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**Review Paper**

# Innovative Practices for the Teaching of Mathematical Concept of Surface Area

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***ABSTRACT:****This paper presents Innovative practices for the teaching of the mathematical concept of surface area in a fifth-grade elementary-school class using tools of RhodeScript Theory, such as the open problem, representations, new technologies, etc.More specifically, this paper aims to present the application of the above mathematical tools of RhodeScript Theory and their use in teaching practice, specifically on the concept of area, in order to improve students' performance on this concept. The good practices presented in this paper were applied in real classrooms, the subject area used for the four teaching interventions designed and implemented is Measurements, which was chosen based on both the difficulties faced by students in the above subject area and the individual concepts and the students' prerequisite knowledge related to the time period of the teaching intervention. The results of the study showed that the application of RhodeScript theory helped students to reduce cognitive barriers in the concept under consideration. It was also observed that students' interest in mathematics increased and students' statements indicated a positive attitude towards mathematics.*
***KEYWORDS:****RhodeScript Theory, Good practices, Fifth grade, Teaching interventions*

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## I. INTRODUCTION

International assessment programmes show low student achievement in mathematics in many countries around the world with different educational systems, including Greece. Therefore, many researchers worldwide have studied students' difficulties in mathematics and have proposed methods to reduce them. In particular, the findings of these studies show the critical aspects of how mathematics is taught [1], [2], [3], [4], [5] and [6], the use of representations when teaching mathematical concepts [7], [8] and [9], and the attitudes and beliefs of teachers and future teachers towards mathematics [10], [11], [12] and [13].

Considering the above, this paper presents good practices that were implemented with teaching interventions in authentic classroom conditions in a fifth-grade elementary-school class using mathematical tools of RhodeScript Theory, such as the open problem, representations and new technologies in order to improve students' performance and reduce their difficulties on the concept of surface area.

The thematic area that was used for the four teaching interventions that were designed and implemented is Measurements, which was selected on the basis of both the difficulties faced by students in the above thematic area and the individual concepts and the students' prerequisite knowledge related to the time period of the teaching intervention.

## II. THEORETICAL FRAMEWORK

### 2.1. Students' difficulties with area

According to a study conducted with 7th grade students [14], it seems that students associate the concept of area with closed geometric shapes and consider that its calculation requires the possibility of dividing/separating the shape into smaller equal shapes. For this reason, the tactic of calculating area that is mostly followed is to divide a given geometric figure into small squares and count them. Children of this age, however, are unable to understand that the area of one shape can serve as a unit of measurement for another. In the case where the given shape is not symmetrical, the strategy employed is the tactic of filling or padding. In this tactic and during the process of dividing the shape into squares, it is not understood that the above must be equal. The surface is often treated as something static that exists as a given, not combined with personal solution strategies.

Another study by authors [15] showed that students who had been taught the mathematical formula used three strategies: 1. 2. use of varied calculation steps, where the results were mostly incorrect, and students followed more than one step for the calculation. For example, they moved, separated and joined squares if the shape was drawn on square paper. 3. Inaccurate strategy. Strategies based on performing arithmetic operations based on mathematical formulas (in the specific case of area) can help to indirectly compute the area of unknown shapes that are not easily found otherwise (i.e., divide the unknown shape into known ones) [16]. Students who fail in tasks of calculating the area of a surface insist on using strategies whose generalization leads to failure.

Research has found that the use of material in measuring area plays an important role in both understanding the concept and calculating it. In terms of students' choice of material, it was shown that at around 5 years of age, students show an apparent preference for using standard units of measurement over non-standard ones [17] and [18]. Indeed, research conducted on length measurement showed that pupils aged 6 to 8 years prefer to use a ruler - even if broken - rather than non-standard units of measurement (e.g., a piece of rope) [19].

**2.2. The RhodeScript Theory**

The teaching approach to mathematical concept of area in the 5th grade of primary school presented in this paper is based on the use of mathematical tools of RhodeScript Theory, as they have been researched and applied by authors[20]. RhodeScript Theory is a theory structured on eleven basic mathematical practices, that is, on 10+1 tools. The teaching framework was named "RhodeScript", a word derived from the initial letters of the names of the mathematical tools in English.

1. **R**epresentations.
2. **H**istory of mathematics.
3. **O**pen problem
4. Breach of **D**idactical contract.
5. **E**stimation and mental Computation.
6. **S**patial ability and geometric transformations
7. **C**ounterexamples
8. **R**ealistic Mathematics Education
9. **I**nterdisciplinarity.
10. **P**osing problem

10+1 **T**echnology

RhodeScript Theory aims to strengthen mathematical literacy through a variety of practices, methods and tools that push students to understand mathematical concepts differently in situations that are meaningful to them, so that they engage in processes of knowledge discovery by externalizing and exchanging multiple strategies for problem solving [21].

In particular, the teaching interventions in this paper exploited the open problem, which is a mathematical problem where more than one correct solution can arise [22] and [23]. Practicing on this mathematical tool brings the innovation of collaborative spirit, multidimensional thinking and perspective, since the mathematical problem is not amenable to a single correct solution and is more "free" to be processed, and pluralism.

Another mathematical tool that will be used is representations. There are two kinds of representations, external and internal [24]. In this paper we will be concerned only with external representations (symbols, shapes, diagrams, texts), which are concerned with the vivid representation and visualization of mathematical concepts in real time, to make them understandable and more accessible to students.

Furthermore, another mathematical tool used is new technologies as they offer added value to the educational process. Future teachers should be prepared and familiar with new technologies to use them in the educational process effectively [25].

## III. THE RESEARCH

**3.1. Purpose of the research**

The present study, considering the difficulties of students in teaching of the mathematical concept of area of the square, the rectangle, and the right triangle, is an attempt to approach the concepts didactically using the mathematical tools of RhodeScript Theory, to reduce students' difficulties on these concepts. In particular, the objectives of the paper are to investigate:

- Whether the application of RhodeScript Theory to the teaching of the mathematical concept of area can reduce the difficulties of 5th grade students in calculating the area of a square, rectangle, and right triangle using their linear dimensions.
- Whether the application of RhodeScript Theory to the teaching of the mathematical concept of area can reduce the difficulties of 5th grade students in distinguishing the equality and equivalence of two shapes.

### 3.2. Research methodology

**Sample**

The population of the study consisted of 38 fifth grade students from a school in Rhodes, Greece.

**Survey tools**

To achieve the objectives of the study, teaching interventions, totaling 4 teaching hours, were conducted in April 2023, and the Lesson Study method [26] was used to observe the teaching interventions, as well as questionnaires/evaluation sheets prepared by the researchers and given to the students after the teaching interventions.

## IV. THE TEACHING INTERVENTIONS

This chapter presents the teaching interventions for the mathematical concept of area of a square, rectangle and right triangle for a fifth-grade elementary-school class using the mathematical tools of RhodeScript Theory, as researched, and applied by [20] and [21], to improve students' performance in the concepts under consideration. The teaching proposal is based on the good practices implemented in the context of our tutorials and had a very remarkable response from the student audience. The teaching interventions presented are related to the phases of control, stimulation, discovery, consolidation, and extension. For each project described below, the phase of the instructional intervention in which it was applied, the mathematical tools of RhodeScript Theory that were utilized, and a description of the project.

### 4.1. The teaching interventions

**Activity:** The distinction between equality and equivalence of two figures
**Phase:** Loading
**Mathematical tools of RhodeScript Theory:** Technologies, Representations
**Student Activity Description - Instructional Management Suggestion:** In class, using a projector, the teacher presents two shapes, a rectangle, and a triangle, in the dynamic geometry software environment GeoGebra (Figure 1). He then draws a vector and makes a transfer of one shape (e.g., the triangle) by that vector. After verbally explaining that he has constructed a picture of the shape in this way, he tells the students that they should predict whether the original and the new shape occupy the same area. After the students' predictions have been made, the teacher varies the vector until the two shapes are identical. He will explain that in this way "two identical shapes are called equal and that they occupy the same area". He will then ask students to predict whether there are shapes that, while not identical, can occupy the same area. The teacher will then explain to students that in geometry the concept of two shapes being identical has the same meaning as the concept of equality. Furthermore, the equality of the area occupied by two shapes in the plane has the meaning of equivalence and not equality. Next, they will be asked to think about and answer the following question: Is there a common method by which we can compare the area occupied in the plane by two geometric shapes? As students are not expected to formulate a clear procedure, the teacher will ask students to compare the two shapes with the help of an appropriate smaller shape.

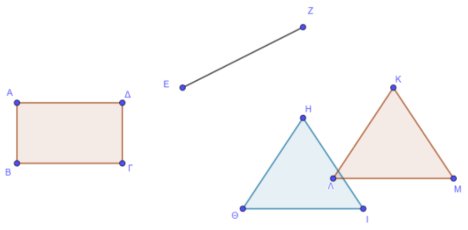

**Figure1:** The triangle and the rectangle in the digital environment of the dynamic geometry software GeoGebra.

**Activity:** Indirect comparison with a square
**Phase:** Discovery
**Mathematical tools of RhodeScript Theory:** New Technologies, Representation
**Student Activity Description - Instructional Management Suggestion:** On the interactive whiteboard, the teacher will use the dynamic geometry software Geogebra to draw a rectangular parallelogram and a triangle and then display the grid on an orthogonal axis system (Figure 2). The teacher asks the students:

1. Count the number of squares contained in each figure.
2. To compare the area occupied by the two shapes and confirm the comparison they made in the previous activity.

In addition, the teacher asks students to carefully count the whole squares contained in each shape and estimate how many whole squares correspond to the remaining parts of the squares in each shape. Students are expected to find different results, and the teacher should guide them to think about and suggest ways to make better measurements of the squares. Gradually students with the help of the teacher are guided to the formulas for the area of a square, rectangle and right triangle.

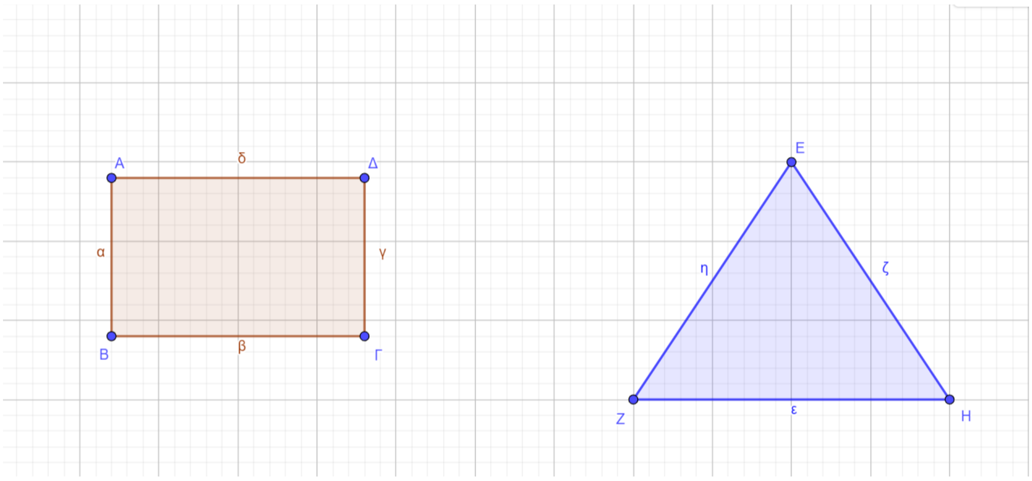


**Figure2:** The triangle and the rectangle in the digital environment of the dynamic geometry software GeoGebra.

**Activity:** Types of calculating the area of shapes.
**Phase:**Comprehension
**Mathematical tools of RhodeScript Theory:** Open Problem
**Description of student activity - Suggested instructional management:** The teacher will ask students to find how many squares it contains:

- A rectangle with one side 8 units (side of a square) and another side 4 units.
- A right triangle with one vertical side 6 units and another vertical side 8 units.
- A rectangular parallelogram whose one side is 10 units, and the opposite side is at a distance (height) of 4 units. (Figure 3)

The teacher instructs the students to use the board to conduct experiments and verify their calculations. The teacher will ask the students to express verbally mainly but also numerically the rules they used to find the area. To facilitate the expression, he will explain to the students what we call height (height: we call the linear segment from a vertex to the line of the opposite side) and base (base: the opposite side from the height of a right triangle is called the base) in the given figures and encourage them to use these terms in expressing the rules.

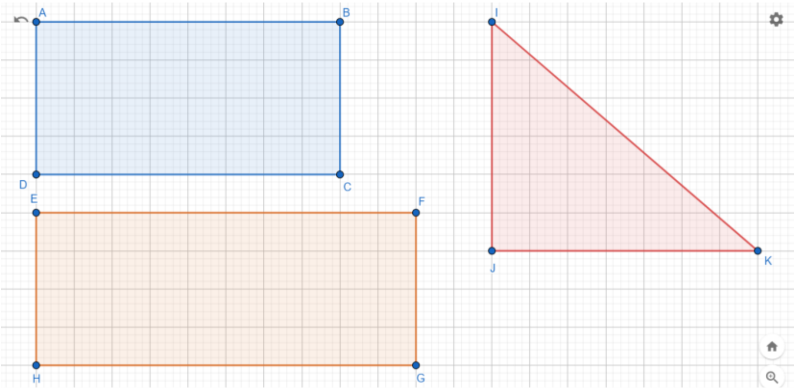


**Figure 3**: The shapes for finding the area in the digital environment of the dynamic geometry software GeoGebra.

**Activity:** Drawing a shape with a given area.
**Phase:**Expansion
**Mathematical Tools of RhodeScript Theory:**Technologies, Open Problem
**Student Activity Description - Instructional Management Suggestion:**The interactive whiteboard will display the Geogebra desktop with a grid and no axes (Figure 4). The teacher asks the students to work in groups of two and draw on their squared paper or on their personal computer desktop several shapes containing exactly 12 squares. For students who focus on only one shape - usually rectangles - that contain 12 squares (Figure 5), the teacher encourages them to find other types of shapes, such as oblique parallelograms or right triangles.



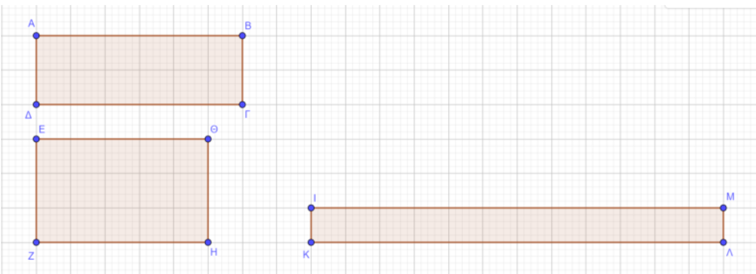


**Figure 4:** Multiple solutions of the problem by a team in the digital environment of the dynamic geometry software GeoGebra, which utilizes only rectangles.

## 4.2. Results

Regarding the teaching intervention onmathematical concept of area in the fifth grade of primary school, during the stimulation phase on the equivalence and equality of two shapes, there was a strong participation of the students, as they observed the shapes via representations in an interactive way and seemed interested.

In the discovery phase with the indirect comparison of two squares, it was observed that this activity contributed to a deeper understanding of area. Students were able to calculate it and even expressed the desire to repeat the activity with different shapes.

In the consolidation phase, which involved drawing a shape with a given area, students were able to complete the activity very quickly and worked in groups. The gradual discovery of knowledge with the help of the GeoGebra software and of the open problem seemed to favour the understanding of area.

In the extension phase with the formulas for calculating the area of shapes, students were now able to verbally explain their thinking.

## V. DISCUSSION-CONFERENCES

In this paper, good practices in the form of teaching interventions for teaching ofthe mathematical concept of surface area to a fifth-grade elementary-school class using RhodeScript Theory mathematical tools, such as the open problem, representations, new technologies, were presented in order to strengthen mathematical literacy through a variety of practices, methods and tools that push students to understand mathematical concepts in a different way in situations that are meaningful to them, so that they can engage in processes of mathematical literacy.

The results of the interventions showed that students who participated in activities in which RhodeScript Theory was applied acquired a positive attitude towards mathematics without any element of fear and aversion, recognized its usefulness, developed their interest in it, thus enhancing their mathematical perception and mathematical creativity.

Also, based on the evaluation sheets, it appeared that the students were helped to reduce their difficulties on the concept of area, as reported in the literature [15] and [17] and [18].

Practices emerged that helped students to understand the concepts of patterns and area in an experiential way. In this way, the process becomes enjoyable, and knowledge is not passively transmitted to students, but is gradually acquired and discovered by them.

Of course, the way a mathematical concept is taught, although very important, is not the only factor responsible for students' difficulties in the concept of area, as textbooks and teachers' knowledge are a major reason for students' difficulties in mathematics. Therefore, the harmonious combination of these three elements of books, teaching, teacher should be the pillar that will support the effort to reduce students' difficulties in the concept under consideration.

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