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Review Paper

Bianchi Type – Ix Inflationary CosmologicalModels with Flat Potential for Barotropic Fluid Distribution in General Relativity

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ABSTRACT: *In the present study, we have investigated some Bianchi Type-IX inflationary cosmological models with flat potential for barotropic fluid distribution. To get a determinate solution, a supplementary condition $A_1 = A_2^n$, between metric potentials, is used where A_1 and A_2 are function of time alone. We observe that the Hubble parameter H, shear scalar σ^2, energy density ρ, pressure p, the coefficient of bulk viscosity ξ and the expansion θ all diverge at $T = 0$. The spatial volume increases with time, representing an inflationary scenario. The deceleration parameter $q < 0$ for barotropic, models representing an accelerated universe. The physical and geometrical behaviors of the model related with inflationary scenario are also discussed.*

KEYWORDS: *Barotropic fluid, inflationary universe, Bianchi type-IX, cosmology.*

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I. INTRODUCTION

Bianchi type-IX cosmological models are very popular for relativistic studies. These models are also used to examine the role of certain anisotropic sources during the formation of large scale structures as we seen the universe today. Bulk viscous Bianchi Type-IX string dust cosmological model with time dependent Λ term by assuming string tension density is equal to rest energy density and the coefficient of bulk viscosity is inversely proportional to the expansion investigated by Parikh, S., et. al. [21]. Some tilted Bianchi type-IX dust fluid cosmological model is investigated by Bagora, A. [1] and she also discussed a special model in terms of t only. Inflationary solutions for Bianchi type-IX space time in presence of a massless scalar field with a flat potential and some solutions for pure massive string following the Takabayashi equation of state $\rho = (1 + w)\lambda$ solved by Chakraborty, S. [13]. Bali, R. and Dave, S. [3] have investigated some Bianchi type-IX string cosmological models with bulk viscous fluid distribution in general relativity and also they have discussed a particular case for $n = 0$.

Bianchi type-IX cosmological models are interesting because these models allow not only expansion but also rotation and shear, and in general these are anisotropic. Recently, Jat, L. L., et. al. [17] calculated the Bianchi type-IX inflationary cosmological model with flat potential for perfect fluid distribution in general relativity. Nimkar, A. S. and Wath, J. S. [20] obtained stability of Bianchi type-IX cosmological model in the presence of energy momentum tensor for matter and the holographic dark energy in the framework of scalar tensor theories of gravitation by using variation law for Hubble parameter. Sharma, S., et. al. [26] investigated a spatially symmetric Bianchi type-IX line element with flat potential for string cosmology under consideration of hybrid expansion law for the scalar factor and an appropriate relation between the metric coefficients. Ghate, H. R. and Sontakke, A. S. [14] have been obtained the Einstein field equations for Bianchi type-IX cosmological model in the presence of perfect fluid with disordered radiation.

Tyagi, A., et. al. [29] have been studied some homogeneous Bianchi type-IX string cosmologicalmodel with cosmological term Λ in the presence of bulk viscous fluid with electromagnetic field. Bali, R. and Kumawat, P. [7] have been investigated some Bianchi type-IX stiff fluid tilted cosmological models with bulk viscosity, they also investigated the tilted cosmological model in presence of bulk viscosity in terms of cosmic time t for $n = 2$. Patil, V. R. and Bhojne, S. A. [22] have examined the nature of Bianchi type-IX string cosmological model in presence of viscous fluid and electromagnetic field in general relativity by considering magnetic permeability as variable quantity. Bali, R. and Yadav, M. K. [11] calculated the Bianchi type-IX viscous fluid cosmological model which is a closed FRW model having positive curvature and η a θ is used to get a deterministic model and coefficient of bulk viscosity is assumed as constant.

The primordial acceleration in which the universe undergoes rapid exponential expansion is known as inflation. A study of inflationary cosmology provides a better explanation of the formation of galactic structure from the early universe. The inflation field energy is represented by two-component cosmic fluid consisting of a vacuum fluid and a Zel'dovich fluid, there exist models in which the viscous Zel'dovich fluid removes the initial singularity of the corresponding viscosity free models reviewed by Gron, O. [15]. Bali, R. and Jain, V. C. [4] have investigated Bianchi type I inflationary cosmological modelin the presence of massless scalar field with a flat potential. Singh, C. P. and Kumar, S. [28] extend their work to anisotropic and homogeneous Inflationary Bianchi type-II space–time in the presence of masslessscalar field with a flat potential in general relativity and they used the special law of variation of Hubble'sparameter to solve the Einstein's field equations. Inflationary cosmological models in Bianchi Type III Space time with variable bulk viscosity and flat potential by assuming $\zeta\theta = constant$ examined by Bali, R. and Poonia, L. [8].

The inflationary phenomenon has astrophysical significance to understand the cosmos evolution. Bali, R. and Kumari, P. [5] have investigated inflationary cosmological model with flat potential and bulk viscosity taking Bianchi Type VI_0 space-time as a source and they also found that the observations of inflationary cosmology i.e. slow role parameters, third slow parameter and anisotropic parameter are in excellent agreement. Maity, D. and Saha, P. [18] have analysed in detail a class of super-gravity inspired phenomenological inflationary models with non-polynomial potential and also they have performed model independent analysis of reheating in terms of the effective equation of state parameter. Venkateshwarlu, R. and Satish, J. [31] have been considered to an analogue of inflationary model in Brans-Dicke string cosmology for a spatially homogeneous and anisotropic LRS Bianchi type-I cosmological model in Brans-Dicke scalar-tensor theory of gravitation in the presence of cosmic string source. There is a great interest in the inflationary universe scenario since this scenario solves different problems of modern cosmology like homogeneity, the isotropy, flatness of observed universe and primordial monopole problems.

In modern research, Einstein theory becomes a subject of interest due to its attainment in explaining accelerated cosmic expansion of the physical universe. Many cosmological problems like isotropy, homogeneity, monopole and flatness can be successfully explained by inflationary theory. Inflationary cosmological model for synchronized Bianchi space VI in existence of flat potential under scalar fields which is purely mass less and in which $V(\phi)$ is constant derived by Sharma, S., et. al. [25] by assuming the appropriate relation between metric coefficients to find deterministic solutions. Vazquez, J. A., et. al. [30] briefly presented some of the relevant short-comings the inflationary cosmology is dealing with, and a short review is carried out about the scalar fields as a promising solution. The inflationary scenario is not the only paradigm of early universe cosmology which is consistent with current observations, nature at a fundamental level by superstring theory, a cosmology without an initial space-time singularity will emerge and a structure formation scenario which does not include inflation may be realized by Brandenberger, R. H. [12].

For realistic picture of the universe in terms of barotropic fluid distribution with variable bulk viscosity for spatially homogeneous anisotropic Bianchi Type I inflationary scenario is investigated by Bali, R. [2]. Perkovic, D. and Stefanci, H. [23] tested of barotropic fluid dark energy follows directly from the function for the Equation of state parameter, which compatibility is based on the functional form of the speed of sound squared and also found that fundamental requirement eliminates a large number of parameterizations as barotropic fluid dark energy models and puts strong constraints on parameters of other dark energy parameterizations. Bali, R. and Kumawat, P. [6] found barotropic perfect fluid distribution with heat conduction for Bianchi type IX tilted cosmological model in general relativity by assuming a supplementary condition between metric potentials. The barotropic fluid condition in Bianchi type IX cosmological models in the frame work of Lyra geometry is investigated by Bali, R., et. al. [10]. Hernandez, H., et. al. [16] sketched an algorithm starting from a barotropic EoS and setting an ansatz on the metric functions to generate exact anisotropic solutions.

A barotropic fluid is a flow in which the pressure is a function of the density only and in cosmology we often make the assumption $p = \gamma\rho$. Bali, R. and Singh, S. [9] have been examined barotropic fluid distribution with variable bulk viscosity and decaying vacuum energy density for an inflationary scenario in Bianchi type V space-time and they also discussed the importance of Bianchi type V model where the anisotropy dies away during the inflationary era. The barotropic perfect fluid condition for tilted Bianchi type I cosmological model is formulated by Mandowara, S., et. al. [19] and they have also assumed that the Universe is field with barotropic perfect fluid $p = \gamma\varepsilon$, where γ is a constant. Barotropic massive string universe with decaying vacuum energy density for magnetized Bianchi type VI0 calculated by Pradhan, A. and Bali, R. [24]. Singh, A. et. al. [27] have been investigated a barotropic fluid condition for the behaviour of cosmologies in homogeneous and isotropic background and they also studied the continuity equation.

In this paper, we have investigate some Bianchi Type-IX inflationary cosmological models with flat potential for barotropic fluid distribution. To get a determinate solution, a supplementary condition $A_1 = A_2^n$, between metric potentials, is used where A_1 and A_2 are function of time alone. We observe that the Hubble parameter H, shear scalar σ^2, energy density ρ, pressure p, the coefficient of bulk viscosity ξ and the expansion θ all diverge at $T = 0$. The spatial volume increases with time, representing an inflationary scenario. The deceleration parameter q < 0 for barotropic, models representing an accelerated universe. The physical and geometrical behaviors of the model related with inflationary scenario are also discussed.

II. THE METRIC AND FIELD EQUATIONS

We consider Bianchi type-IX line element in form

$$ds^2 = -dt^2 + A_1^2dx^2 + A_2^2dy^2 + [A_2^2\sin^2 y + A_1^2\cos^2 y]dz^2 - 2A_1^2 \cos y\, dxdz \qquad (1)$$

in which $A_1(t)$ and $A_2(t)$ are cosmic scale functions.

The Lagrangian is that of gravity minimally coupled to Higgs scalar field (ϕ) with effective potential $V(\phi)$ given by

$$L = \int \sqrt{-g}\,[R - \frac{1}{2}g^{ij}\partial_{ij}\phi\partial_{ij}\phi - V(\phi)]d^4x \qquad (2)$$

The energy momentum tensor (T_{ij}) for scalar field in presence of viscosity is given by

$$T_{ij} = (\rho + p)v_i v_j - pg_{ij} + \partial_i\phi\partial_j\phi - [\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)]g_{ij} + \xi\theta(g_{ij} + v_i v_j) \qquad (3)$$

where V is the effective potential, ϕ is Higgs field, ξ is the coefficient of bulk viscosity and θ is the expansion in the model.

Also v^i, the unit time like vector satisfying the following condition:

$$v_i v^i = -1$$

The co-moving coordinate system is chosen as

$$v^1 = 0 = v^2 = v^3,\ v^4 = 1$$

The Einstein's field equations (in the gravitational unit $8\pi G = c = 1$) in case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \qquad (4)$$

The energy conservation law coincides with the equation of motion for ϕ and we have

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) = -\frac{dV}{d\phi} \qquad (5)$$

where scalar field ϕ is the function of t-alone.

The Einstein field equation (3) for the metric (1) and energy momentum tensor (4) leads to the following system of equations

$$2\ddot{A_2}/A_2 + \dot{A_2}^2/A_2^2 + 1/A_2^2 - 3A_1^2/4A_2^4 = -\dot{\phi}^2/2 + V(\phi) - (p - \xi\theta) \qquad (6)$$

$$\ddot{A_1}/A_1 + \ddot{A_2}/A_2 + \dot{A_1}\dot{A_2}/A_1A_2 + A_1^2/4A_2^4 = -\dot{\phi}^2/2 + V(\phi) - (p - \xi\theta) \qquad (7)$$

$$2\dot{A_1}\dot{A_2}/A_1A_2 + \dot{A_2}^2/A_2^2 + 1/A_2^2 - A_1^2/4A_2^4 = \rho + \dot{\phi}^2/2 + V(\phi) \qquad (8)$$

The equation (5) for scalar field (ϕ) leads to

$$\ddot{\phi} + [\dot{A_1}/A_1 + 2\dot{A_2}/A_2]\dot{\phi} = -dV/d\phi \qquad (9)$$

where the $(\cdot)$ indicates derivative with respect to time t.

III. SOLUTION OF FIELD EQUATIONS

We are interested in inflationary solution so flat region is considered. Thus we have $V(\phi)$ is constant.

i.e. $V(\phi) = K$ (10)

From equations (9) and (10), we get

$$\ddot{\phi} + [\dot{A_1}/A_1 + 2\dot{A_2}/A_2]\dot{\phi} = 0 \qquad (11)$$

above equation leads to

$$\dot{\phi} = l/A_1A_2^2 \qquad (12)$$

where l is constant of integration.

The average scale factor (R) for line element (1) is given by

$$R^3 = A_1A_2^2 \qquad (13)$$

By equation (7) and (8), we have

$$\ddot{A_1}/A_1 + \ddot{A_2}/A_2 + \dot{A_2}^2/A_2^2 + 3\dot{A_1}\dot{A_2}/A_1A_2 + 1/A_1^2 = \rho + 2k - (p - \xi\theta) \qquad (14)$$

The above equation (14) has $A_1, A_2, \phi, V, p, \rho, \xi$ and θ unknown parameters. To obtain the deterministic solution, we assume the following conditions:

(i) Barotropic fluid condition

$$p = \gamma\rho; 0 \le \gamma \le 1$$
$$\theta = 3H, \qquad \rho = 3H^2, \qquad \xi = \rho^{1/2} \qquad (15)$$

(ii) Shear (σ) is proportional to the expansion (θ), which leads to

$$A_1 = {A_2}^n \tag{16}$$

Now equations (14), (15) and (16) together lead to

$$\dot{A_2} + [(n+1) - (\frac{1-\gamma}{3} + \frac{1}{\sqrt{3}})\frac{(n+2)^2}{(n+1)}] {A_2}^2/A_2 = -{A_2}^{-1}/(n+1) + 2kA_2/(n+1) \tag{17}$$

which leads to

$$2\dot{A}_2 + (a/A_2){A_2}^2 = b_1 A_2^{-1} + b_2 A_2 \tag{18}$$

where

$$a = 2[(n+1) - (\frac{1-\gamma}{3} + \frac{1}{\sqrt{3}})\frac{(n+2)^2}{(n+1)}], \quad b_1 = \frac{-2}{(n+1)} \quad \text{and} \quad b_2 = \frac{4k}{(n+1)} \tag{19}$$

To get solution, we assume that $\dot{A_2} = f$. Thus equation (18) leads to

$$df^2/dA_2 + (a/A_2) f^2 = b_1 A_2^{-1} + b_2 A_2 \tag{20}$$

which on integration equation (20) leads to

$$f^2 = b_1/a + b_2 {A_2}^2/(a+2) + b_3 {A_2}^{-a} \tag{21}$$

where b_3 is constant of integration.

$$f = \dot{A_2} = dA_2/dt = [b_1/a + b_2 A_2^2/(a+2) + b_3 {A_2}^{-a}]^{1/2} \tag{22}$$

After suitable transformation of co-ordinates, the metric (1) leads to the form

$$ds^2 = -dT^2/[a_1 + a_2T^2 + b_3T^{-a}] + T^{2n}dX^2 + T^2dY^2 + [T^2sin^2Y + T^{2n}cos^2Y]dZ^2$$
$$-2T^{2n}\cos Y\, dXdZ \tag{23}$$

where $a_1 = \frac{b_1}{a}$, $a_2 = \frac{b_2}{(a+2)}$, $x = X,\ y = Y,\ z = Z$ and $A_2 = T$.

IV. PHYSICAL AND GEOMETRICAL ASPECTS

The Higgs field (ϕ), the pressure (p), Energy density (ρ), the spatial volume (R^3), the expansion (θ), shear (σ), the directional Hubble parameter (H_x, H_y, H_z), Hubble parameter (H), the anisotropic parameter (Δ), and the deceleration parameter (q) for the model (23) are given by

$$\phi = l \int dT/T^{+2}[a_1 + a_2T^2 + b_3T^{-a}]^{1/2} + L \tag{24}$$

where L is constant of integration.

$$R^3 = {A_2}^{n+2} = T^{n+2} \tag{25}$$

$$H_x = \dot{A_1}/A_1 = n\dot{A_2}/A_2$$

$$H_x = n[a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}]^{1/2}$$

and $H_z = H_y = \dot{A_2}/A_2$

$$H_z = H_y = [a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}]^{1/2}$$

The average Hubble parameter (H) is found to be

$$H = \left(\frac{n+2}{3}\right)[a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}]^{1/2} \tag{26}$$

$$\theta = 3H = (n+2)[a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}]^{1/2} \tag{27}$$

$$q = -1 + \frac{3[2a_1 T^{-2} + (a+2) b_3 T^{-(a+2)}]}{2(n+2)[a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}]} \tag{28}$$

$$\Delta = \frac{2(n-1)^2}{(n+2)^2} \tag{29}$$

$$\sigma = \left[\frac{(n-1)}{\sqrt{3}}\right][a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}]^{1/2} \tag{30}$$

$$\sigma/\theta = \frac{(n-1)}{\sqrt{3}(n+2)} \neq 0 \ (constant)$$

The energy density ρ is given by Barotropic fluid condition equation (15)

$$\rho = \left[\frac{(n+2)^2}{3}\right][a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}] \tag{31}$$

The bulk coefficient ξ is given by equation (15)

$$\xi = \rho^{1/2} = \left[\frac{(n+2)}{\sqrt{3}}\right][a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}]^{1/2} \tag{33}$$

Now by equation (32) and (33) $(p - \xi\theta)$ is

$$(p - \xi\theta) = \left(\gamma - \frac{1}{3}\right)(n+2)^2[a_1 T^{-2} + a_2 + b_3 T^{-(a+2)}] \tag{34}$$

V. CONCLUSION

We found that, at initial epoch the model starts evolving with zero volume and expands for large time for $n > -2$. It represents inflationary scenario of universe in Bianchi type - IX cosmological models with flat potential for barotropic fluid distribution in general relativity.

Initially when $T = 0$, the Hubble's parameter H, expansion scalar θ, shear scalar σ^2, energy density ρ and pressure p diverge. The rate of expansion (θ) decreases with increase of time. We also observe that it approaches to zero as $T \to \infty$ and stops when $n = -2$.

The rate of Higgs field (ϕ) is initially large, but decreases as time increases for $n > -2$ and constant for $T \to \infty$. The model has point type singularity at $T = 0$.

Since, $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ indicating that the model does not approach isotropy for large time. It is also observed that, $\Delta \neq 0$ which indicates the model is anisotropic and represents the early stages of the universe.

When $T \to \infty$ then deceleration parameter tends to -1, so the model represent accelerating phase of universe.

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