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Research Paper

Effect of Heat Transfer Coefficient on the Heat Transfer Process of Solidification with Sinusoidal Convective Upper Boundary Condition

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ABSTRACT: *Freezing/solidification process involves continuous transfer of heat in the material medium to change from liquid to solid. The problems that describe these processes satisfy certain initialand boundary conditions along with the Stefan condition at the interface. These problems referred as Stefan problems or Moving boundary problems or Phase change problemsin which there exists continuously changing moving fronts.At any time, tracking the position of themoving fronts and the temperature plays an important role to solve these non-linear problems. Phase change problems has applications in the field of energy conservation techniques, Material Science,industrial units and many more.In this paper, solidification of water is studied to understand the mechanics of heat transfer problems. A convective upper boundary condition is considered as sinusoidal function of time, to study the changes in the temperature fields, which in general treated as constant. Understanding the effect of heat transfer coefficient on the movement of moving fronts and the temperature distributionhelp to design ice storage units in energy conservations technologies.*

KEYWORD: *Moving boundary problems, phase change, Heat transfer coefficient, frozen region, dehydrated region.*

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I. INTRODUCTION

Moving boundary Problems are phase change problems that changes the phase of the medium from one state to another state. In particular, solidification is a phase change process from liquid to solid. A moving boundary that separates these phases evolves in the process and continuously changes its position with respect to time [1]. Tracking or finding the position of this moving boundary and the temperature distribution in the medium are the two important parameters that describe the mechanism. Stefan problems are non-linear and there exists very limited analytical solutions. In this study, a one-dimension rectangular medium is considered with water as its medium for simple understanding of the process.Water is an ideal choice for thermal storage systems because of its availability, high latent heat, non-flammability, and non-toxicity. Also temperature in the medium is assumed to be uniform, with constant thermal properties [2].In this problem, at the surface convective boundary condition is a function of time that is a periodically oscillating sinusoidal function of time, which is usually in much Stefan type of problems treated as a constant. The effect of heat transfer coefficient h with sinusoidal upper boundary condition on the movement of the interfaces of dehydrated and frozen zones along with the temperature distributions in these regions is studied. Forward difference method is applied to solve the simultaneous first order differential equations. Initial conditions are used to solve the moving interfaces positions at the nodal points in the fixed time intervals. Scilab software is used to solve the proposed finite difference method. There is been extensive research in this field to understand the mechanism with appropriate boundary and initial conditions.

Mariela C. Olguin [4] produced an analytical solution for the freezing problem for water content materials. Both heat and mass transfers are coupled to study the freezing mechanics for the two-phase solidification problem with vapor concentration. This study is an extension work of this research paper.

Controlled melting and solidification problems are of great practical applications. The paper [3] describes the effect of various modes of heat transfer in a cylindrical encapsulation both in horizontal and vertical orientations to study the melt fraction and heat transfer rates. Correlations was drawn between the Fourier, Stefan, Rayleigh and Nusselt numbers that help to analyze the melting characteristics.

In phase change problems, the geometry of the PCM plays an important role. The study by [8]describes analytical investigation of the freezing process in an annular region. Variable Heat transfer coefficient was determined on the surface of the solidification front as a function of moving front. The numerical results are compared with the experimental results with good agreement. These models help in designing heat accumulators and to understand the solidification process.

Energy storage in the form of latent heat for energy conservation using phase change materials is the current active research area that is drawing many researchers in this field. An extensive study is carried [7] to enhance the thermal conductivities of the PCM's using the external fields. The paper is a review the effect of external fields like magnetic field, electric field on the process of nucleation, reducing the time of nucleation to improve the heat transfer rate. In addition, the study focuses on improvement of heat transfer characteristics of PCM's, future scope, insufficiency in the current research.

Numerical simulations are compared with the experimental results performed on PCM RD 42 to see the effect of natural convection along with the volume change [6]. Lagrangian- Eulerian method is applied to implement volume change by adding a force field to the regular Navier- Stokes equations. An Enhanced model with convection and volume changes unlike the default modelfor phase change problems, which neglects convection flow and volume change has drawn results that are more accurate.

II. MATHEMATICAL MODELLING

To model the freezing process the region is divided into sub-regions unfrozen, frozen and dehydrated, separated by the moving interfaces or boundaries. Frozen and dehydrated regions are considered as active regions in which conduction mode of heat transfer dominates over convection heat transfer. Freezing process starts at the top of the surface and proceeds downwards. In the dehydrated region, a very thin layer exists in which the water vapor evaporates in the form of sublimation (solid to vapor) which progresses with the sublimation temperature (T_{sub}). $T_1(x,t) and\, T_2(x,t)$arethe temperature distributions and $x = s_1(t)\ and\ x = s_2(t)$ are the interface positions at the dehydrated and frozen regions respectively.

The governing differential equations that describe the system are:

Differential equation at the dehydrated region is

$\rho_1 c_1 \frac{\partial T_1}{\partial t} = k_1 \frac{\partial^2 T_1}{\partial x^2} \quad 0 < x < s_1(t) \qquad t > 0$ ………(1)

Differential equation at the frozen region is

$\rho_2 c_2 \frac{\partial T_2}{\partial t} = k_2 \frac{\partial^2 T_2}{\partial x^2} s_1(t) < x < s_2(t) \qquad t > 0$ ……….(2)

Boundary condition at the moving boundary $x = s_1(t)$ is

$T_1(x,t) = T_2(x,t) = T_{sub}(t)$ ………..(3)

$k_2 \frac{\partial T_2(s_1(t))}{\partial t} - k_1 \frac{\partial T_1(s_1(t))}{\partial t} = L_1 m_1 \dot{s}_1(t)$……..(4)

Free boundary conditions at the moving freezing front $x = s_2(t)$ is

$T_2(s_2(t), t) = T_{if}$……..(5)

$k_2 \frac{\partial T_2(s_2(t))}{\partial x} = L_2 m_2 \dot{s}_2(t)$………… (6)

$T_{sub}(t)$, $T_{if}$and T_s are sublimated,initial freezing and surrounding temperatures respectively. $L_1, L_{2,} m_1, m_2, c_{1,} c_2, \rho_1, \rho_2, k_1, k_2$are latent heat , mass per unit volume, volumetric heat capacity, density and thermal conductivities in dehydrated and frozen regions, respectively.

Convective boundary conditions at the fixed interface $x = 0$ is

$k_1 \frac{\partial T_1(0,t)}{\partial x} = h(T_1(0,t) - T_s) \qquad for \quad t > 0$ ………. (7)

Initial conditions at time $t = 0$ are

$s_1(0) = s_2(0) = 0$ ……… (8)

$T = T_{if} \qquad for\ x \geq 0$

Assuming the temperatures in dehydrated and frozen regions as

$T_1(0,t) = f(t)$……………(9)

$T_1(x.t) = A(t) + xB(t) \qquad 0 < x < s_1(t) \qquad t > 0$………. (10)

$T_2(x.t) = D(t) + xE(t)s_1(t) < x < s_2(t) \qquad t > 0$………. (11)

By using the initial and boundary conditions, the constants $A(t), B(t), D(t)\ and\ E(t)$

are evaluated using the initial and boundary conditions, the expressions for two moving fronts $s_1(t)$, $s_2(t)$in terms of simultaneous differential equations are

$$\dot{s}_2(t) = \frac{K_m L_1}{m_2 L_2} \frac{\left[\frac{Ma\, e^{\left(b - \frac{c}{T_{sub}(t)}\right)}}{RC_a T_{sub}(t)} - 1 \right]}{1 + \frac{K_m}{D_{ef}} S_1(t)} + \frac{h}{m_2 L_2} (f(t) - T_s) \quad \text{............ (12)}$$

$$\dot{s}_1(t) = \frac{K_m}{m_1} \frac{\left[\frac{Ma\, e^{\left(b - \frac{c}{T_{sub}(t)}\right)}}{RC_a T_{sub}(t)} - C_a \right]}{1 + \frac{K_m}{D_{ef}} s_1(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(13)}$$

$$s_2(t) = \frac{k_2 T_{if}}{m_2 L_2} \frac{\left[1 - \frac{T_{sub}(t)}{T_{if}} \right]}{(s_2(t) - s_1(t))} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(14)}$$

Introducing the non-dimensional parameters described below

$$\delta_1 = \frac{m_2 L_2}{k_2 T_{if}},\ \delta_2 = \frac{m_1}{K_m C_a}\ ,\ \delta_3 = \frac{m_1 L_1}{h T_{if}}\ ,\ \delta_4 = \frac{Ma}{C_a R T_{if}}\ ,\ \delta_5 = \frac{m_2 L_2}{h T_{if}}$$

Equations (12), (13), (14) are reduced as follows

$$\dot{s}_2(t) = \frac{\delta_3 \delta_4 T_{if}}{\delta_2 \delta_5} \frac{\left[\frac{e^{\left(b - \frac{c}{T_{sub}(t)}\right)}}{T_{sub}(t)} - 1 \right]}{1 + \frac{K_m}{D_{ef}} S_1(t)} + \frac{(f(t) - T_s)}{\delta_3 T_{if}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(15)}$$

$$\dot{s}_1(t) = \frac{\delta_4 T_{if}}{\delta_2} \frac{\left[\frac{e^{\left(b - \frac{c}{T_{sub}(t)}\right)}}{T_{sub}(t)} - 1 \right]}{1 + \frac{K_m}{D_{ef}} s_1(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(16)}$$

$$T_{sub}(t) = T_{if}(1 - \dot{s}_2(t) \delta_1 (s_2(t) - s_1(t))) \ldots\ldots\ldots \text{(17)}$$

$$s_2(0) = s_1(0) = 0 \ \ldots\ldots\ldots \text{(18)}$$

Also $T_1(x,t)$ and $T_2(x,t)$ has the following expressions.

$$T_1(x,t) = T_{sub}(t) + \frac{h}{k_1}\left(x - s_1(t)\right)(f(t) - T_s),\ 0 < x < s_1(t), \quad t > 0 \ldots\text{..(19)}$$

$$T_2(x,t) = T_{if}\left(1 - \delta_1 \dot{s}_2(t)(s_2(t) - x)\right),\ s_1(t) < x < s_2(t), \quad t > 0 \ \ldots\ldots \text{(20)}$$

Equation (15), (16), (17) are simultaneous differential equations which are solved using the Scilab software by discretizing the equations using the finite difference method. The solution obtained explains the effect of the heat transfer coefficient on the process of movement of dehydrated and frozen depths and the temperature distributions in these regions at any given time interval. The sinusoidal function is a smooth curve with periodic oscillations that approaches zero as the time increases. Damped sine wave functions has engineering applications that explain the energy decrease in the form of heat after each oscillation. We choose

$$f(t) = T_{if} + A\sin(\omega t) \ldots\ldots\ldots\ldots\ldots\ldots\text{..(21)}$$

Here A is the surface temperature oscillation amplitude and ω is the oscillation frequency. Same form of oscillating boundary condition given by equation (21) was earlier used by Rizwan-Uddin [5]. The periodical sinusoidal function at the upper boundary can predict the periodicity of the waveform and the effect of the amplitude on the temperature distribution for a particular surrounding temperature. Near the beginning of the freezing process, thickness of ice is very thin and has little impact on heat transfer. As freezing progresses, however, the ice becomes thicker and significantly impedes heat transfer. Temperature of the fluid must decrease near the end of the freezing process in order to maintain the same freeze rate with this decreasing heat transfer. Obtained results explain the effect of heat transfer coefficient h on $T_1(x,t), T_2(x,t), s_1(t),\ s_2(t)$. All the calculations are performed at 1 atm pressure.

III. RESULTS AND DISCUSSION

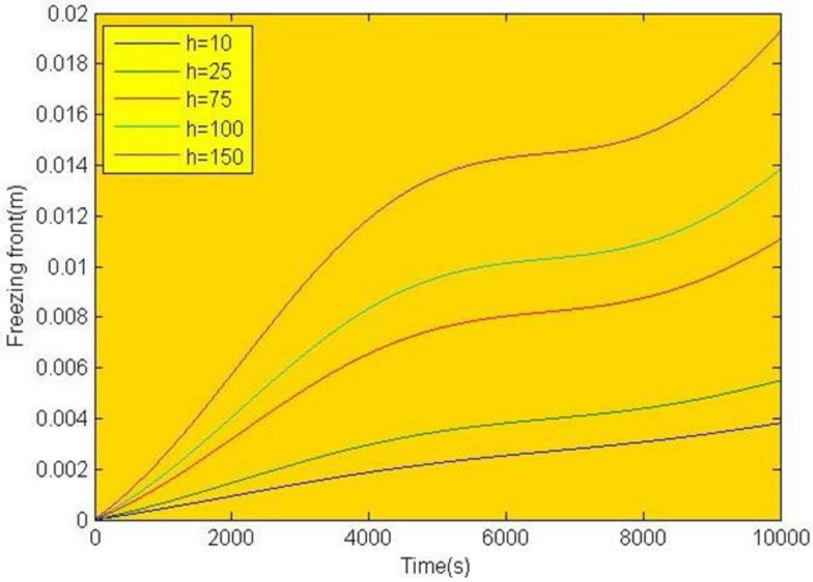

Figure 1: Influence of the heat transfer coefficient on the evolution of freezing front,
$f(t) = T_{if} + A\sin(\omega t)\,, A = 10, T_{if} = 243$

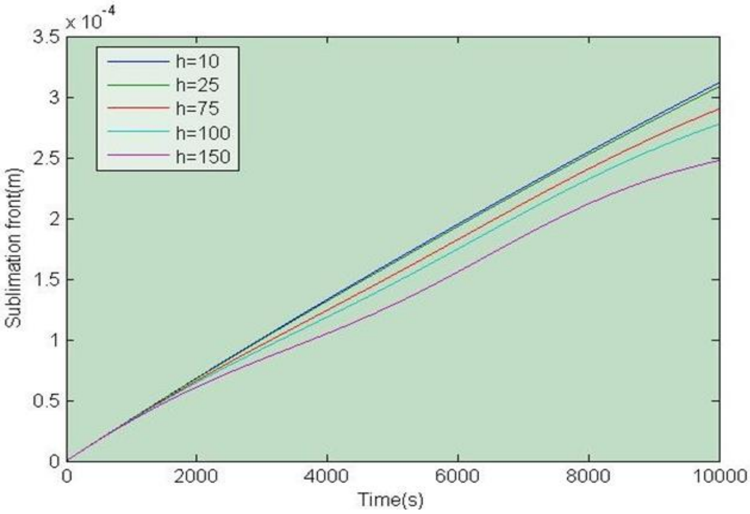

Figure 2: Influence of the heat transfer coefficient on the evolution of dehydrated front,
$\boldsymbol{f(t) = T_{if} + A\sin(\omega t)\,, A = 10, T_{if} = 243}$

Figure 1 and figure 2 show the increasing trend of$s_1(t),\ s_2(t)$, at different time intervals. Both the boundaries are increasing linearly for different values of h . The growth rate is more in the initial time-periodboth for freezing and for dehydrated fronts and is relatively low at the later period. This is because in the initial stages,heat transfer rate is more and hence both the regions grow at faster rate. Once the dehydrated region evolves, which has lower values of thermal conductivity and diffusion of water, which lowers the heat transfer rates from the freezing surface.Influence of$h$, is seen more on freezing front $s_2(t)$ than on the dehydrated front $s_1(t)$. It is observed that as h increases the depth of the frozen region increases. Higher the value of h, higher is the growth rate of freezing front. In addition, the amplitude of the waveform is higher for higher values of h ($h = 150$). For equal system characteristics and freezing conditions, dehydrated moving front $s_1(t)$, is lesser than at-least two orders of magnitude of, freezing moving front, $s_2(t)$. Ice dehydration is a very thin surface layer under normal freezing times for any real system.

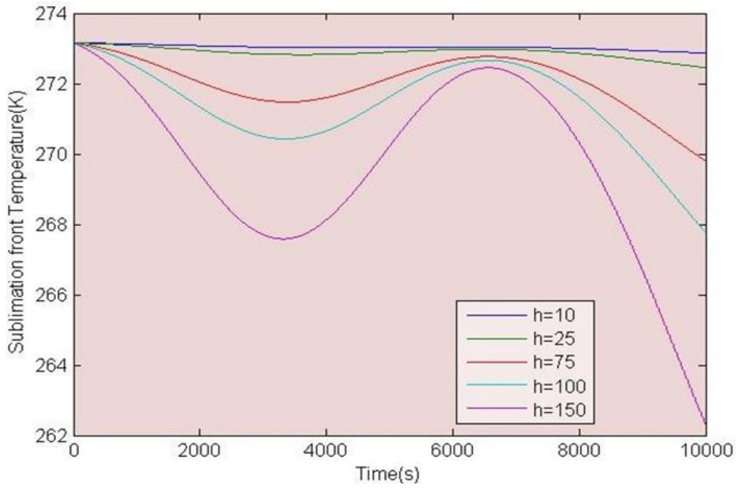

Figure 3: Influence of the heat transfer coefficient on the evolution of sublimation front temperature, $f(t) = T_{if} + A\sin(\omega t), A = 10, T_{if} = 243$

Figure3 show the sublimation front temperature, for different heat transfer coefficients. As h is increasing the sublimation temperature T_{sub} is decreasing. There is a significant effect of upper boundary conditions on the sublimation front temperatures. It is observed that when the temperature at the upper boundary is sinusoidal periodic boundary condition sublimation temperature decreases. Sublimation temperature characteristic curve amplitude is increasing with increasing h in sinusoidal form of boundary condition.

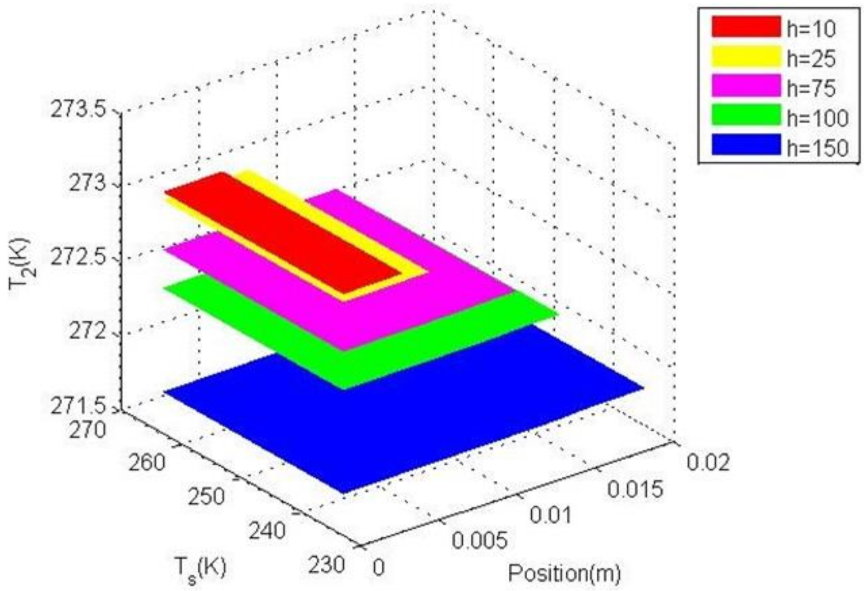

Figure 4: Temperature profile in the frozen region as a function of h, and T_s $f(t) = T_{if} + A\sin(\omega t), A = 10, T_{if} = 243$

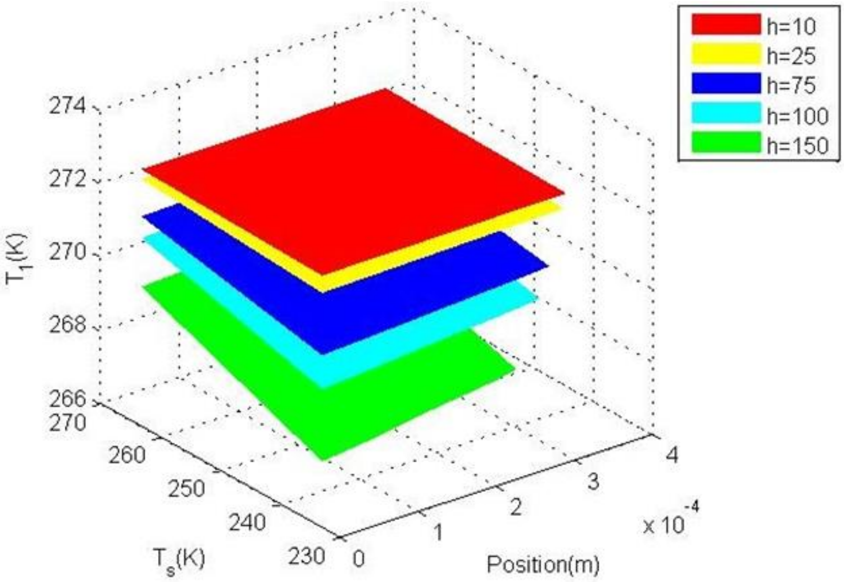

Figure 5: Temperature profile in the dehydrated region as a function of h, and T_s
$$f(t) = T_{if} + A\sin(\omega t)\,, A = 10, T_{if} = 243$$

Figure4 and figure5 shows the temperature profile in the frozen and dehydrated regions for different h, andT_s. It is noticed that for higher values of$h$, considerable decrease in frozen region and dehydrated region temperaturesand the range of the temperature distribution is significantly affected by the parameter h.

IV. CONCLUSIONS

- For the same freezing conditions, dehydrated moving fronts_1, is approximately lesser of two orders of magnitude of freezing moving front, s_2.
- Higher the value of h higher is the growth rate of the frozen region.
- Dehydrate and frozen regions show more steep in the temperature distribution for higher value of h.

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