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Research Paper

Characterizations Of 3- Minamally Nonouterplanar Semitotal-Block Graphs:

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ABSTRACT: *In this paper, we obtain characterization of graphs whose semitotal-block graphs is 3- minimally nonouterplanar in terms of forbidden sub-graphs.*
KEYWORDS: *semitotal-block graphs, minimally nonouterplanar, planarity, outerplanarity, homeomorphic, isomorphic*

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I. INTRODUCTION:

In [2] kulli introduced the concept of the semitotal-block graphs and total-blockgraps.In [3] and [4], the planarity and outerplanarity of these graph valued functions were discussed. In[5], one finds the minimally nonouterplanarity of these graph valued functions. In [1], D.G.Akka and M.S.Patil finds the 2-minimally nonouterplanarity of these graph valued functions. In [6], M.H. Muddebihal, jayashree.B.Shetty and Shabbir Ahmed finds the 3-minimally nonouterplanarity of these graphs valued functions. In this paper we obtain the characterizations of graphs whose semitotal-block graphs are 3–minimally nonouterplanar in terms of forbidden subgraphs.

The following definitions will be noted for later use. A graph G is called a block if it has more than one vertex, is connected and has no cutvertices block of a graph G is a maximal subgraph of G which itself a block.
If B={u_1, u_2,_ _ _ _, u_r ; $r \geq 2$} is a block of G, then we say that vertex u_1 and block B are incident with each other as are u_2 and B so on.
If two distinct blocks B_1 and B_2 are incident with a common cutvertex, than they are adjacent blocks. The vertices and blocks of a graph are called the members.

The following will be useful in the proof of our results

Theorem A [6], the semitotal –block graph $T_b(G)$
Of a connected graph G is 3-minimally, nonouterplanar if and only if.[1] or[2] or [3] holds.
1) G has exactly three cycles and each cycle is a block
Or
2) G is either P_4+K_1 of K_4 -x.Cn
Or
3) G is a Cycle C_n ($n \geq 6$) together with a diagonal edge joining a pair of vertices of length (n-3).

II. FORBIDDEN SUBGRAPHS

By using theorem A, we now characterizations of graphs whose 3-minimally nonouterplanar semitotal-block graphs in terms of forbidden subgraph as follows.

Theorem 1: A connected planar graph G is 3-minimally nonouterplanar semitotal-blockgraph if and only if G has no subgraph homeomorphic to

1) A graph G which has exactly four blocks and each block is a Cycle.
   Or
2) G has exactly three blocks in which one blocks is isomorphic to K_4 -x and two blocks are Cycles
   Or
3) G has exactly two blocks in which each block is isomorphic to K_4 -x remaining blocks of G are edges
or
4) P_5+K_1 or H_1 or H_2 {see fig 1}

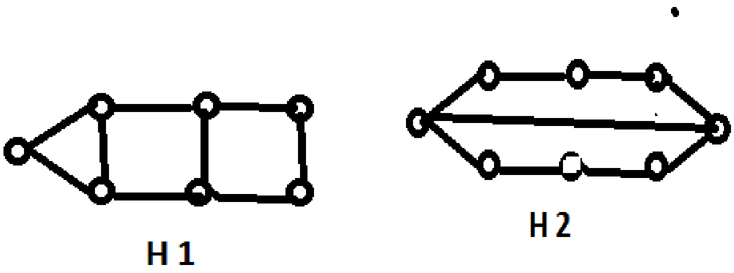

Figure-1

Proof: - If G is a connected planar graph with 3-minimally nonouterplnar semitotal-block graph. Therefore we need to show that all graphs isomorphic to P_5+K_1 or G has exactly four blocks and each block is homeomorphic to cycle C_p with $P\geq3$ vertices or G has exactly three blocks in which one block is isomorphic to K_4 -x and two blocks which are homeomorphic to cycles C_p with $P\geq3$ vertices or G has exactly two blocks in which each block is isomorphic to K_4 -x and remaining blocks of G are edges or G has homeomorphic to H_1 or H_2 For all the above conditions, G have no 3-minimally nonouterplanar semitotal-block graph. This follows from theorem A.

Since graph isomorphic to P_5+K_1 is a block which contains exactly four interior regions and is also a maximal outerplanar in condition 1,G has four cycles of length C_n [$n\geq3$] as blocks, in condition 2,G has four cycles as three blocks, in condition 3, G has four Cycles as exactly two blocks each isomorphic to K_4 -x and remaining blocks are edges, graph homeomorphic to H_1 has a cycle C_n [$n\geq7$] together with two diagonal edges each joining a pair of vertices of length two and three and graph homeomorphic to H_2 has a cycle C_n [$n\geq8$] together with a diagonal edge joining a pair of vertices of length at least (n-4)

To prove sufficiency, assume G is 3-minimally nonouterplanar graph which does not contain subgraph homeomorphic to P_5+K_1 or G has exactly four blocks and each block is a cycle or G has exactly three blocks in which one block is isomorphic to K_4 -x and two blocks are cycles or G has exactly two blocks in which each block is isomorphic to K_4 -x remaining blocks of G are edges or H_1 or H_2.
We consider the following cases

Case1. Suppose G has four cycles. Then we have two following subcases of case1.

Subcase 1.1: Assume G is a block which contains four cycles. Then G has a subgraph homeomorphic to P_5+K_1, a contradiction. Thus G has a P_4+K_1 as a block which contains three cycles.

Subcase 1.2: Assume G has exactly two blocks in which each block is isomorphic to K_4 -x and remaining blocks of G are edges. Let v_1 and v_2 are two cutvertices in G such that vertices v_1 and v_2 lies on exactly two blocks each. In which one block is K_4 -x and other block is an edge. Then G has subgraph homeomorphic to condition 3.

Subcase1.3: Assume G has exactly three blocks in which one block is isomorphic to K_4 -x and remaining two blocks are cycles C_n [$n\geq3$]. Let G has exactly one cut vertex of degree ≥6, such that vertex v lies on three blocks in which one block is isomorphic K_4-x and remaining two blocks are cycle C_n [$n\geq3$]. Than G has a subgraph is homeomorphic to condition 2.

From the above subcases 1.2 and 1.3 we conclude that G has exactly two blocks such that one block is isomorphic to K_4 -x and other one is a cycle C_n [n≥3].

Subcase 1.4: Assume G has exactly four cycles as blocks. Than we consider the following subcases of subcase 1.4

Subcase1.4.1: suppose G has four blocks. Let v be a cut vertex in G such that v lies on four blocks in which each block contains a cycle. Than G has a subgraph homeomorphic to condition 1, a contradiction.
Subcase 1.4.2: Supposes G has exactly four blocks as cycles and remaining blocks are edges. Let $v_{1,}$ v_2 and v_3 are cutvertices in G such that vertex is v_1 is lies on exactly three blocks in which each block contains a cycle, the vertex v_2 lies on exactly two blocks in which one block is a cycle and other is a edge. Similarly vertex v_3 lies an exactly two blocks in which one block is a cycle and other is an edge. Than G has a subgraph homeomorphic to condition 1, a contradiction.

Subcase 1.4.3: Suppose G has exactly four blocks as cycles and remaining blocks are edges. Let v_1, $_2$, v_3, and v_4, are the cutverties in G, such that each cutvertex v_i [i=1 ,2, 3, 4] lies on exactly two blocks in which one block is a cycle and other block is an edge. Than G has a subgraph homeomorphic to condition 1, a contradiction.

From the above subcases 1.4.1, 1.4.2 and 1.4.3. We conclude that G has exactly three cycles as blocks.

Case 2: Suppose G has at least two diagonal edges. Then there are two subcases to consider depending on whether the two diagonal edges exits in one cycle or in two different edge disjoint cycles.

Subcsase 2.1 Assume two diagonal edges exist in one cycle. Than G has a subgraph homeomorphic to H_1, a Contradiction.

Subcase 2.2 Assume two diagonal edges exist in different edge disjoint cycles. Than G has subgraph homeomorphic to condition3, a contradiction. In each subcase we have a contradiction. Hence G has exactly one diagonal edge. Than we discuss in the following case.

Case 3: Suppose G has exactly one diagonal edge exist in a cycle C_n [n≥8] together with a diagonal edge joining a pair of vertices of length [n-4]. Than G has a subgraph homeomorphic to H_2, a contradiction.

From above cases we conclude that G is a Cycle C_n [n≥6] together with a diagonal edge joining a pair of vertices of length [n-3].
Thus by theorem A, G has 3-minimally nonouterplanar semitotal-block graph.
Henc the Proof.

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