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Research Paper

Application of Beta-Binomial and Negative Binomial Models To Road Traffic Accidents in Nigeria

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ABSTRACT

Road traffic accident in Nigeria is known to be a major cause of untimely death. The ultimate goal of this research is to apply Beta Binomial model and Negative Binomial model in tracking the pattern of the roads traffic accident data and this was done based on the secondary data collected from Federal Road Safety Corps, on the fatal, serious and minor road traffic accidents for eight consecutive years.

Akaike information criteria (AIC) and Bayesian information criteria (BIC) were used to show the efficiency of Negative Binomial model over Beta Binomial. From the result of the analysis, Negative binomial model performed better in its efficiency and consistent result shown by AIC for the fatal, serious and minor road traffic accidents, it was clear that the Negative Binomial model was the best fit for the data. It was observed from the result that in general the number of fatality in road traffic accidents gradually increases with time as the years go by, the number of people killed by road accidents increases. The kind of vehicle involved in an accident and over speed was found to be associated with the highest factor that influences the rate of road traffic accidents.

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I. INTRODUCTION

1.1 Background to the study

In some applications of Bernoulli trials, the underlying success probability could change from one trial to another, that is, likelihood based techniques has to be changed. This is common in conditions where there are unmodeled influences that affect all of the components of the binomial sum which count the number of successes in a fixed number N of Bernoulli trials. This takes place, as an instance, in organic or agriculture applications, when we have batch effects (see as an instance, Kleinman (2008); Crowder (2008); Haseman and Kupper (2009);Akomolafe.et.al(2009), Williams (2005) or Morgan (2002)). A litter effect is related to the tendency that members of a group respond in a more similar way to some treatment than members of other groups. These random outcomes have to be covered within the modeling of the information set besides the same old covariates to account for this overdispersion. The beta-binomial model was introduced by Pearson (2005) and more officially defined by Skellam (2008). It's far a famous technique for explicitly account for the over dispersion. We are able to discover several packages of this version in diverse areas, such as Chatfield and Goodhardt (2006), who described the buying behaviour of the consumer, and Gange et al. (2006), who studied the effect of coverage adjustments on appropriateness of clinic admissions. In addition, Aeschbacher et al. (2007) showed that a beta- binomial distribution furnished a higher match than the usual distributions in organic experiments using mice's while the statistics used were based totally on a large wide variety of counts of dead.

The prevalence of a motor car crash can, in concept, be represented via a Binomial process. As such, every car that enters an intersection or travels through a section both crashes or does no longer crash. This procedure is referred to as a Bernoulli trial and the summation of those trials (i.e, the cars) gives rise to the Binomial distribution e.g., B(n, p), where p refers to the probability of occurrence and n is the variety of trials). For typical vehicle crashes wherein the occasion has a totally low probability of incidence and a massive range of trials present (e.g. million coming into vehicles, automobile-miles-traveled, and many others.), it could be shown that the Binomial distribution can be approximated by using a Poisson distribution.

In most road traffic accidents in Nigeria, the principle reason is the negligence of drivers, although mechanical faults and road deficiencies play primary roles in coincidence occurrences.

The importance of effective transport to socio-economic and political improvement has lengthy been set up (Addo, 1979; 2012; Abane, 2010; 2012; Abane et al., 2010). The supply of road networks and transportation systems connect settlements and open them as much as investment possibilities. Indeed, modern financial activities have been followed by means of a huge boom in mobility as a result of higher stages of accessibility. But, in maximum developing international locations, transport structures have posed a non-stop assignment to satisfying mobility needs to help financial improvement (Leinbach, 1986), while acclaiming the significance of transport development, it's far important to bear in thoughts that as with every human sports, offering transportation isn't a panacea to solving all development issues. The role of transportation in socioeconomic development is neither direct nor automated (Abane, 2012), further refers back to the reality that funding in transportation has the ability to spell doom and often compels beneficiaries to whinge, wishing such a thing had no longer been covered inside the device. This thing, he says, consists of diverse disasters, which he listed as follows:

i. Air pollution in our towns, cities and the nation at large.

ii. Excessive road traffic congestion, which makes travelling in my opinion and financially inconvenient and time-ingesting.

iv. Vehicular crashes that maim and kill both old and young, and an increase inside the world's disability adjusted existence Years.

Widely, Abane's postulations imply that road traffic crashes (RTCs), in any other case referred to as road traffic accidents (RTAs), are normally accidental and preventable. This is in accord with the statement of the World Health Organization(WHO) in 2009 that road traffic accidents are increasingly more becoming a hazard to public health and country wide development in lots of developing nations (WHO, 2009). Mainly, RTCs make a contribution to poverty with the aid of inflicting deaths, injuries, disabilities, grief, and loss of productiveness and cloth harm (WHO,2002). As a result of the amazing effect of road accidents on human lives, homes and the environment, several researchers has come out with the reasons, causes and recommendations to road accidents.

Researchers in recent times have been modeling road injuries with crash prevention models in numerous parts of the sector. However, it's far extraordinarily tedious to just road accident is defined as any pastime that distracts the everyday trajectory of moving cars in a way that causes instability within the unfastened glide of the car. Road accidents in Nigeria has been a critical challenge to many observers which have worked in different vicinity acquiring statistics from different international locations because of the versions inside the different factors as it affects different countries (Fletcher et al., 2006). There has now not been plenty statistical research work conducted with appreciate to accidents on roads in Nigeria and this could had been because of insufficient statistics available on accidents on roads and its effect on human lives and the surroundings as a whole within the country Salifu (2004) developed a forecasting model for traffic crashes for road junctions without a street symptoms, Afukaar and Deborah (2007) alternatively modeled traffic crashes for signalized urban junctions in Nigeria and Akachukwu (2011) moreover modeled road traffic crashes on rural highways in location. Road traffic accidents have constantly been attributed to human mistakes which includes excessive alcoholic content in blood taken by many drivers, over dashing, incorrect-overtaking among others. It has additionally been related to poor road networks, poor surfacing of roads, witchcraft and loss of life death nature of a few automobiles which ply the roads. It is however surprising that there spite of the numerous factors identified by means of researchers as causes of road accidents in Nigeria and its results on human lives and properties; no one has modeled the reasons of accidents on city roads and their contributions to the demise of casualties in Nigeria to authenticate the contributions of every of these elements to the casualties. it's miles therefore in light of this that this research is performed in an effort to decide the appropriate rely regression version that fits coincidence information with admire to the predicted quantity of men and women who can be killed through road accidents on city roads in Nigeria and moreover use the regression version to analyze the key predictors of accident that consequences in the death of casualties.

Road traffic accidents occur when a vehicle collides with another vehicle, pedestrian, animal, road debris, or other stationary obstruction, such as a tree or utility pole. Worldwide, road traffic accidents lead to death and disability as well as financial cost to both society and the individual involved.There is generally increasing incidence, morbidity and mortality rates of road traffic accidents . People are injured in road accidents everyday more so in developing countries like Nigeria, the problem is that the enormity of the problem is not appreciated and enough preventive measures are not taken.

1.2 OVERVIEW OF BETA BINOMIAL MODEL

The beta-binomial model has been widely used as an analytically tractable alternative that captures the overdispersion of a binomial random variable, *X*, which is a sum of Bernoulli random variables with success probability *p* and intraclass correlation *ρ*, [Skellam \(1948\).](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5736152/#R8) When *p* is assumed to follow a beta distribution, *p* ∼ Beta (α, β) and then unconditional on *p*, the *X* follows a beta-binomial distribution (BBD). Here, we denote this as *Z*. Similar to a beta distribution, a probability mass function of *Z* takes on a few different shapes: for example, Bell-shape, J-shape, inverse J-shape, and U-shape/Bimodal. Although the success probability and intra class correlation of *Z* are identical to those of *X*, there may exist significant discrepancy in distribution between them as the bimodal shape is more likely to reflect the intra class correlation between Bernoulli random variables within *X*. In such cases, inference based on the beta-binomial model approach is biased.

Many researchers have contributed to the theory of beta binomial distribution and its applications in various fields, among them are Akomolafe (2009), Pearson (1925), Skellam (1948), Lord (1965), Greene (1970), Massy et. al. (1970), Griffiths (1973), Williams (1975), Huynh (1979), Wilcox (1979), Smith (1983), Lee and Sabavala (1987), Hughes and Madden (1993), and Shuckers (2003), are notable.

If we consider X, the number of successes in n Bernoulli experiments, in which p is the probability of success in an individual trial, the variability of X often exceeds the binomial variability $np(1-p)$. This is known as overdispersion and is caused by the violation of any of the hypotheses of the binomial model: independence of observations or variability of the parameter p. A possible way to analyze this is to develop a parametric model for the excess dispersion. Since the parameter p varies from individual to individual, it is considered to be a continuous random variable P with density function $f_P(p)$ for $0 \angle p \angle 1$. It has frequently been proposed that P has a beta(*α*, *β*) distribution and, hence, X has a beta–binomial (BB) distribution (Chatfield and Goodhart, 1970; Williams, 1975; Lindsey and Altham, 1998).

A few methods have been developed for the goodness-of-fit (GOF) test of a beta-binomial model [\(Brooks](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5736152/#R2) *et. al*, [1997](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5736152/#R2) an[d Garren](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5736152/#R4) *et. al*, 2000).

1.3 OVERVIEW OF NEGATIVE BINOMIAL MODEL

In the sequence of independent Bernoulli $\,(\,p\,)$ trials let the random variable X denote the trials at which the

$$
r^h
$$
 success occurs, where r is a fixed integer. Then
\n
$$
p(X = x \setminus r, p) = {x-1 \choose r-1} p^r (1-p)^{x-r}, x = r, r+1...
$$

And we say that X has a negative binomial (r, p) distribution. The negative binomial distribution is sometimes defined in terms of the random variables Y=number of failures before r^{th} success. This formulation is statistically equivalent to the one given above in terms of X=trials at which the r^{th} success occurs, since $Y = X - r$.

1.4 SIGNIFICANCE OF THE STUDY

The study on Beta-Binomial and Negative Binomial models and its applications to road traffic accident will be of immense benefit to policy makers so as to come out with strategies to reduce the numerous deaths caused by vehicular accidents, Federal Road Safety Corps, health authorities, as well as the specialization area. The study will serve as a repository of information to other researchers that desire to carry out similar research on the above findings so as to contribute to the body of the existing literature on Beta-Binomial model and its application.

1.5. STATEMENT OF THE PROBLEM

Range of methods are being carried out to test and identify road traffic accident of vehicle but no automatic gadget has been advanced up till now that can carry out the project without human assistance. The essential issues visible at the present context are :

Road accidents are increasing day by day with prime motive being the over speeding of the vehicle. The government of Nigeria has over the years made strenuous efforts to finance and develop the Road network nationwide. An evaluation of the country's budgets shows that road creation and maintenance are allocated huge percentage of the yearly expenditure, no matter the big investment in road infrastructure development in Nigeria, the country is yet to achieve the benefits of its investments, in large part because of the increasing loss of life toll on the roads. The state of affairs of increasing death toll on the roads has come to be so worrisome that the road surroundings is now defined as a 'loss of life trap' and increasingly more perceived as a main development and public-health issue, causing pain and hardship to most Nigerians (FRSC, 2002).

1.6. **AIM AND OBJECTIVE OF THE STUDY**

The main aim of this research work is to apply Beta-Binomial model to road traffic accident. The specific objectives are:

- (a) Investigate the tracking pattern of beta-binomial and negative-binomial models.
- (b) Compare the efficiency of beta-binomial and negative-binomial models.
- (c) Examine the factors influencing the rate of road traffic accidents and detecting the most significant factors.

1.7. JUSTIFICATION OF THE STUDY

Nigeria's central problem in road safety has been diagnosed as indiscipline, especially among motorists/drivers. Indiscipline results in recklessness and other unacceptable behaviours that cause general dismiss for traffic regulations and rules. Indeed, within the past decade a global census has revealed that Nigeria's road safety institutions need to be strengthened in the legal, administrative, and financial spheres to enable them to deliver positively, the purpose is that such institutional capability-building has to this point been insufficient.

This study consequently seeks to contribute to a deeper understanding of the factors responsible for road traffic accidents and associated injuries. For road transport policy makers, the findings of this observations will aid, guard and guide the planning and evaluation road safety measures and strengthening the capacity of appropriate institutions in approaches that might be replicated elsewhere in the country. Findings from this research will serve as benefit to health authorities and road traffic law enforcement agencies of various form as well as forming the basis for future research.

1.8 LIMITATIONS OF THE AVAILABLE STATISTICS

Details of traffic crashes are not available at the state level. It is believed that not all accidents are reported to the Federal road safety corps for records to be made on them due the human nature and Delta state hospitality attitude. Also, it is possible that the Federal road safety corps might not have filled the accident report form for all accidents which might have been reported to them. Even as the serious accident data may be close to the actual number, the fatal and minor data are gross underestimates. Underreporting of road traffic accidents is a serious and global problem, the spectrum of accidents from road crashes varies from fatal, serious and minor road traffic accident.

A minor accident and fatal accidents are not reported to the federal road safety corps due to several reasons.

1.9. DEFINITION OF TERMS

This look at makes use of the following applicable definitions

1.9.0 Over-dispersion

This is the presence of greater variability in a data set that would be expected based on a given statistical model. **1.9.1 Fatality**

This refers back to the death of a crash victim within 30 days of the incidence of the crash.

1.9.2 Road traffic accidents/crash

This refers to a collision between motors, a car and an object, or a automobile going for walks

into a ditch. Such accidents are those who purpose damage, harm on a public road,with at least one individual injured or killed.

2.1. INTRODUCTION

II. LITERATURE REVIEW

This chapter reviews the relevant theories and ideas associated with the reasons of road traffic accidents. Many factors contributes to road traffic accidents on primary roads worldwide. An account of the salient elements is important in comprehending the significance and scale of the accident hassle. That is against the history that a visitors twist of fate hassle wishes to be in reality defined, nicely analyzed, and contextualized whilst in search of its answers. Researchers has studied and research associated with coincidence examine road protection improvements for a particular location or pick out stretch in an extraordinary manner. Some of the evaluations are achieved regarding twist of fate records analysis, identification of Black Spot and coincidence Prediction model development.

Time series analysis was used by Mekky (1985) to study the effect of rapid increase in the motorization levels on the rate of fatalities in some developing countries. Many researchers have dived into the investigation of traffic crash patterns in different countries in order to understand its relationship with the fatality rate of road accident. Among such researcher are Dinesh (1985), who investigated crash patterns in Delhi, Emanalo et al (1987) developed the trend curves for road accidents, casualties and other vital quantities in Zambia, Pramada and Sarkar (1993) studied the variations in the pattern of road accidents in various States and Union Territories of India, Johnson (1997) studied the change in the number of accidents between before-year and after-year, Thole"n (1999) and Velin et al (2002) have also compared the variations in the change in the number of accidents in all the control sites of public roads in the Region West Sweden which were not surfaced during the study period.

Tanner (1953) proposed a model of accidents that occurs in 3-way junction part of the road in a given period and concluded that it is approximately proportional to the root of the product of the two way major road traffic volume and the turning flows from the minor road.

Hakkert and Mahalel (1978) in their investigations to estimate the number of accidents which occur at intersections of the road came out with the ""product of intersection flow" model. Also, Leong (1973) suggested a model which states that ""product of flow" each raised to the power less than one and this model was confirmed by Hauer et al (1989). McGuigan also investigated the root product flow model after Tanner (1953) and tested the sum of inflows 20 relationships and concluded that the use of the root product flows" model better model fits accident data as compared with the sum of inflows model which cannot be justified universally.

However, Mountain and Fawaz (1996) identified that the sum of flow model has some inconsistencies in relation to its ability to predict more than zero accidents between conflicting streams of traffic even when one of inflows is zero. Furthermore, the sum of inflows" model has the possibility of predicting equal numbers of accident for a given value of total inflows with no regards to the distribution of flows between the major and minor arms of the junctions, Salifu (2004).

In correcting these errors associated with the sum of inflows model and the product of flows model, Salifu (2004) reported that the cross product of flows model, sum of crossing flow product and the sum of encounter flow product models produce much more better fit to accident data than the simple flow models such as the total junction traffic inflow model. He stated that the most influential traffic exposure model for X-junction is sum of crossing flow product model, Salifu (2004).

Jacobs and Cuttings (1986) investigated into the previous accident models developed by earlier researchers to improve upon them. A study was conducted to find the effects of speed limits on road accidents by Fieldwick (1987) who identified that speed limits have significant impact on road safety and severity in both rural and urban roads.

John and Adams (1987) assessed Smeed"s law and provided more insight in the analysis of road accident data using Smeed"s model. Minter (1987) also discussed the applications of two accident models which were developed by Wright and Towel for road safety problems and came out finally with a new model for estimating road accidents in United Kingdom.

Pramada and Sarkar (1997) used road length as an additional parameter and established a model for road accidents with the length of road covered by the vehicle as a factor. Jamal and Jamil (2001) presented a general model to predict road accident fatalities in Yemen. Pramada (2004) used road accident data to compare the models developed by Smeed and Andreassen and confirmed that the two models worked well.

Agent et al. (2002) studied the effect of design exceptions on crash rates in the state of Kentucky. They found that the most common design exception was for a design speed lower than the posted speed limit followed by a lower than standard sight distance, curve radius or shoulder width. With an average of about 39 design exceptions per year in Kentucky, they concluded (based on observations of crash rates) that design exceptions did not result in projects with high crash rates relative to average statewide rates. Unfortunately, in this and many other studies, the amount of data available (which is limited because of the small number of design exceptions granted per year and the highly detailed roadway and accident information required) has made it difficult to develop statistically defensible models to assess the safety impacts of design exceptions in a multivariate framework.

Lee and 4 Mannering, 2002); negative binomial with random effects models (Shankar et al., 1998); Conway– Maxwell–Poisson generalized linear models (Lord et al., 2008); negative binomial with random parameters (Anastasopoulos and Mannering, 2009) and dual-state negative binomial Markov switching models (Malyshkina et. al, 2009a). For the severity of accidents, quantifying the effects of roadway characteristics on vehicleoccupant injuries have been undertaken using a wide variety of models including multinomial logit models, dual-state multinomial logit models, nested logit models, mixed logit models and ordered probit models (O'Donnell and Connor, 1996; Shankar and Mannering, 1996; Shankar et al., 1996; Duncan et al., 1998; Chang and Mannering, 1999; Carson and Mannering, 2001; Khattak, 2001; Khattak et al., 2002; Kockelman and Kweon, 2002; Lee and Mannering, 2002; Abdel-Aty, 2003; Kweon and Kockelman, 2003; Ulfarsson and Mannering, 2004; Yamamoto and Shankar, 2004; Khorashadi et al., 2005; Lee and Abdel-Aty, 2005; Eluru and Bhat, 2007;Savolainen and Mannering, 2007; Milton et al., 2008; Malyshkina and Mannering, 2009).

Cropper and Kopits (2003) in their study have predicted that fatalities in India would reach a total of about 198,000 before starting to decline in 2042. Mohan (2009) studied that road traffic fatalities have been increasing at about 8% annually for the last 10 years and show no signs of decreasing. Sivakumar and Krishnaraj (2012) studied and revealed some alarming facts. In 2010, India recorded 134,000 road accident deaths highest in the world. The World Bank trends put this figure at 200,000 annually. About 520,000 road accident injuries and

490,000 road accidents occurred in 2010 i.e. about 56 accidents per hour (one accident per minutes). If a person meets with a road accident in India, there is an over 30% chance of death. Around 53% of the people who die in India are males in the most productive age group of 20 to 50 years. The number of people killed has increased four times from 1970 to 2009. A major contributor to traffic deaths in India is drunk driving, which is responsible for 70% of road fatalities. India accounts for about 10 per cent of road accident fatalities worldwide. An estimated 1,275,000 persons are grievously injured on the road every year. Professionalism in driver training is absent, proportion of untrained drivers is continually on the rise and a positive driving culture is lacking.

Seva, Flores, Gotohio, and Paras (2013) studied the motorcycle accidents in the Philippines considering personal and environmental factors. The variables considered by them for study were age, lighting conditions, traffic movement, road character, junction type, day, surface conditions, and driving behaviour. Logistic regression was used to predict the likelihood of an accident from the variables considered and a logit model was thus developed. According to their study, three variables were found to be significant predictors of motorcycle accidents. They were age, driving behaviour, and junction type. Wald's and Hosmer_Lemeshow test were used by them as the logistic regression for the goodness of fit.

2.2. HISTORY OF ROAD TRAFFIC ACCIDENTS IN THE WORLD

Road traffic accidents have become a serious global challenge. Long before cars were invented, Road traffic accidents and injuries occurred involving carriages, carts, animals, and people. The numbers grew exponentially as cars, buses, trucks, and other motor vehicles were introduced and became common. On 30 May 1896, a cyclist in New York was the first recorded case of injury involving a motor vehicle, and on 17 August in the same year a London pedestrian was the first recorded motor vehicle death (Peden, 2004).

According to the WHO (2015) in its third global status report, road traffic injuries claim more than 1.2 million lives each year and have a huge impact on health and development.

The report showed that lower- and middle-income countries are the hardest hit, with double the fatality rates of high-income countries and 90% of global road traffic deaths.

Thus road traffic accidents are not a new problem only in industrialized countries but in developing countries as well (Zwi, 1995). Researchers believe that road traffic death rates in many high-income countries have stabilized or declined in recent decades, while they are getting worse in developing countries (WHO, 2009). These observations give credence to the supposition that low- and middle-income countries are the most affected by the burden of the world's injuries and fatalities attributable to RTAs (WHO, 2009).

Although middle-income and low-income countries operate less than 50% of the world's vehicles, they account for over 90% of reported RTC fatalities. Middle- and low-income country RTC rates are estimated at 19.5 and 21.5 per 100,000 population, respectively, compared with 103 per 100,000 population for high-income countries (WHO, 2009). The WHO further reports that a substantial proportion of those who die through RTCs are pedestrians, cyclists, and users of motorized two-wheelers; these are generally considered the 'vulnerable' road users. Most of the deaths affect persons aged between 5 and 44 years (WHO, 2009). The figures show that developing countries are losing substantial proportions of children and working persons through RTCs, and this has obvious implications for sustainable development.

2.3. ROAD TRAFFIC ACCIDENTS IN NIGERIA

In Nigeria today, hardly a day goes by without the occurrence of a road traffic accident leading to generally increasing incidence of morbidity and mortality rates as well as financial cost to both society and the individual involved. Information on some of these traffic accidents get to the news rooms of media houses and are aired while majority goes unreported. Nigeria has the highest road accidents rate as well as the largest number of death per 10,000 vehicles. Sheriff, M.A. (2009). One may be tempted to believe that the level of awareness on the causes of road traffic accidents is very low among Nigerians. Put differently, Nigerian roads have become killing fields without protection for their users. Travelers have a sigh of relief if they make their destinations. Eze, B. (2012). Contrary to the general belief that Nigerians posses very low level of awareness on the causes of road traffic accidents, previous research has shown that Nigerians knows quite a lot about what could cause road traffic accidents. Johnson, J.O. (2010).

Nigeria has the status of a developing country where road facilities are grossly inadequate to carter for the teeming population of road users. The discovery of oil in Nigeria came with its own problems. Prior to the 'Oil boom' in Nigeria, road accidents were rather rare. The oil boom brought along with it an increase in disposable income of the people which in turn increased vehicle ownership and brought about 'rapid' industrialization. This undoubtedly calls for improved road network accessibility, roads were therefore built albeit without dire attentions to standard, these developments were not matched by adequate measures and control. Sheriff, M.A. (2009). Consequently, the roads grew to be a death trap for Nigerian citizens and road users. This is significant when the fact that majority of these injuries and deaths can be prevented. It becomes worrisome with the fact that the incidence is increasing. Eze, B. (2012). Effective interventions include designing safer infrastructure and incorporating road safety features into land-use and transport planning; improving the safety features of vehicles.

To a very large extent, it is not entirely the poor deplorable condition of Nigerian roads that causes incessant road traffic accidents but a large proportion can be attributed to the carelessness and negligence of its road users.

Nigeria is an unenviable pinnacle spot in records of avenue visitors harm fatality costs (WHO & world financial institution, 2009). Lagos state, Port Harcourt, Delta state are some of the top ten towns reporting the highest number of fatalities (WHO, 2000). Such increase in premature deaths is yet another instance of the widening gap between different states, according to WHO (2000), Nigeria have demonstrated very excessive fatality prices in comparison with different countries. The evaluation suggests that RTCs and injuries represent fundamental health, economic, and developmental challenges to Nigeria, in particular those states stated. Of the anticipated 10,000 human beings killed in RTCs, 99% passed off in Lagos state, Port Harcourt, Delta state (Felix, 2004).

Despite the fact that the number of automobiles in Nigeria is increasing, the fatality charge due to traffic accident is extraordinarily excessive. With increasing motorization in Nigeria, road crashes and accidents are anticipated to develop at a faster charge, threatening the monetary and human improvement of this bad but promising us of a (Okon & Reich, 2003; Lagarde, 2007). It's far therefore necessary to lessen the number of accidents in maximum in Nigeria. Every other placing characteristic of RTCs and accidents in Nigeria is its excessive involvement in and impact on the most vulnerable road users: pedestrians and passengers in public delivery.

The literature overview indicates that pedestrians account for greater than 40% of casualties in maximum state. As an example, among 1992 and 1991, pedestrians accounted for 55% of road traffic deaths in Delta state (kingsley et al., 2003). Pedestrian casualties also accounted for 46% of road deaths in Delta state between 1994 and 1998 (Onwubuya, 2003). The state of affairs is an amazing burden on the maximum susceptible road users and their households, specially the bad and the less-knowledgeable. This development has emerge as a fitness as well as an monetary difficulty dealing with Nigeria. Afukaar (2003) asserted that RTCs and injuries have slowly advanced within the beyond decade in higher-earnings States.

The belief right here is that road traffic fatality quotes in Africa are predicted to increase with the aid of 80% between 2000 and 2023, if most important adjustments in guidelines and strategies do now not take region within the foreseeable destiny (Ruben, 2004.) negative traffic protection has a corresponding impact on financial growth in Nigeria (Kopits &Cropper, 2005). A booming financial system with growing motor vehicle possession is frequently followed via accelerated visitors collisions and accidents. However, crashes can increase even at some point of monetary stagnation or downturn, given certain conditions. Within the last couple of many years in the 20th century, the economies of most states skilled sturdy boom, but a few states are even poorer nowadays than they had been 30 years ago. Nigeria has had the bottom Gross home Product (GDP) for decades (Ikejiaku, 2009). Insufficient financial activities and monetary resources mean people cannot have the funds for to buy new and safe vehicles, while governments are more likely to keep vintage automobiles and defer upkeep. It could also imply that governments delay funding the maintenance of existing infrastructure, not to mention constructing new infrastructure. The conclusion is that road fatalities continue to increase.

Application of Beta-Binomial And Negative Binomial Models To Road Traffic Accidents In Nigeria

Figure 2.3: **A typical road traffic gridlock in delta state, Nigeria**

2.4. CAUSES OF ROAD TRAFFIC ACCIDENTS

Accident is defined as anything which happens by chance, anything occurring spontaneously . Road traffic accident is therefore an unexpected phenomenon that occurs as a result of the operation of vehicles . Accidents can be fatal, resulting in the deaths of the road user or minor. Accident don't just happen, they are caused. In other words, every accident in relation transport is not just a mere occurrence but has been instituted as a result of one factor or the other. A good awareness and knowledge of causes of road traffic accidents will help us to avoid them. Eventually this will bring about the desired goal of safety consciousness of road users in our society.

The causes of road traffic accidents therefore fall under three major categories viz– Human factors, Mechanical factors and the Environmental factors.

Of these three categories, the human factors are said to be responsible for over 80 percent of all traffic crashes because the drivers' operational ability is very critical to the causes and prevention of traffic accidents.

2.4.1 The Human Factor

The human factors constitute about 80% of the cause of road traffic accidents recorded in the country. The major components of human factor are drivers, pedestrian, law enforcement agent and the engineer. Most drivers on Delta state road are very rude, discourteous and have scant regard for human life. This has led to daily avoidable carnage on delta state roads with many losses of lives. Almost to the point of indisputability is the fact that, of virtually all the significant factors contributing to the alarming proportion of accidents on Delta state roads, the human factor tops the list. Indicators to verify the claim are evident:

- Prevalent disregard of road traffic signs by road users
- Lack of proper training of drivers
- Irresponsible driving habit particularly among teenage drivers
- Inexperience and incompetent drivers

 Over speeding, dangerous driving and total disrespect of traffic regulations especially concerning speed limits

- Drink driving or driving under the influence of drugs including herbal concoctions laced with spirit
- Lack of respect and consideration for other road users
- Impatience and negligence
- Overloading of vehicles
- Fatigue
- Poor vision.
- Corruption

2.4.2. Environment Factor

Each the physical and social environments are key chance elements in RTAs, due to the fact they affect road users' behavior and the condition and motion of vehicles. As an instance, pot-holed roads have an effect on driving force behavior and the condition of cars. Poor lighting fixtures influences visibility for each drivers and pedestrians. Unfavorable weather conditions, such as raining seasons, can make road use specially difficult. The values and cultural and spiritual ideals within the social surroundings, which includes fatalism or ideals that accidents are simply punishments from the gods for wrongdoers, indirectly have a strong impact on road users conduct. The environment factor is still a strong debate within the context of Deltians as to whether the high incidence of road accidents should actually be attributed to bad roads. Or, if they are not a paradoxical function of the good and modern highways that the country invested on so much. The contention is against the backdrop that despite the construction of new roads in the state, appreciable reduction has not been witnessed in accidents rates but rather seem to be increasing. In other words, there is need to focus on other factors, particularly the human elements contributing to the disaster.

Environmental factor include:

- Bad road;
- Weather conditions;
- Dangerous bend;
- Broken down/ abandon vehicles
- Animals not under control
- Obstruction on the road.

2.4.3. The Mechanical Factor

The vehicle also constitutes one of the major factors of road traffic accident. Road safety however goes beyond periodic check or prompt repair of vehicles. It should be a daily routine of care and check of all components of a vehicle. The main vehicle factors are defects in tyres, brakes and inputs all arising from poor maintenance of the vehicle. The global economic recession have badly affected the quality of products in the Nigerian markets such that people now favour the use of sub-standard products like Tokunbo tyres, spare parts and Tokunbo vehicles. These, coupled with over speeding and reckless driving, negate the principles of safety when considered against the phenomenon of used vehicles. Any of those parts that malfunction can eventually affect smooth driving, which in the end, can lead to serious accident. The different component of mechanical factor that resulted into accident are:

- Brake failure
- Burst tyres
- Engine failure
- Use of fake and substandard spare parts
- Defective and Dazzling lights
- Poorly maintained vehicles.

In essence, a deficient vehicle, an unserviceable car, or a poorly maintained automobile are all dangers with high probability to cause accidents on the roads.

Figure 2.5: **Road Traffic Accident in Delta State**

2.5. RISK THEORY

Risk can be defined, for instance, as a subjective assessment o of the probability for a specific occurrence of a negative event, and how concerned an individual is with the consequences of this event (Sjorberg, 1983:8 Rundmo & Iversen, 2004). As a result, the combination of perceived opportunity and severity of results pertains to how a person perceives risk. According to Dejoy (1989), road traffic, risk is a function of four elements. The first is the exposure – the amount of movement or travel within the system by different users or a given population density. The second is the underlying probability of a crash, given a particular exposure. The third is the probability of accident, given a crash. The fourth element is the outcome of accident.

Another definition is that risk can also be defined by human blunders, kinetic power, tolerance of human frame and submit-crash care (Bustide et al, 1989).

The second theoretical approach used in RTA research is the risk theory. A variety of factors has been suggested to predict risk perception. Rundmo and Iversen (2004) identified that poverty and poor countries exhibit a higher risk of tolerance culture. People may neglect risk because they are influenced by other existing risks, and in a high-risk society such as a poor country, people experience more severe risks generally.

In reviewing existing literature on traffic accidents, Zuckerman (1979) points to the fact that while rates of accidents have fallen in industrialized countries, it is rather on the increase in developing countries. He explains that as developing countries are characterized by poverty, the majority of the people living in these countries are exposed to various risk situations everyday. Risk is also associated with personality traits and attitudes. Some people prefer a higher risk level; they are the so-called sensation-seekers and are found in all societies and cultures (Sjorberg, 1983).

Several variables are idea to influence chance perceptions some of the public. statistics about hazard from various social members of the family and the media, for example, are thought to shape how individuals and societies method potential dangers (Slovic, 1987). A consequence is that the general public does no longer continually partner threat objectively with extra dangerous activities (Moen et al., 2005). As an instance, at some stage in a vacation in Kaduna state, the statistical opportunity of being accident in roads is extra than being struck by means of a terrorist assault. Nonetheless, many Northern travelers tend to worry greater about terrorism than visitors accidents. This case demonstrates that human belief of risk have to be seemed as a multidimensional concept, and notion of risk isn't continually congruent with objective statistical calculations. While people perceive chance, several factors warrant consideration, the first is the possibility of a poor event and the severity of consequences of such an event. Further, processing theories and appraisal theories account for a way applicable theories have an impact on such judgments. The results of research finished formerly on RTAs have shown that the more the results of the bad occasion, the more the impact might be gift while thinking about the chance supply, and the greater the precautionary movement essential to avoid accidents (Rundmo & Iversen, 2004). According to Thompson et al.(2002), chance reimbursement is the name given to a concept which states that [an] individual provided with a protecting tool together with vehicle seatbelts will act or behave in a more risky manner because of the multiplied experience of protection from the seatbelt and thereby nullifies the protection afforded by using the seatbelt.

Consistent with Adams (1995) and Wilde (1998), an individual's chance-taking choices constitute a balancing act in which perceptions of hazard are weighed towards the propensity to take a threat. The propensity to take a hazard is motivated by using anticipated rewards, at the same time as perceived threats or risk increase, humans reply by means of being greater cautious. There may be consequently a balancing behaviour motivated by perceived hazard and the propensity to take risk, which in flip affects accidents and rewards. If the perceived danger of a situation exceeds our target stage, we will act to lessen it; and if the perceived risk is lower than the target level, we are able to generally tend to raise our danger returned to our target degree (danger optimization) through undertaking more risky actions. Wilde's term for this system is hazard compensation, an idea which indicates that humans commonly regulate their behaviour in response to a perceived degree of threat and turn out to be greater careful when they sense extra danger and much less careful if they feel more blanketed. In his view, chance homeostasis is therefore an extreme form of behavioral variation.

Dejoy (1989) notes that the target degree of twist of fate hazard is decided via four categories of motivating factors. The first category accommodates factors associated with the anticipated benefits of comparative risks behavior alternatives. this is expressed with the aid of gaining time with the aid of dashing when roads are excellent (hazard repayment) (Adams, 1999). The second one class accommodates elements associated with the anticipated value of a relatively risky behavior alternative; for example, automobile repair expenses or insurance surcharges for being at fault will gasoline the price of a road accident (Wilde, 1998). The third category comprises factors related to the expected benefit of comparatively safe behavior alternatives, which Rundmo (1999), for instance, cites as the psychology of insurance discount for accident free driving. The fourth category comprises factors related to the expected cost of comparatively safe behavior alternatives using an uncomfortable seatbelt is cited as an example of positive behavior that reduces RTAs (Wilde, 1998).

2.6. SYSTEM OF TRAFFIC LAWS, CONTROLS AND REGULATIONS

Ineffective traffic laws and policies purpose chaos at the roadways. One result is Road traffic accidents. From the three-factor behavioral model by Jørgensen and Abane (1999), the promulgation of traffic laws and policy and their enforcement (or in any other case) play an essential function in lowering crash causation. That is due to the fact they have an impact on all of the three remaining elements: behavior, automobiles, and surroundings. Passing traffic laws and enforcing them are important to traffic safety research, for these laws guide behavior and regulate the conduct of road users. As an example, a gadget of rules prescribing pedestrians' proper of manner, consisting of a zebra crossing, obliges motorists to yield to pedestrians. Another traffic policy is the 'vision zero' should be applied in Nigeria, emphasizing zero tolerance for Road traffic accidents by providing effective preventive tools and is vital for accident reduction. Vision zero will have a far-achieving, positive effect on road traffic safety control in Nigeria.

The goal of traffic law structures and enforcement is to modify the site traffic environment and make sure gadget maintenance. law via site visitors signaling structures, pace limits, and pace controls, as well as the existence of police patrols and checkpoints can reduce accidents through influencing road-user behavior, Jorgensen and Abane (1999) argue that traffic law schemes aren't systematically implemented in Nigeria, and the police provider is generally no longer properly-educated, is unwell-equipped, and is ill-prompted to enforce guidelines; those shortcomings are not functions of towns in evolved nations. Anthony reported the effectiveness of helmets in pedal cyclists and motor cyclists (Anthony, 1986). Obligatory use of helmets in Delta state confirmed the same right consequences (Astrom et al., 2006). The effectiveness of helmet use is dependent on the speed of the motorcyclist. In line with Astrom et al. (1986), it is more shielding at low speeds of 45 km per hour or less, but much less effective at higher speeds. Henry discovered that obligatory seatbeltwearing became useful (Henry, 1991). Seatbelts for each children and adults averted approximately 50–60% of all fatalities that could have resulted from motor site visitors injuries (Rivara, 1986).

It is a properly-documented fact that the use of infant restraints, especially child protection seats, can reduce mortality in young sufferers of RTCs (CSA, 1983). Restraints prevent about 90% of fatalities within the 0 to 4-12 months-antique-age organization (Rivara, 1986).

2.6.1. Behavior

The machine of laws and regulations immediately affects avenue-person behavior, such as the behavior of drivers, pedestrians, cyclists, and passengers. Also, the characteristics of the vehicle and the surroundings impact pedestrians and motive force behavior in methods that may cause injuries. Road user behavior equally affects the environment and the vehicle. As an instance, a fatigued or distracted driving force can turn out to be worried in a RTA due to the fact his mental nation is challenged, and the possibility of a crash is elevated when distracted driving combines with bad conditions on the road and non-enforcement of traffic laws. In different phrases, a driver with a propensity for danger-taking is more likely to use a street in a manner that poses a danger to fellow road users if the opposite conditions have interaction to facilitate higher danger-taking.

Behavioral intervention and tighter policies also are crucial measures (Jayasuria,1991; Graham, 1993). But, legislative and different counter measures proved powerful in Nigeria (Asogwa, 1992). promotion of street protection thru the use of targeted media campaigns at community levels can efficiently lessen motor visitors injuries (Tripop,1994).

2.6.2. Automobiles

The situation of vehicles, on the road can be accountable for accident. Examples of such automobile situations consist of used (old) vehicles, the conversion of cargo vehicles into passenger ones by tampering with the original make of the vehicle, poor maintenance of vehicles, or maintenance with used-car parts. The direction strains within the above framework show that a car isn't best suffering from behaviour and the environment, but also by the device of legal guidelines, controls, and rules. A clear working example is the regulation in Nigeria that lets in using imported used spare elements; this may worsen the accident trouble, both the bodily and social environments are key hazard factors in motor vehicle crashes.

These influence road-user behavior and automobiles. The structures version is all encompassing as it is going past the domain of behavior, vehicles, and environment to consist of the structures in standard as a separate assemble. Thus, the relevance of the model in understanding the topic under consideration can be seen at three different levels. First, the theory helps us to identify the system of traffic laws, regulations, and the mode of enforcement designed to ensure traffic safety in delta state. Second, the model helps us to identify the multiple causes and the interplay of risk factors and the prevention of traffic accidents that occur in the study area. Third, the model assists us in identifying or understanding the three major factors that contribute to RTAs. This model makes it possible for us to isolate key variables for the purposes of planning and policy, and it indicates how the factors interact in general, thus allowing us to understand that most RTCs are caused by a combination of various circumstances.

2.7 PASSENGER BEHAVIOUR

People travelling in the bus forced the driver to keep full volume of music, at times driver attention gets divert with argument when they are forced to increase the speed to reach the destination on time. Furthermore, Person at times distract the driver by discussing sensitive topic about political or social issue which makes driver agitated ultimately lead to accidents and injuries.

2.8 PEDESTRIANS

Type of road is a major factor of road accidents including pedestrians. Especially children on the road left unattended by the parents without their supervision crossing roads in heavy traffic, where traffic lights and traffics rule observed very rare. This is also a result of serious injuries and death among this age group.

3.0. INTRODUCTION

III. RESEARCH METHODOLOGY

This chapter deals with a detailed description of the methods used for this research and explained the theory behind the various models for the analysis. It addresses the possible probability distributions of accident data (count data) on fatal, serious and minor road traffic accident in Nigeria using Delta state as a case study, it also provided the description of the software packages used for the analysis and modeling.

3.1. BETA DISTRIBUTION

The Beta distribution is a continuous probability distribution having two parameters α and β . One of it's most common uses is to model one's uncertainty about the probability of success of an experiment defined by an interval [0,1]. Suppose a probabilistic experiment can have only two outcomes, either success, with probability **p**, or failure, with probability $(1-p)$. Suppose also that is unknown and all its possible values are deemed

equally likely. This uncertainty can be described by assigning to a uniform distribution on the interval. This is appropriate because, being a probability, can take only values between 0 and1; furthermore, the uniform distribution assigns equal probability density to all points in the interval, which reflects the fact that no possible value of is, a priori, deemed more likely than all the others.

Now, suppose that we perform independent repetitions of the experiment and we observe successes and failures. After performing the experiments, we naturally want to know how we should revise the distribution initially assigned to, in order to properly take into account the information provided by the observed outcomes. In other words, we want to calculate the conditional distribution of, conditional on the number of successes and failures we have observed. The result of this calculation is a Beta distribution. In particular, the conditional distribution of ,conditional on having observed successes out of trials, is a Beta distribution with parameters α and β .

Definition

The Beta distribution is characterized as follows.

Definition Let *X* be a [continuous random variable.](https://www.statlect.com/glossary/absolutely-continuous-random-variable) Let its [support](https://www.statlect.com/glossary/support-of-a-random-variable) be the unit interval:

$$
R_{X} = [0,1]
$$

Let $\alpha, \beta \in R_{++}$. We say that X has a Beta distribution with shape parameters α and β if and only if its probability density function is

$$
f_x(x) = \begin{cases} \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1} & \text{if } \alpha > 0 \end{cases}
$$

If $\beta \succ 0$

where B(**x**) is the [Beta function.](https://www.statlect.com/mathematical-tools/beta-function) A random variable having a Beta distribution is also called a Beta random

3.1.1 Expected Value

The expected value of a Beta random variable *X* is

variable. The following is a proof that
$$
f_x(x)
$$
 is a legitimate probability density function.
\n**3.1.1 Expected Value**
\nThe expected value of a Beta random variable X is
\n
$$
E(x) = \frac{\alpha}{\alpha + \beta}
$$
\n(3.11)

3.1.2 Variance

The variance of a Beta random variable is

3.1.2 Variance
The variance of a Beta random variable is

$$
var(x) = \frac{\alpha \beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}
$$
(3.12)

Higher moments

The k-th moment of a Beta random variable *X* is

$$
(\alpha + \beta + 1)(\alpha + \beta)^2
$$

Higher moments
The k-th moment of a Beta random variable X is

$$
U_x(k) = E(x^k) = \frac{B(\alpha + K, \beta)}{B(\alpha + \beta)} = \prod_{n=0}^{k-1} \frac{\alpha + n}{\alpha + \beta + n}
$$
(3.13)

3.1.3 Moment Generating Function

The moment generating function of a Beta random variable *x*is defined for any *t* and it is

The K di mionenic of a Beta random variable X is
\n
$$
U_x(k) = E(x^k) = \frac{B(\alpha + K, \beta)}{B(\alpha + \beta)} = \prod_{n=0}^{k-1} \frac{\alpha + n}{\alpha + \beta + n}
$$
\n(3.13)
\n3.1.3 Moment Generating Function
\nThe moment generating function of a Beta random variable x is defined for any t and it is
\n
$$
M_x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \frac{B(\alpha + K, \beta)}{B(\alpha + \beta)} = 1 + \sum_{k=1}^{\infty} \frac{t^k}{k!} \frac{\alpha + n}{\alpha + \beta + n}
$$
\n(3.14)
\nThese above formula for the moment generating function might seem impractical to compute, because it

These above formula for the moment generating function might seem impractical to compute, because it involves an infinite sum as well as products whose number of terms increase indefinitely. However, the function

$$
M_x(t) = \sum_{k=0}^{\infty} \frac{t}{k!} \frac{B(\alpha + K, p)}{B(\alpha + \beta)} = 1 + \sum_{k=1}^{\infty} \frac{t}{k!} \frac{\alpha + n}{\alpha + \beta + n}
$$
(3.14)
These above formula for the moment generating function might seem impractical to compute, because it
involves an infinite sum as well as products whose number of terms increase indefinitely. However, the function

$$
f_1(\alpha, \alpha + \beta, t) = 1 + \sum_{k=1}^{\infty} \frac{t^k}{k!} \prod_{n=0}^{k-1} \frac{\alpha + n}{\alpha + \beta + n}
$$
(3.15)
The distribution function of a Beta random variable x

The distribution function of a Beta random variable *x*

The distribution function of a Beta random variable x
\n
$$
f_x(x) = \begin{cases}\n0 & \text{If } x < 0 \\
\frac{B(x, \alpha, \beta)}{B(\alpha + \beta)} & \text{If } 0 \le x \le 1 \\
1 & \text{If } x > 0\n\end{cases}
$$

3.2. BINOMIAL DISTRIBUTION

 ϵ

The binomial distribution with parameters *n* and *p* is the discrete probability distribution of the number of successes in a sequence of *n* independent experiments, each asking a yes-no question, success/yes/true/one (with probability *p*) or failure /no/false/zero (with probability $q = 1 - p$). A single success/failure experiment is also called a Bernoulli trial or Bernoulli experiment and a sequence of outcomes is called a Bernoulli process; for a single trial, i.e., $n = 1$, the binomial distribution is a Bernoulli distribution. The binomial distribution is the

basis for the popular binomial test of statistical significance. The binomial distribution is frequently used to model the number of successes in a sample of size *n* drawn with replacement from a population of size *.*

3.2.1 Probability Mass Function

In general, if the random variable *X* follows the binomial distribution with parameters $n \in N$ and $p \in [0,1]$, we write $X \sim B(n, p)$. The probability of getting exactly *k* successes in *n* trials is given by the probability mass function. $A \cdot A$ **.1 Probability Mass Function**
general, if the random variable *X* follows the binomial distribution with parameters *n* ∈ N and *p* ∈ [0,1], we
ite *X* ~ B(*n*, *p*). The probability of getting exactly *k* success *i.2.1 Probability Mass Function*
n general, if the random variable *X* follows the binomial distribution with pa
vrite $X \sim B(n, p)$. The probability of getting exactly *k* successes in *n* trials
unction.
 $f(k, n, p) = P_r(k;$

$$
f(k,n,p) = P_r(k;n,p) = P_r(X=k) = \binom{n}{k} P^K (1-P)^{n-k}
$$
\n(3.21)

for $k = 0, 1, 2, ..., n$, where

$$
\binom{n_k}{k} = \frac{n!}{k!(n-k)!}
$$

[Binomial Coefficient:](https://en.wikipedia.org/wiki/Binomial_coefficient) The [binomial coefficient,](https://en.wikipedia.org/wiki/Binomial_coefficient) hence the name of the distribution. The formula can be understood as follows. *k* successes occur with probability p^k and $n - k$ failures occur with probability $(1 - p)^{n - k}$. However, the *k* successes can occur anywhere among the *n* trials, and there are $\binom{n}{k}$ κ) different ways of

distributing *k* successes in a sequence of *n* trials.

In creating reference tables for binomial distribution probability, usually the table is filled in up to *n*/2 values. This is because for $k > n/2$, the probability can be calculated by its complement as $f(k, n, p) = f(n-k, n, 1-p)$

$$
f(k,n,p) = f(n-k,n,1-p)
$$

Looking at the expression $f(k, n, p)$ as a function of k , there is a k value that maximizes it. This k value can be

found by calculating
\n
$$
\frac{f(n-k,n,p)}{f(k,n,p)} = \frac{(n-k)p}{(k+1)(1+p)}
$$

3.2.2 Cumulative Distribution Function

The cumulative distribution function can be expressed as *k*

3.2.2 Cumulative Distribution Function
The cumulative distribution function can be expressed as

$$
f(k, n, p) = P_r(X \le k) = \sum_{i=0}^{|k|} {n \choose i} P^i (1-P)^{n-i}
$$
(3.22)

3.2.3 Expectation

If $X \sim B(n, p)$, that is, X is a binomially distributed random variable, n being the total number of experiments and p the probability of each experiment yielding a successful result, then the expected value of *X* is If $X \sim B(n, p)$

and p the pr
 $E(x) = np$
 $U = \sum_{i=0}^{n} x_i p_i$

$$
U = \sum_{i=0}^{n} x_i p_i
$$
\n
$$
U = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}
$$
\n(3.23)

$$
U = \sum_{i=0}^{n} x_i p_i
$$

\n
$$
U = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}
$$

\n
$$
= np \sum_{k=0}^{n} k \frac{(n-1)!}{k!(n-k)!k!} p^{k-1} (1-p)^{(n-1)-(k-1)}
$$
\n(3.25)

$$
U = \sum_{k=0}^{n} k \binom{n}{k} p^{k} (1-p)^{n-k}
$$
\n
$$
= np \sum_{k=0}^{n} k \frac{(n-1)!}{k!(n-k)!k!} p^{k-1} (1-p)^{(n-1)-(k-1)}
$$
\n
$$
= np \sum_{k=0}^{n} \frac{(n-1)!}{(n-1)-(k-1)!(k-1)!} p^{k-1} (1-p)^{(n-1)-(k-1)}
$$
\n(3.26)

$$
= np \sum_{k=0}^{n} k \frac{(n-1)!}{k!(n-k)!k!} p^{k-1} (1-p)^{(n-1)-(k-1)}
$$
\n
$$
= np \sum_{k=1}^{n} \frac{(n-1)!}{(n-1)-(k-1)!(k-1)!} p^{k-1} (1-p)^{(n-1)-(k-1)}
$$
\n(3.26)\n
$$
= np \sum_{\ell=0}^{n} {n-1 \choose \ell} p^{\ell} (1-p)^{(n-1)-\ell}
$$
\n(3.27)

$$
= np \sum_{\ell=0}^{n} \binom{n-1}{\ell} p^{\ell} (1-p)^{(n-1)-\ell} \tag{3.27}
$$

with $\ell = k - 1$

*Corresponding Author: EHIKWE 29 | Page

Application of Beta-Binomial And Negative Binomial Models To Road Traffic Accidents In Nigeria
with
$$
m = n - 1
$$

$$
= np \sum_{\ell=0}^{n} {m \choose \ell} p^{\ell} (1-p)^{m-\ell} = np \sum_{\ell=0}^{n} {n-1 \choose \ell} p^{\ell} (1-p)^{(n-1)-\ell}
$$
(3.26)

$$
= np (p + (1-p))^m
$$

$$
= np
$$
(3.27)

3.2.4 Variance

The variance is =np(3.27)

3.2.4 Variance

The variance is

Var (x) =np(1-p) (3.28)

Since $\text{Var}(x_i) = p(1-p)$, we get:

Let X=
$$
x_i
$$
+...+ x_n where all x_i are independently Bernoulli distributed random variables.
\nSince Var (x_i) =p(1-p), we get:
\nVar (x_i) =Var $(x_i$ +...+ x_n) = Var (x_i) +...+Var (x_n) =nVar (x_i) =np(1-p)

3.3 NEGATIVE BINOMIAL DISTRIBUTION

The NBD was first used to assess recurring patterns of accidents in a population (Greenwood and Yule, 1920). A negative binomial random variable is the number X of repeated trials to produce r successes in a negative binomial experiment. The probability distribution of a negative binomial random variable is called a negative binomial distribution. The negative binomial distribution is also known as the Pascal distribution. The negative binomial random variables and distribution are based on an experiment requires:

1. The trials be independent.

2. The outcome is binary (success or failure).

3. The probability of success or failure is independent from trial to trial.

4. The trials continue until you have *r* successes.

One important application of the negative binomial distribution is that it is a mixture of a family of Poisson distributions with gamma mixing weights. The negative binomial can be viewed as a Poisson distribution where the Poisson parameter is itself a random variable, distributed according to a Gamma distribution. Thus the negative binomial distribution is known as a Poisson-Gamma mixture.

The model

negative binomial distribution is known as a Poisson-Gamma mixture.
\nThe model
\nSuppose that we have a series of random counts that follows the poisson distribution:
\n
$$
f(y_i \setminus \lambda_i) = \frac{e^{-\lambda_i} \lambda_i}{y_i}, \qquad y \ge 0, \lambda \ge 0
$$
 (3.31)
\n $\lambda_i = \exp(\beta_0 + \sum_{j=1}^{k} x_{ij} \beta_j + \varepsilon_i)$
\n $\lambda_i = e^{\sum_{j=1}^{k} x_{ij} \beta_j} e^{(\beta_0 + \varepsilon_i)}$
\n $\lambda_i = e^{(\beta_0 + \sum_{j=1}^{k} x_{ij} \beta_j)} e^{\varepsilon_i}$
\n $\lambda_i = u_i v_i$
\nWhere $e^{(\varepsilon_i)} \square$ gamma(a^{-1} ; a^{-1}); $e^{(\beta_0 + \varepsilon_i)}$ is defined as a random intercept;
\n $u_i = \exp(\beta_0 + \sum_{j=1}^{k} x_{ij} \beta_j)$ is the log-link between the Poisson mean and the covariates or independent
\nvariable x's; and β 's are the regression coefficients. The marginal distribution of y_i can be obtained by
\nintegrating the error term, v_i ,

Application of Beta-Binomial And Negative Binomial Models To Road Traffic Accidents In Nigeria
\n
$$
f(y_i : u_i) = \int_0^{\infty} g(y_i; u_i, v_i) h(v_i) dv_i
$$
\n
$$
f(y_i : u_i) = E_v[g(y_i; u_i, v_i)]
$$
\n(3.32)
\nwhere h(v_i) is a mixing distribution. In the case of the Poisson-Gamma mixture, g(v_i : u_i, v_i) is the Poisson

where $h(v_i)$ is a mixing distribution. In the case of the Poisson- Gamma mixture, $g(y_i; u_i, v_i)$ is the Poisson Binomial distribution.

Assume that the variance \mathbf{v}_i follows a two parameter gamma distribution;

distribution and
$$
h(v_i)
$$
 is the Gamma distribution. This distribution has a close form and leads to the Negative Binomial distribution.
\nAssume that the variance v_i follows a two parameter gamma distribution;
\n
$$
f(v_i; a, \delta) = \frac{\delta^a}{\Gamma(a)} v_i^{\delta - 1} e^{-v_i \delta}, \qquad a > 0, \delta > 0, v_i > 0
$$
\n(3.33)

Where $\rm E(V_i)$ $E(v_i) = \frac{a}{\delta}$ and $var(v_i) = \frac{a}{\delta^2}$

Where
$$
E(v_i) = \frac{a}{\delta}
$$
 and $var(v_i) = \frac{a}{\delta^2}$
Setting $\Box = \delta a$, we have
 $E(v_i) = 1$ and $var(v_i) = \frac{1}{a}$ (3.34)

The mean and variance are the following (Lord and Park (2014))

The mean and variance are the following (Lora and
\n
$$
E(y_i, a, u_i) = u_i
$$
 and $var(y_i; a, u_i) = u_i + \frac{u_i^2}{a}$
\nThe next steps consist of defining the log-likelihood
\n
$$
In \left[\frac{\Gamma(y_i + a)}{\Gamma(a)} \right] = \sum_{i=1}^{y-1} In(j+a)
$$

The next steps consist of defining the log-likelihood function and it can be shown that, (Lord and Park (2013))

The mean and variance are the following (Lord and Park (2014))
\n
$$
E(y_i, a, u_i) = u_i \text{ and } var(y_i; a, u_i) = u_i + \frac{u_i^2}{a}
$$
\nThe next steps consist of defining the log-likelihood function and it can be shown that, (Lord and Park (2013))
\n
$$
\text{In}\left[\frac{\Gamma(y_i + a)}{\Gamma(a)}\right] = \sum_{j=0}^{y-1} \text{In}(j+a)
$$
\n(3.35)
\nTherefore, the log-likelihood function becomes (Lord and Park (2013));
\n
$$
\text{InL}\left[\alpha, \beta\right] = \sum_{i=1}^{n} \left\{\left[\sum_{j=0}^{y-1} \text{In}(j+a)\right] - \text{In } y_i! - (y_i + a) \text{In}(1 + a^{-1}u_i) + y_i \text{In}a^{-1} + y_i \text{In}u_i\right\}
$$
\n(3.36)

Therefore, the log-likelihood function becomes (Lord and Park (2013));

Therefore, the log-likelihood function becomes (Lord and Park (2013));
\n
$$
InL[\alpha, \beta] = \sum_{i=1}^{n} \left\{ \left[\sum_{j=0}^{y-1} In(j+a) \right] - In y_i! - (y_i + a) In(1 + a^{-1} u_i) + y_i Ina^{-1} + y_i Inu_i \right\}
$$

3.4 BETA BINOMIAL MODEL

Beta Binomial model is a family of discrete probability distributions on a finite support arising when the probability of a success in each of a fixed or known number of Bernoulli trials is either unknown or random. The beta binomial distribution is the binomial distribution in which the probability of success at each trial is not fixed but random and follows a beta distribution.

3.4.1 **Assumption of beta binomial model**

- i. The trials are not independent.
- ii. The probability of success is not fixed but varies from one trial to trial.
- iii . P follows beta distribution.

The basic statistical structure of beta binomial, (pdf of binomial)
\n
$$
P(X = x / p) = {n \choose x} p^x (1-p)^{n-x}, x=1,2,...,n
$$

Where x is the independent Binomial random variable conditional on p which follow Beta distribution
\n
$$
f(p/\alpha, \beta) = [B(\alpha, \beta)]^{-1} p^{\alpha-1} (1-p)^{b-1}, 0 < p < 1
$$

Where, *a* and *b* are unknown positive constants (need to be estimated) and $B(a,b)$ is the Beta function. Given the above conditional probability function, the distribution X_i is expressed by integrating the Binomial and Beta distributions.

Suppose $X|p \rvert$ *Bin*(*n*,*p*) and $p \sim Beta(\alpha, \beta)$.

$$
P(X = x; n, \alpha, \beta)
$$

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= $\binom{s}{i} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(x+\alpha)\Gamma(n-x+\beta)}{\Gamma(\alpha+\beta+n)}$ (3.41)		
$x = k/p, n \sqcup Bin(n, p)$	the mean value	
$E(x) = k$	recall that the mean Binomial random variable	
$E(x = k) = \sum \varphi(x = k)$	$\sum k \int p(x = k, p) dp$	
$E(x = k) = \sum \varphi(x = k)$	$\sum k \int p(x = k/p, n) . f(p, n) dp$	
$\sum k \int p(x = k/p, n) . f(p, n) dp$	$\sum k \int p(x = k/p, n) . f(p, n) dp$	
$E(x = k) = \int npf(p, n) dp$	$\sum (3.44)$	
$f(p, n) \geq Beta(p, \alpha, \beta, n)$	$\sum (x = k) = \int npf(\alpha\mu, \alpha, \beta) dp$	(3.44)
$f(p, n) = Beta(p, \alpha, \beta, n)$	$\sum (x) = \pi \int pbeta(\alpha(p, \alpha, \beta) dp$	The mean of beta binomial is given as
$E(x) = \left(\frac{n\alpha}{\alpha + \beta}\right)$	$\sum (x^2/\beta) \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha + \alpha)\Gamma(n - x + \beta)}{\Gamma(\alpha + \beta + n)}$	
conditional r.v	$x = k/p, n \equiv \binom{n}{k} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha + \beta + n)}{\Gamma(\alpha + \beta + n)}$	(3.46)
conditional r.v </td		

$$
x = k / p, n \Box \binom{n}{k} p^{k} (1-p)^{n-k}
$$

using the second moment

$$
E(x^2 / p, n) = np(1-p) + n^2 p^2
$$
 (3.46)

using the second moment
\n
$$
E(x^2 / p, n) = np(1-p) + n^2p^2
$$
\n(3.46)
\n
$$
Var(x) = E(x^2) - [E(x)]^2
$$
\n(3.47)
\n
$$
E(x) = \left(\frac{n\alpha}{\alpha + \beta}\right)
$$

$$
Var(x) = E(x2) - [E(x)]2
$$

\n
$$
E(x) = \left(\frac{n\alpha}{\alpha + \beta}\right)
$$

\n
$$
E(x)2 = \sum_{k} k2 p(x = k)
$$
\n(3.48)

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\n
$$
\sum_{k} k^{2} \int p(x=k/p,n) \cdot f(p;n,\alpha,\beta) dp
$$
\n
$$
\int f(p;n,\alpha,\beta) \sum_{k} k^{2} \int p(x=k/p,n) dp
$$
\n
$$
\int p(p-1-p)+n^{2} p^{2} \cdot Beta(p;\alpha,\beta) dp
$$
\n
$$
E(x^{2}) = nE(P)+(n^{2}-n)E(P^{2})
$$
\n
$$
E(x^{2}) = \frac{n\alpha}{\alpha+\beta} + (n^{2}-n) \left[\frac{\alpha\beta}{(\alpha+\beta)^{2}(\alpha+\beta+1)} + \frac{\alpha^{2}}{(\alpha+\beta)^{2}} \right]
$$
\n
$$
Var(x)=Var(x) = E(x^{2}) - [E(x)]^{2}
$$
\n
$$
Var(x) = Var(x) = E(x^{2}) - [E(x)]^{2}
$$
\n
$$
Var(x) = \frac{n\alpha\beta}{\alpha+\beta} + \frac{(n^{2}-n)\alpha\beta}{(\alpha+\beta)^{2}(\alpha+\beta+1)} + \frac{n^{2}\alpha^{2}}{(\alpha+\beta)^{2}} - \frac{n\alpha^{2}}{(\alpha+\beta)^{2}} - \frac{n^{2}\alpha^{2}}{(\alpha+\beta)^{2}}
$$
\n
$$
Var(x) = \frac{n\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^{2}(\alpha+\beta+1)}
$$
\n
$$
Var(x) = \frac{n\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^{2}(\alpha+\beta+1)}
$$
\n
$$
Var(x) = \int_{\alpha}^{n} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} - \frac{Var(\beta(\alpha+\beta+n))}{\Gamma(\alpha+\beta+n)}
$$
\n
$$
E(x) = \left(\frac{n\alpha}{\alpha+\beta}\right) - Var(x) = \frac{n\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^{2}(\alpha+\beta+1)}
$$
\nThe value of the sample mean and sample variance
\n
$$
\overline{X} = \frac{1}{N} \sum_{i} x_{i}
$$
\n
$$
S_{x} = \frac{1}{N} \sum_{i} x_{i}
$$
\n
$$
S_{y} = \frac{1}{N} \sum_{i} (x_{i} - \overline{x})^{2}
$$
\n
$$
S_{z} = \frac{1}{N} \sum_{i} (x_{i} - \over
$$

$$
E(x^{2}) = nE(P) + (n^{2} - n)E(P^{2})
$$

\n
$$
E(x^{2}) = \frac{n\alpha}{\alpha + \beta} + (n^{2} - n) \left[\frac{\alpha\beta}{(\alpha + \beta)^{2}(\alpha + \beta + 1)} + \frac{\alpha^{2}}{(\alpha + \beta)^{2}} \right]
$$
\n(3.51)

$$
E(x^{2}) = nE(P) + (n^{2} - n)E(P^{2})
$$

\n
$$
E(x^{2}) = \frac{n\alpha}{\alpha + \beta} + (n^{2} - n) \left[\frac{\alpha\beta}{(\alpha + \beta)^{2}(\alpha + \beta + 1)} + \frac{\alpha^{2}}{(\alpha + \beta)^{2}} \right]
$$

\n
$$
Var(x) = Var(x) = E(x^{2}) - [E(x)]^{2}
$$
\n(3.51)

$$
\begin{aligned}\n\text{Var}(x) &= \text{Var}(x) = E(x^2) - \lfloor E(x) \rfloor \\
&= \frac{n\alpha}{\alpha + \beta} + \frac{\left(n^2 - n\right)\alpha\beta}{\left(\alpha + \beta\right)^2 \left(\alpha + \beta + 1\right)} + \frac{n^2\alpha^2}{\left(\alpha + \beta\right)^2} - \frac{n\alpha^2}{\left(\alpha + \beta\right)^2} - \frac{n^2\alpha^2}{\left(\alpha + \beta\right)^2} \\
\text{Var}(x) &= \frac{n\alpha\beta(\alpha + \beta + n)}{\left(\alpha + \beta\right)^2 \left(\alpha + \beta + 1\right)}\n\end{aligned}\n\tag{3.52}
$$

3.4.2 Moment method of beta binomial model
\nGiven the pmf
\n
$$
P_x(X) = {n \choose x} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(x+\alpha)\Gamma(n-x+\beta)}{\Gamma(\alpha+\beta+n)}
$$
\n
$$
E(x) = \left(\frac{n\alpha}{\alpha+\beta}\right) \quad Var(x) = \frac{n\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}
$$
\nThe value of the sample mean and sample variance
\n
$$
\overline{X} = \frac{1}{N} \sum_i x_i
$$
\n(3.53)

The value of the sample mean and sample variance

$$
\overline{X} = \frac{1}{N} \sum_{i} x_i
$$
\n(3.53)

$$
\overline{X} = \frac{1}{N} \sum_{i} x_{i}
$$
\n
$$
S_{x} = \frac{1}{N} \sum_{i} (x_{i} - \overline{x})^{2}
$$
\n
$$
\left(\frac{n\alpha}{\alpha + \beta}\right) = \overline{X}
$$
\n(3.54)

$$
S_x = \frac{1}{N} \sum_i (x_i - \overline{x})^2
$$
\n
$$
\left(\frac{n\alpha}{\alpha + \beta}\right) = \overline{X}
$$
\n(3.55)\nequating the variance\n
$$
\frac{n\alpha\beta(\alpha + \beta + n)}{(\alpha + \beta)^2(\alpha + \beta + 1)} = S_x
$$
\n(3.56)

equating the variance

$$
(\alpha + \beta)
$$

equating the variance

$$
\frac{n\alpha\beta(\alpha + \beta + n)}{(\alpha + \beta)^2(\alpha + \beta + 1)} = S_x
$$
 (3.56)

$$
\frac{\alpha}{\alpha+\beta} = \frac{\overline{x}}{\pi}; \quad \alpha+\beta=\frac{n\alpha}{\overline{x}}
$$
\nfinally

\n
$$
\beta = \alpha \left(\frac{n}{\overline{x}}-1\right)
$$
\nSubstituting into equation 2

\n
$$
\frac{n\overline{x}}{n} \cdot \alpha \left(\frac{n\alpha}{\overline{x}}-1\right) \left(\frac{n\alpha}{\overline{x}}-n\right) = S_x
$$
\n
$$
\frac{n\alpha}{n} \cdot \left(\frac{n\alpha}{\overline{x}}\right) + n\overline{X}^2 \cdot (n-\overline{x}) = m^2S_x + n\overline{X}S_x
$$
\n
$$
= \overline{X}n\alpha(n-\overline{x}) + n\overline{X}^2(n-\overline{x}) = m^2S_x + n\overline{X}S_x
$$
\n
$$
= \overline{X}n\alpha(n-\overline{x}) - n^2S_x = \overline{n}S_x - n\overline{X}^2(n-x)
$$
\n
$$
\alpha = \frac{\overline{X}(n\overline{x}-\overline{X}^2-S_x)}{nS_x + \overline{X}^2-n\overline{X}} \qquad (3.57)
$$
\n
$$
\hat{\beta} = \frac{(n-\overline{X})(n\overline{x}-\overline{x}^2-S_x)}{nS_x + \overline{X}^2-n\overline{X}} \qquad (3.58)
$$
\nThe beta (R, β) random variable.

\nThe solution is used to model the number of success in a binomial trials when the probability of the Beta (R, β) random variable.

\nThe initial distribution is used to model the number of success in a binomial trials when the probability of the Beta (R) in the right of the variance of beta following theorem and derivatives:

\n
$$
Var(x) = E\left[n\rho(1-\rho)\right] + Var\left[\frac{E(x/\rho)}{P}\right] = \overline{E\left[n\rho(1-\rho)\right]} + Var\left(n\rho\right) \qquad \text{and } \overline{E\left[n\rho(1-\rho)\right]} + Var\left(\frac{\rho}{\rho}\right) \qquad (3.59)
$$
\nThis result shows that Vary is the variance associated with Binomial distribution and p the variance of beta variable \overline{B} which is the average probability of the vehicle crash.

\n
$$
\overline{E} = \frac{\alpha}{\alpha+\beta}, \text{Var}(p) = \frac{\alpha}{\alpha+\beta
$$

$$
\hat{\beta} = \frac{(n - \bar{X})(n\bar{X} - \bar{X}^2 - s_X)}{nS_x + \bar{X}^2 - n\bar{X}}
$$
\n(3.58)

of success p is the Beta (α, β) random variable.

following theorem and derivation:

The beta binomial distribution is used to model the number of success in n binomial trials when the probability of success p is the Beta (a,
$$
\beta
$$
) random variable.
\nThe ability of the Beta Binomial distribution to explain the extra variability can be demonstrated using the following theorem and derivation:
\n
$$
Var(x) = E[Var(x/p)] + Var[E(x/p)]
$$
\n
$$
= E[np(1-p)] + Var(np)
$$
\n
$$
= n\left[E(p) - E(p)^{2}\right] + n^{2}Var(p)
$$
\n
$$
= n\overline{p}(1-\overline{p}) + n(1-n)Var(p)
$$
\n(3.59) This result shows that Var(x) is the variance associated with Binomial distribution and p the variance of beta

random variable \bar{p} which is the average probability of the vehicle crash.

random variable *p* which is the average probability of the vehicle crash.
\naccording to the characteristics of Beta distribution,
\n
$$
\overline{P} = \frac{\alpha}{\alpha + \beta}, \text{Var}(p) = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}
$$
\n
$$
\text{Var}(p) = \left(\frac{1}{(\alpha + \beta + 1)}\right) \frac{\alpha}{(\alpha + \beta)} \frac{\beta}{(\alpha + \beta)} = \alpha \overline{P}(1 - \overline{p}) \tag{3.61}
$$
\n
$$
\alpha = \overline{P}(\alpha^{-1} - 1) \text{ and } \beta = (1 - \overline{p})(\alpha^{-1} - 1) \text{ where } \alpha = (\alpha + \beta + 1)^{-1}
$$

$$
\alpha = \overline{P}(\alpha^{-1} - 1)
$$
 and $\beta = (1 - \overline{p})(\alpha^{-1} - 1)$ where $\alpha = (\alpha + \beta + 1)^{-1}$

The log-likelihood function of the Beta-binomial probability distribution function can be written as a function of *p*,α and n. While *p* can be linked with explanatory variables through the logit link.

Application of Beta-Binomial And Negative Binomial Models To Road Traffic Accidents In Nigeria
\n
$$
\overline{P} = \frac{1}{1 + \exp(-Y\beta)}
$$
\n(3.62)
\nWhere, **X** = YB is the simplest safety performance function (a vector), with the coefficients B.

likelihood function of the Beta Binomial distribution.

Where,
$$
\mathbf{X} = \mathbf{Y} \boldsymbol{\beta}
$$
 is the simplest safety performance function (a vector), with the coefficients $\boldsymbol{\beta}$.
To estimate the coefficients $\boldsymbol{\beta}$, the maximum likelihood estimation (MLE) method can be applied to the log-
likelihood function of the Beta Binomial distribution.
Finally the Beta Binomial model has the following form:

$$
\hat{U}_i = E(\hat{x}_i) = \hat{n}_i \hat{p}_i = \frac{\hat{n}_i}{1 + \exp(-Y\beta)}
$$
(3.63)

The efficiency of the stated models will be compared using Akaike Information Criterion(AIC) and Bayesian information criterion (BIC) in order to choose the model with the best performance in terms of mean square effort or mean absolute error, the model with the lower value is the best fit.

3.5 AKAIKE INFORMATION CRITERION (AIC)

The Akaike Information Criterion(AIC) is a way of selecting a model from a set of models. The chosen model is the one that minimizes the Kullback-Leibler distance between the model and the truth. Its based on information theory, but a heuristic way to think about it is as a criterion that seeks a model that has a good fit to the truth but few parameters. It is defined as:

$AIC = -2 \left(\ln \left(\text{likelihood} \right) \right) + 2K$.

Where likelihood is the probability of the data given a model and K is the number of free parameters in the model. AIC scores are often shown as \Box AIC scores. Or difference between the best model (smallest AIC) and each model (so the best model has a \Box AIC of zero).

The second order information criterion, often called AICc, takes into account sample size by, essentially, increasing the relative penalty for model complexity with data sets. It is defined as:
AICc= -2 $\left(ln\left(likelihood\right) \right) + 2K * \left(\frac{n}{n-k-1} \right)$

increasing the relative penalty for model complexity with data:
AICc=
$$
-2\left(In\left(likelihood\right)\right)+2K*\left(\frac{n}{n-k-1}\right)
$$

Where n is the sample size, as n gets larger. AICc converges to AIC $n-k-1 \rightarrow n$ as n gets much bigger than

, and so $\left(\frac{n}{n-k-1} \right)$ *n* $\left(\frac{n}{n-k-1}\right)$ approaches 1, and so there's really no harm in always using AICs regardless of sample

size. In model selection in comparative methods, sample size often refers to the number of taxa(Butler and King, 2004;O'Meara et al.,2006).

3.6 BAYESIAN INFORMATION CRITERIA

The Bayesian information criterion (BIC) is a criterion for model selection among a finite set of model; the model with the lowest BIC is preferred. It is based on the likelihood function and it is closely related to the Akaike information criterion (AIC). It is a model selection among a class of models with different number of parameters and a measure of consistence of the model. BIC provides a large estimator of a transforming of the Bayesian posterior probability associated with the approximating model . By choosing the fitted candidate model corresponding to the minimum value of BIC, one is attempting to select the candidate model corresponding to the highest Bayesian posterior probability.
BIC = $-2InL(\theta_K / y) + kIn(n) = -2Inf(y / \theta_k) + kIn(n)$ corresponding to the highest Bayesian posterior probability.

$$
BIC = -2InL(\theta_K / y) + kIn(n) = -2Inf(y/\theta_k) + kIn(n)
$$

BIC could be advocated when the primary goal of the modeling application is descriptive.

3.7 CHISQUARE GOODNESS OF FIT

The chi-square goodness of fit test is used to determine whether observed sample frequencies differ significantly from expected frequency specified in the null hypothesis. The degree of freedom are equal to the number of level k of the categorical variable minus 1: df=k-1. The expected frequency counts at each level of the categorical variables are equal to the sample size times the hypothesized proportion from the null hypothesis where E_i is the expected frequency count for the i^{th} Level of the categorical variable, n is the total sample size and p_i is the hypothesized proportion of the observations in level i. the test statistics square random variable (χ^2) defined by the following equations

$$
\chi^2 = \Sigma \left[\left(O_i - E_i \right)^2 / E_i \right]
$$

Where O_i is the observed frequency count for the i^{th} level of the categorical variable, and E_i Is the expected

frequency count of the ith level of the categorical variable.

To examine factors that influence the rate of road traffic accidents such as, speed, kind of vehicles, age of the victim, type of traffic, driver's license holder and others will be considered, it's consistency will be visually measured.

IV. DATA ANALYSIS AND RESULT

4.0 INTRODUCTION

This chapter focuses on the preliminary analysis of the road traffic accident data in Delta State and the detailed analysis of the data using Beta Binomial model as well as comparing it with negative Binomial model.

4.1 SOURCE OF DATA

The data is a secondary data collected from the state secretariat, Federal Road Safety Commission Delta state sector command and accident prone stretches on the area.

The FRSC was established by the government of the Federal Republic of Nigeria vide Decree 45 of 1988 as amended by Decree 35 of 1992, with effect from 18th February, 1998. The commission was charged with responsibilities of, among others, policymaking, organization and administration of road safety in Nigeria. This study considered road traffic accident from 2010 to 2018.

4.1 THE DAYS OF FATAL ROAD TRAFFIC ACCIDENT

The days in which people were killed through fatal road traffic accident from 2010 to 2018 was modeled using beta binomial and the various AICs and BICs were presented in tables.

4.1.1 : Parameter Estimates Of Beta Binomial Model For Days Of Fatality

The table 4.1.1 shows the parameter estimates of the selected model. The AIC of this model was 489.61: the null deviance was 294.483 on 69 degrees of freedom and residual deviance of 90.604 on 54 degrees of freedom following the chi-square distribution (χ^2) with one degree of freedom. The dispersion parameter was found to be 1.9931 and P-value of 1.9590543e-07 which indicated that the model is significant. The dispersion parameter of the above model is far greater than 1, an indication of over dispersion in the data. This means that the parameters of the model have been over estimated and will not give true reflection of number of people likely to be killed through road traffic accident in a given day of the week for a particular year. To eliminate this error, Negative Binomial model was used to validate the model and the result shows in the

table 4.1.2

Table 4.2: Parameter Estimates of Negative Binomial Model for Days of Fatality

Coefficient	Estimate	Std.Error	Z value	$Pr(\geq z)$
(intercept)	3.98228	0.08458	47.080	$< 2e - 16***$
wkdavMondav	-0.12838	0.7774	-1.652	0.09863
Wkday Tuesday	-0.40939	0.08316	-4.923	$8.52e-07***$
Wkday Wednesday	-0.61076	0.08780	-6.956	$3.50e-12***$
Wkday Thursday	-0.34001	0.08171	-4.161	$3.17e-05***$
Wkday Friday	-0.002368	0.06543	-2.2871	0.003214
wkdavSaturdav	-0.24915	0.07992	-3.117	$0.00183***$
Year2010	-0.11407	0.09586	-1.190	0.23404

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The table below shows the assessment criteria for beta binomial and negative binomial models

The dispersion parameter for negative binomial model was 1.8948 which is closer to 1 and a good sign of reduced over dispersion in the data if not totally eliminated as compared with 1.9931 for beta binomial model as shown in the table above.

Also, it could be observe d from the above table that AIC and BIC of the negative binomial model is 489.61 and 702.51 which is smaller than the beta binomial model and this test state that model with the lowest AIC and BIC will produce a best result when fitted to the data. From the table 4.2.3 above, it can be deduced that Negative Binomial model fits the fatal accident data. Thus, Negative Binomial Model is efficient to describe the fatal road traffic accident.

The R package takes the first category in a data as the base level by default and as such Sunday and 2010 were picked as the base levels for comparison in the analysis of the parameter estimates in the model. From table 4.2 the intercept was found to be 3.98228 which was very significant at 95% significant level.

4.2 THE MONTHS Of THE YEAR OF SERIOUS ROAD TRAFFIC ACCIDENTS

The months in which people were seriously injured in road traffic accident from 2010 to 2018 was modeled using beta binomial model and the various models with their AICs and BICs presented in the tables below

Injured Inrough Road Trainc Accidents in Months Of The Years					
Coefficient	Estimate	Std.Error	Z value	$Pr(\geq z)$	
(Intercept)	1.6967	0.1350	12.570	$<2e-16***$	
Monthfeb	0.1994	0.1120	1.780	0.075021	
Month mar	0.0339	0.1165	0.291	0.770977	
Month apri	0.2273	0.1113	2.042	$0.041181*$	
Monthmay	0.2162	0.1116	1.938	0.052663	
Monthjun	0.3985	0.1074	3.712	$0.000205***$	
Monthjul	0.2162	0.1108	2.248	0.024592*	
Monthagus	0.3985	0.1053	4.747	$2.06e-06***$	
Monthsept	0.2490	0.186	3.143	$0.001673**$	
Monthoct	0.4997	0.1076	3.619	0.000296***	
Monthnov	0.34143	0.1103	2.451	$0.014240*$	
Monthdec	0.3892	0.1121	1.728	0.084069	
Year2010	0.2703	0.1326	5.316	1.06e-07	
Year 2011	0.1938	0.1246	9.156	$<2e-16***$	
Year 2012	0.7048	0.1272	7.685	$1.52e-14$	
Year 2013	1.1408	0.1197	12.727	$< 2e-16***$	
Year 2014	0.9779	0.1200	12.456	$< 2e-16***$	
Year 2015	1.5235	0.1254	8.664	$<$ 2e-16***	
Year 2016	1.4949	0.1428	2.178	0.029422	
Year 2017	1.0868	0.1303	6.255	3.99e-10***	
Year 2018	0.3109	0.1315	5.748	$9.01e-09***$	

Table 4.2.1: The Parameter Estimates Of Beta Binomial Model For The Number Of People Seriously Injured Through Road Traffic Accidents In Months Of The Years

The table 4.2.1 shows the parameter estimated of the beta binomial model. The AIC of this model was 841.27: the null deviance was 692.84 on 199 degree of freedom and residual deviance of 247.25 on 99 degree of

freedom following the chi-square distribution (χ^2) with one degree of freedom. The dispersion parameter was found to be 3.6781 and P-value of 0.006 which indicates that the model is significant.

The parameters of the model have been over estimated and will not give a true reflection of number of people likely to be seriously injured through road traffic accident in a given month of a particular year. To estimate this error, Negative binomial model was used to validate the model and the result is shown in the table 4.4.2 below.

The table below shows the assessment criteria for beta binomial model and negative binomial model.

Table 4.2.3: Parameter Comparison Beta Binomial And Negative Binomial Models For Goodness Of Fit for Month Of Serious Road Traffic Accident

The dispersion parameter for negative binomial model is 2.6486 which is better than that of beta binomial and a good sign of reduced over dispersion in the data if not totally eliminated.

Also, it could be observed from the above table that AIC and BIC of the negative binomial model is 812.12 and 522.56 which is smaller than that of beta binomial model of 841.27 and 549.49 and this test state that model with the lowest AIC and BIC will produce a best result when fitted to the data. From the table 4.2.3 above, it can be deduced that Negative Binomial model fits the serious accident data. Thus, Negative Binomial Model is efficient to describe the serious road traffic accident.

4.3 **THE NUMBER OF PEOPLE WHO HAD MINOR ROAD TRAFFIC ACCIDENT FROM 2010 to 2018**

The number of people who was involved in minor road traffic accident were analyze as follows

Table4.3: The parameter estimate of Beta Binomial Model for the number of people involved in minor road traffic accident

Coefficient	Estimate	Std. Error	Z value	$Pr(\geq z)$
(Intercept)	3.551770	0.059868	59.326	$<$ 2e-16
Monthfeb	-0.059058	0.085945	-0.780	0.49198
Month mar	-0.074384	0.084743	-0.291	0.38865

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Month apri	-0.003591	0.083442	-0.042	0.96620
Monthmay	0.059148	0.090050	0.938	0.47842
Monthjun	-0.233049	0.096599	3.712	$0.00965**$
Monthjul	-0.472156	0.097119	-2.248	$1.02e***$
Monthagus	-0.489548	0.106471	-4.747	$1.02e-06***$
Monthsept	0.771399	0.03463	-3.143	$4.32e-13***$
Monthoct	0.204991	0.00546	-0.019	$2.02e-02$
Monthnov	0.03452	0.02324	-2.451	0.0034
Monthdec	0.42554	0.04367	1.728	$<$ 3e016***
Year2010	0.30245	0.03247	-5.316	0.005431
Year 2011	-0.20499	0.02132	9.156	0.00012
Year 2012	0.039036	0.02328	7.685	$<$ 2e-16***
Year 2013	0.01067	0.04450	0.0727	0.04353
Year 2014	0.108945	0.01980	0.4256	0.0452
Year 2015	0.551770	0.03121	1.064	0.00341
Year 2016	0.21132	0.203991	2.158	0.00034
Year 2017	0.12328	0.03352	5.255	0.00044
Year 2018	0.04906	0.22554	8.848	$< 2.e-13**$

Table 4.3.2: the parameter estimates of Negative Binomial Model for the number of people involved in minor road traffic accident

From the table 4.3.2 it is observed that the parameter estimate has been reduced .Therefore, there us need to compare the goodness of fit for both beta binomial and negative binomial regression as shown in table 4.3.3 below;

Looking at the results presented in table 4.3.3, it is clear that the Negative Binomial model is actually the best model which fit the minor road traffic accident data because the dispersion parameter has reduced from 1.6781 which was giving by the beta binomial model to 1.0552.

The AIC of the beta binomial also reduced from 1041.27 to 717.23 in the negative binomial model. Therefore, negative binomial indicating a better estimate of the minor road traffic accident model.

R package takes the first category in a data as the base level by default and as such Monthfeb

And the year 2010 were picked as the base level for comparison in the analysis of the parameter estimates in the negative binomial model.

The intercept was found to be 3.032725 which was very significant at 95% significant level with p vale of <2e-16 following Pearson Chi-square (χ^2) distribution with 54 degrees of freedom as shown in table 4.3 above.

4.4 EXAMINE THE FACTORS INFLUENCING THE RATE OF ROAD TRAFFIC ACCIDENTS The factors contributing to road traffic accident cannot be ruled out

4.4.1 **Physical factors influencing the rate of road traffic accident**

From table 4.4.1 above, it could be seen that Speed of vehicle during the accident occurs 828 times from 2010 to 2018 which constitute 36.7% of the total number of occurrence via road traffic accident in the same period. The was followed by road characteristics 409 representing 18.1% of the number of times it occurs.

The further descriptive statistics of average number of occurrence of the factors influencing road traffic accident is as shown in the table 4.4.2 below

Table4.4.3 Descriptive Statistics

It is discovered that the minimum value is 14.00 for Pavement condition and the maximum is 828.00 for Speed

of vehicle during the accident with range 814.00 and standard error 93.21345.

V. SUMMARY OF FINDINGS, CONCLUSION AND RECOMMENDATION

5.0 SUMMARY

Road traffic accidents has played and is still playing very important role in human life. On one hand it is loss of property and on the other hand it is a loss of humanity and vicious cycle of poverty with its impact on overall economy of the country. This is a preventable issue which could save many lives with little efforts by abiding with the traffic rules and regulations.

On the basis of the AIC values in tables 4.2.2 the estimated AIC for beta binomial model is 490.97 whereas it is 489.61 for the negative binomial model for fatal road traffic accident, for serious road traffic accident 841.27 for the beta binomial model and 812.12 for negative binomial model, finally, minor road traffic accident has 1041.27for beta binomial model and 717.23 for negative binomial model, the smallest AIC value is that of the negative binomial model. There is a need to take a step on government side, health care professional's side and individual side. Based on the result from the analysis above.

5.1 **CONCLUSION**

Accident is a common phenomenon, it does not segregate on the basis of time and place of occurrence. Road traffic accident in Nigeria is a very serious issue requiring a holistic attention and approach towards curbing its occurrence considering the magnitude of the problem it presents to every Nigerian road users. This research applied beta binomial and negative binomial models to road traffic accident in Delta state.

Therefore, the best model for the number of road traffic accident in Delta state road safety command is best modeled and described using the negative binomial model. It can be concluded that negative binomial model provides a better fit with relative lower AIC and BIC than the beta binomial model.

5.2 RECOMMENDATION

Road Traffic accidents will not only take human lives and damage properties but also can lead to social problems. It is important to prevent or at least minimize road traffic accidents. Keeping this in mind and the causes of road traffic accident, the following are the proposed recommendations to prevent/minimize road traffic accidents.

1. It is necessary to provide proper education to drivers and proper driving habits should be enforced.

2. The process of issuing driving licenses must be tough and the license should be issued after proper examinations.

3. The roads should be maintained properly. The black spots(where repeated accidents have happened) should be looked into for proper possible remedial measures.

4. Over speeding offenders should be dealt with and animals that used to cross the road should be checked by putting appropriate measures.

5. Sign boards and traffic signals must be provided wherever it is necessary.

6. The accident data base of the country should be expanded to include more variables so that researchers could really determine the actual factors contributing to road accident.

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