.4$

Then IFMT of the IFS A is given by $T = \left\{\langle 1, 0.38, 0.58\rangle, \langle \omega, 0.42, 0.48\rangle, \langle \omega^2, 0.54, 0.34\rangle\right\}$

3.2 Definition:

An intuitionistic fuzzy set (IFS) $A = \left\{\left\langle x, \mu_A(x), \nu_A(x)\right\rangle : x \in R\right\}$ of a ring R is said to be intuitionistic L-fuzzy bi-ideal of R (ILFBI) of R if

$$\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$$

$$\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$$

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$$

$$\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$$

$$\mu_A(xry) \geq \mu_A(x) \wedge \mu_A(y)$$

$$\nu_A(xry) \leq \nu_A(x) \vee \nu_A(y) \quad \text{for all } x, y, r \in R.$$

3.3 Theorem:

Let μ and ν be an intuitionistic L-fuzzy bi-ideal of a ring R. Then the intuitionistic fuzzy magnified translation $\mu^A_{(\beta,\alpha)}(x), \nu^A_{(\beta,\alpha)}(x)$ is also an intuitionistic L-fuzzy bi-ideal of R.

Proof:

Let μ and ν be an intuitionistic L-fuzzy bi-ideal of a ring R.Now for all $x, y, r \in R$,

$$\mu^A_{(\beta,\alpha)}(x - y) = \beta\,\mu_A(x - y) + \alpha$$

$$\geq \beta\,\{\mu_A(x) \wedge \mu_A(y)\} + \alpha$$

$$= (\beta\,\mu_A(x) + \alpha) \wedge (\beta\,\mu_A(y) + \alpha)$$

$$\mu^A_{(\beta,\alpha)}(x - y) \geq \mu^A_{(\beta,\alpha)}(x) \wedge \mu^A_{(\beta,\alpha)}(y)$$

Again,

$$\mu^A_{(\beta,\alpha)}(xy) = \beta\,\mu_A(xy) + \alpha$$

$$\geq \beta\,\{\mu_A(x) \wedge \mu_A(y)\} + \alpha$$

$$= (\beta\,\mu_A(x) + \alpha) \wedge (\beta\,\mu_A(y) + \alpha)$$

$$\mu^A_{(\beta,\alpha)}(xy) \geq \mu^A_{(\beta,\alpha)}(x) \wedge \mu^A_{(\beta,\alpha)}(y)$$

Also,

$$\mu^A_{(\beta,\alpha)}(xry) = \beta\,\mu_A(xry) + \alpha$$
$$\geq \beta\{\mu_A(x) \wedge \mu_A(y)\} + \alpha$$
$$= (\beta\,\mu_A(x) + \alpha) \wedge (\beta\,\mu_A(y) + \alpha)$$
$$\mu^A_{(\beta,\alpha)}(xry) \geq \mu^A_{(\beta,\alpha)}(x) \wedge \mu^A_{(\beta,\alpha)}(y)$$
$$\nu^A_{(\beta,\alpha)}(x-y) = \beta\,\nu_A(x-y) + \alpha$$
$$\leq \beta\{\nu_A(x) \vee \nu_A(y)\} + \alpha$$
$$= (\beta\,\nu_A(x) + \alpha) \vee (\beta\,\nu_A(y) + \alpha)$$
$$\nu^A_{(\beta,\alpha)}(x-y) \leq \nu^A_{(\beta,\alpha)}(x) \vee \nu^A_{(\beta,\alpha)}(y)$$

Again,

$$\nu^A_{(\beta,\alpha)}(xy) = \beta\,\nu_A(xy) + \alpha$$
$$\leq \beta\{\nu_A(x) \vee \nu_A(y)\} + \alpha$$
$$= (\beta\,\nu_A(x) + \alpha) \vee (\beta\,\nu_A(y) + \alpha)$$
$$\nu^A_{(\beta,\alpha)}(xy) \leq \nu^A_{(\beta,\alpha)}(x) \vee \nu^A_{(\beta,\alpha)}(y)$$

Also,

$$\nu^A_{(\beta,\alpha)}(xry) = \beta\,\nu_A(xry) + \alpha$$
$$\leq \beta\{\nu_A(x) \vee \nu_A(y)\} + \alpha$$
$$= (\beta\,\nu_A(x) + \alpha) \vee (\beta\,\nu_A(y) + \alpha)$$
$$\nu^A_{(\beta,\alpha)}(xry) \leq \nu^A_{(\beta,\alpha)}(x) \vee \nu^A_{(\beta,\alpha)}(y)$$

Thus, $\mu^A_{(\beta,\alpha)}(x), \nu^A_{(\beta,\alpha)}(x)$ is anintuitionistic L-fuzzy bi-ideal of R.

3.4 Theorem:

If μ and ν be an intuitionistic L-fuzzy left (right,two sided) ideal of a ring R,then the intuitionistic fuzzy magnified translation $\mu^A_{(\beta,\alpha)}(x), \nu^A_{(\beta,\alpha)}(x)$ of μ and ν is an intuitionistic L-fuzzy bi-ideal of R.

Proof:

Let μ and ν be an intuitionistic L-fuzzy left ideal of a ring R.

Then for all $x, y \in R$

$$\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$$
$$\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$$
$$\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y) \quad \textit{for all } x, y \in R.$$

Also let $x, r, y \in R$

Then,

$$\mu(xry) \geq \mu((xr)y) \geq \mu(y) \geq \mu(x) \wedge \mu(y)$$
$$\nu(xry) \leq \nu((xr)y) \leq \nu(y) \leq \nu(x) \vee \nu(y)$$

Thus μ & ν will be an intuitionistic L-fuzzy bi-ideal of R.

Now for all $x, y \in R$

$$\mu^{A}_{(\beta,\alpha)}(x-y)=\beta\,\mu_A(x-y)+\alpha$$
$$\geq \beta\{\mu_A(x)\wedge\mu_A(y)\}+\alpha$$
$$=(\beta\,\mu_A(x)+\alpha)\wedge(\beta\,\mu_A(y)+\alpha)$$
$$\mu^{A}_{(\beta,\alpha)}(x-y)\geq \mu^{A}_{(\beta,\alpha)}(x)\wedge\mu^{A}_{(\beta,\alpha)}(y)$$

Again,

$$\mu^{A}_{(\beta,\alpha)}(xy)=\beta\,\mu_A(xy)+\alpha$$
$$\geq \beta\{\mu_A(x)\wedge\mu_A(y)\}+\alpha$$
$$=(\beta\,\mu_A(x)+\alpha)\wedge(\beta\,\mu_A(y)+\alpha)$$
$$\mu^{A}_{(\beta,\alpha)}(xy)\geq \mu^{A}_{(\beta,\alpha)}(x)\wedge\mu^{A}_{(\beta,\alpha)}(y)$$

Also let $x,r,y\in R$

$$\mu^{A}_{(\beta,\alpha)}(xry)=\beta\,\mu_A(xry)+\alpha$$
$$\geq \beta\{\mu_A(x)\wedge\mu_A(y)\}+\alpha$$
$$=(\beta\,\mu_A(x)+\alpha)\wedge(\beta\,\mu_A(y)+\alpha)$$
$$\mu^{A}_{(\beta,\alpha)}(xry)\geq \mu^{A}_{(\beta,\alpha)}(x)\wedge\mu^{A}_{(\beta,\alpha)}(y)$$

Now

$$\nu^{A}_{(\beta,\alpha)}(x-y)=\beta\,\nu_A(x-y)+\alpha$$
$$\leq \beta\{\nu_A(x)\vee\nu_A(y)\}+\alpha$$
$$=(\beta\,\nu_A(x)+\alpha)\vee(\beta\,\nu_A(y)+\alpha)$$
$$\nu^{A}_{(\beta,\alpha)}(x-y)\leq \nu^{A}_{(\beta,\alpha)}(x)\vee\nu^{A}_{(\beta,\alpha)}(y)$$
$$\nu^{A}_{(\beta,\alpha)}(xy)=\beta\,\nu_A(xy)+\alpha$$
$$\leq \beta\{\nu_A(x)\vee\nu_A(y)\}+\alpha$$
$$=(\beta\,\nu_A(x)+\alpha)\vee(\beta\,\nu_A(y)+\alpha)$$
$$\nu^{A}_{(\beta,\alpha)}(xy)\leq \nu^{A}_{(\beta,\alpha)}(x)\vee\nu^{A}_{(\beta,\alpha)}(y)$$

Also let $x,r,y\in R$

$$\nu^{A}_{(\beta,\alpha)}(xry)=\beta\,\nu_A(xry)+\alpha$$
$$\leq \beta\{\nu_A(x)\vee\nu_A(y)\}+\alpha$$
$$=(\beta\,\nu_A(x)+\alpha)\vee(\beta\,\nu_A(y)+\alpha)$$
$$\nu^{A}_{(\beta,\alpha)}(xry)\leq \nu^{A}_{(\beta,\alpha)}(x)\vee\nu^{A}_{(\beta,\alpha)}(y)$$

Thus, $\mu^{A}_{(\beta,\alpha)}(x),\nu^{A}_{(\beta,\alpha)}(x)$ is an intuitionistic L-fuzzy bi-ideal of R.

3.5 Theorem:
If T is an intuitionistic fuzzy magnified translation of an intuitionistic L-fuzzy bi-ideal A of a ring R then

$(i)\ \mu_T(x^{-1})=\mu_T(x)\ and\ \nu_T(x^{-1})=\nu_T(x)$

$(ii)\ \mu_T(x)\leq\mu_T(0)\ \&\ \nu_T(x)\geq\nu_T(0)\ \ for\ all\ x,0\in R.$

Proof:

Let $x\ and\ 0$ be elements of R.

Now,

$$\begin{aligned}\mu_T(x) = \mu^A_{(\beta,\alpha)}(x) &= \beta\,\mu_A(x)+\alpha \\ &= \beta\;\mu_A((x^{-1})^{-1})+\alpha \\ &\geq \beta\,\mu_A(x^{-1})+\alpha \\ &= \mu_T(x^{-1})+\alpha \\ &= \beta\,\mu_A(x^{-1})+\alpha \\ &\geq \beta\,\mu_A(x)+\alpha \\ &= \mu_T(x)\end{aligned}$$

$$\begin{aligned}\nu_T(x) = \nu^A_{(\beta,\alpha)}(x) &= \beta\,\nu_A(x)+\alpha \\ &= \beta\;\nu_A((x^{-1})^{-1})+\alpha \\ &\leq \beta\,\nu_A(x^{-1})+\alpha \\ &= \nu_T(x^{-1})+\alpha \\ &= \beta\,\nu_A(x^{-1})+\alpha \\ &\leq \beta\,\nu_A(x)+\alpha \\ &= \nu_T(x)\end{aligned}$$

$Therefore\, \nu_T(x) = \nu_T(x^{-1})\;\; \forall\, x\ in\, R.$

$$\begin{aligned}\mu_T(0) &= \beta\,\mu_A(0)+\alpha \\ &= \beta\,\mu_A(x-x)+\alpha \\ &\geq \beta\{\mu_A(x)\wedge\mu_A(x)\}+\alpha \\ &\geq (\beta\,\mu_A(x)+\alpha)\wedge(\beta\,\mu_A(x)+\alpha) \\ &\geq \mu_T(x)\wedge\mu_T(x) \\ &= \mu_T(x)\end{aligned}$$

$Therefore\;\; \mu_T(0) \geq \mu_T(x)$

$$\begin{aligned}\nu_T(0) &= \beta\nu_A(0)+\alpha \\ &= \beta\nu_A(x-x)+\alpha \\ &\leq \beta\{\nu_A(x)\;\nu_A(x)\}+\alpha \\ &\leq (\beta\nu_A(x)+\alpha)\vee(\beta\nu_A(x)+\alpha) \\ &\leq \nu_T(x)\vee\nu_T(x) \\ &= \nu_T(x)\end{aligned}$$

$Therefore\;\; \nu_T(0) \leq \nu_T(x)\;\; \forall\, x,0 \in R.$

3.6 Theorem:
Let T be an intuitionistic fuzzy magnified translation of an intuitionistic L-fuzzy bi-ideal A of a ring R.
$If\;\; \mu_T(xy^{-1}) = 1,\; then\;\; \mu_T(x) = \mu_T(y)\;\; and\; if\;\; \nu_T(xy^{-1}) = 0,\; then\;\; \nu_T(x) = \nu_T(y)$

Proof:

Let $x\; and\; y$ be elements of R.

Now,

$$\mu_T(x) = \mu_T(xy^{-1}y)$$
$$\geq \mu_T(xy^{-1}) \wedge \mu_T(y)$$
$$= 1 \wedge \mu_T(y)$$
$$= \mu_T(y)$$
$$= \mu_T(y^{-1})$$
$$= \mu_T(x^{-1}xy^{-1})$$
$$\geq \mu_T(x^{-1}) \wedge \mu_T(xy^{-1})$$
$$= \mu_T(x) \wedge 1$$
$$= \mu_T(x)$$
$$\therefore \mu_T(x) = \mu_T(y) \text{ for all } x \text{ and } y \text{ in } R.$$
$$\nu_T(x) = \nu_T(xy^{-1}y)$$
$$\leq \nu_T(xy^{-1}) \vee \nu_T(y)$$
$$= 0 \vee \nu_T(y)$$
$$= \nu_T(y)$$
$$= \nu_T(y^{-1})$$
$$= \nu_T(x^{-1}xy^{-1})$$
$$\leq \nu_T(x^{-1}) \vee \nu_T(xy^{-1})$$
$$= \nu_T(x) \vee 0$$
$$= \nu_T(x)$$
$$\therefore \nu_T(x) = \nu_T(y) \text{ for all } x \text{ and } y \text{ in } R.$$

IV. CONCLUSION

In this article authors have been discussedintuitionistic L-fuzzy bi-ideals of magnified translation in rings. Using these, various results can be developed under the topic an intuitionistic L-fuzzy bi-ideals of magnified translation.

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