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Review Paper

Quotient-4 Cordial Labeling Of Generalized Jahangir Graphs

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ABSTRACT: *Let G (V, E) be a simple graph of order p and size q. Let* $\varphi : V(G) \rightarrow Z_5 - \{0\}$ *be a function. For each edge set E (G) define the labeling* $\varphi^*: E(G) \rightarrow Z_4$ *by* $\varphi^*(uv) = \left\lceil \frac{\varphi(u)}{\varphi(v)} \right\rceil$ *(mod 4) where* $\varphi(u) \geq \varphi(v)$*. The function* φ *is called quotient-4 cordial labeling of G if* $|v_\varphi(i) - v_\varphi(j)| \leq 1$, $1 \leq i, j \leq 4$, $i \neq j$ *where* $v_\varphi(x)$ *denote the number of vertices labeled with x and* $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3$, $k \neq l$, *where* $e_\varphi(y)$ *denote the number of edges labeled with y. Here the Jahangir graph* $J_{n,m}$ *proved to be quotient-4 cordial graphs.*
KEYWORDS: *Jahangir Graph, Quotient-4 Cordial Labeling, Quotient-4 Cordial Graph.*

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I. INTRODUCTION

Here the graphs considered are finite, simple, undirected and non-trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [6] for more information. The cordial labeling concept was first introduced by Cahit [3]. H- andH2–cordial labeling was introduced by Freeda S and Chellathurai R.S [4]. Mean Cordial Labeling was introduced by Albert William, Indira Rajasingh, and S Roy [1]. A graph G is said to be quotient-4 cordial graph if it receives quotient-4 cordial labeling.Let $v_\varphi(i)$ denotes the number of vertices labeled with i and $e_\varphi(k)$ denotes the number of edges labeled with k, $1 \leq i \leq 4, 0 \leq k \leq 3$.

II. DEFINITIONS

Definition: 2.1[7] Let G (V, E) be a simple graph of order p and size q. Let φ: V (G) → $Z_5 - \{0\}$ be a function. For each edge set E (G) define the labeling φ^*: E (G) → Z_4 by $\varphi^*(uv) = \left\lceil \frac{\varphi(u)}{\varphi(v)} \right\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1$, $1 \leq i, j \leq 4$, i≠j where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3, k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y.

Definition: 2.2[2] Jahangir graphs $J_{n,m}$ for $n \geq 1, m \geq 3$, is a graph on $nm + 1$vertices and $m(n+1)$ edges consisting of a cycle C_{nm} with an additional central vertex say w which is adjacent to m vertices of C_{nm}at distance n to each other on C_{nm} .

III.MAIN RESULT

Theorem: 3.1 A graph $J_{n,m}$ is quotient-4 cordial if $n \geq 1, m \geq 3$.
Proof: Let G be a $J_{n,m}$ graph.
V (G) = $\{w, w_1, w_2 \ldots w_{nm}\}$.
E (G) = $\{w_i w_{i+1} : 1 \leq i \leq nm-1\} \cup \{w_1 w_{nm}\} \cup \{w w_{ni} : 1 \leq i \leq m\}$.
Here $|V(G)| = nm+1, |E(G)| = m(n+1)$.
Define φ :V (G) → {1, 2, 3, 4}.

The value of wis labeled as follows.
φ (w) = 1.
The values of $w_i$are labeled as follows.
Case 1: When $n \equiv 0$ (modulo 8).
Subcase 1.1: When $m \equiv 0,1,2,3,6,7$ (modulo 8).
For $nj + 1 \leq i \leq n\,(j + 1)$, $0 \leq j < m$.
φ (w_i) = 1 if i≡ 0, 3 (modulo 8) and j ≡ 0, 7 (modulo 8).
φ (w_i) = 1 if i≡ 2, 7 (modulo 8) and j ≡ 1, 3, 4, 6 (modulo 8).
φ (w_i) = 1 if i≡ 2, 5 (modulo 8) and j ≡ 2, 5 (modulo 8).
φ (w_i) = 2 if i≡ 5, 6 (modulo 8) and j ≡ 0, 7 (modulo 8).
φ (w_i) = 2 if i≡ 4, 5 (modulo 8) and j ≡ 1, 3, 4, 6 (modulo 8).
φ (w_i) = 2 if i≡ 7, 8 (modulo 8) and j ≡ 2, 5 (modulo 8).
φ (w_i) = 3 if i≡4, 7 (modulo 8) and j ≡ 0, 7 (modulo 8).
φ (w_i) = 3 if i≡ 3, 6 (modulo 8) and j ≡ 1, 6 (modulo 8).
φ (w_i) = 3 if i≡ 3, 4 (modulo 8) and j ≡ 2 (modulo 8).
φ (w_i) = 3 if i≡ 0, 1 (modulo 8) and j ≡ 3, 4 (modulo 8).
φ (w_i) = 3 if i≡ 1, 6 (modulo 8) and j ≡ 5 (modulo 8).
φ (w_i) = 4 if i≡ 1, 2 (modulo 8) and j ≡ 0, 7 (modulo 8).
φ (w_i) = 4 if i≡ 0, 1 (modulo 8) and j ≡ 1, 6 (modulo 8).
φ (w_i) = 4 if i≡ 1, 6 (modulo 8) and j ≡ 2 (modulo 8).
φ (w_i) = 4 if i≡ 3, 6 (modulo 8) and j ≡ 3, 4 (modulo 8).
φ (w_i) = 4 if i≡ 3, 4 (modulo 8) and j ≡ 5 (modulo 8).
Subcase 1.2: When $m \equiv 4, 5$ (modulo 8).
For $5 \leq i \leq nm$, the labeling of $w_i$values are same as subcase 1.1.
φ (w_1) = 3, φ (w_2) = 1, φ (w_3) = φ (w_4) = 4.
Case 2: When $n \equiv 1$ (modulo 8).
Subcase 2.1: When $m \equiv 0,1$ (modulo 8).
For $1 \leq i \leq nm$.
φ (w_i) = 1 if i≡ 2, 7 (modulo 8).
φ (w_i) = 2 if i≡4, 5 (modulo 8).
φ (w_i) = 3 if i≡ 0, 1 (modulo 8).
φ (w_i) = 4 if i≡3, 6 (modulo 8).
Subcase 2.2: When $m \equiv 2$ (modulo 8).
For $1 \leq i \leq nm - 5$, the labeling of $w_i$values are same as subcase 2.1.
φ (w_{nm}) = φ (w_{nm-1}) = 4, φ (w_{nm-2}) =1, φ (w_{nm-3}) = φ (w_{nm-4}) =3.
Subcase 2.3: When $m \equiv 3$ (modulo 8),$m > 3$if $n = 1$.
For $2 \leq i \leq nm - 2$.
φ (w_i) = 1 if i≡ 3, 6 (modulo 8).
φ (w_i) = 2 if i≡ 0, 1 (modulo 8).
φ (w_i) = 3 if i≡ 2, 7 (modulo 8).
φ (w_i) = 4 if i≡ 4, 5 (modulo 8).
φ (w_1) = 3, φ (w_{nm}) = 4, φ (w_{nm-1}) = 2.
Subcase 2.4: When $m \equiv 4$ (modulo 8), $m \neq 4$if $n = 1$.
For $2 \leq i \leq nm - 3$, the labeling of $w_i$values are same as subcase 2.3.
φ (w_1) = 1, φ (w_{nm}) = 4, φ (w_{nm-1}) = 3, φ (w_{nm-2}) = 2.
Subcase 2.5: When $m \equiv 5$ (modulo 8).
For $1 \leq i \leq nm - 1$, the labeling of $w_i$values are same as subcase 2.1.
φ (w_{nm}) = 3.
Subcase 2.6: When $m \equiv 6$ (modulo 8).
For $2 \leq i \leq nm - 5$, the labeling of $w_i$values are same as subcase 2.3.
φ (w_1) = 2, φ (w_{nm}) = φ (w_{nm-1}) = 3, φ (w_{nm-2}) =1, φ (w_{nm-3}) = φ (w_{nm-4}) =4.
Subcase 2.7: When $m \equiv 7$ (modulo 8).
For $1 \leq i \leq nm - 4$, the labeling of $w_i$values are same as subcase 2.1.
φ (w_{nm}) = 3, φ (w_{nm-1}) = φ (w_{nm-2}) = 2, φ (w_{nm-3}) =4.
Case 3: When $n \equiv 2$ (modulo 8).
Subcase 3.1: When $m \equiv 0,3,4,7$ (modulo 8).
For $1 \leq i \leq nm$.
φ (w_i) = 1 if i≡ 0, 3 (modulo 8).

φ (w_i) = 2 if i≡ 5, 6 (modulo 8).
φ (w_i) = 3 if i≡ 1, 2 (modulo 8).
φ (w_i) = 4 if i≡ 4, 7 (modulo 8).
Subcase 3.2: When $m \equiv 1{,}5$ (modulo 8).
For $1 \leq i \leq nm - 2$, the labeling of $w_i$values are same as subcase 3.1.
φ (w_{nm}) = 3, φ (w_{nm-1}) = 4.
Subcase 3.3: When $m \equiv 2$ (modulo 8).
For $1 \leq i \leq nm - 4$.
φ (w_i) = 1 if i≡ 2, 5 (modulo 8).
φ (w_i) = 2 if i≡ 0, 7 (modulo 8).
φ (w_i) = 3 if i≡ 3, 4 (modulo 8).
φ (w_i) = 4 if i≡ 1, 6 (modulo 8).
φ (w_{nm}) = 4, φ (w_{nm-1}) = 2, φ (w_{nm-2}) = 1, φ (w_{nm-3}) =3.
Subcase 3.4: When $m \equiv 6$ (modulo 8).
For $1 \leq i \leq nm - 4$, the labeling of $w_i$values are same as subcase 3.1.
φ (w_{nm}) = 3, φ (w_{nm-1}) = 1, φ (w_{nm-2}) = 4, φ (w_{nm-3}) =2.
Case 4: When $n \equiv 3$ (modulo 8).
Subcase 4.1: When $m \equiv 0{,}2{,}3$ (modulo 8).
For $1 \leq i \leq nm$.
φ (w_i) = 1 if i≡4, 7 (modulo 8).
φ (w_i) = 2 if i≡1, 2 (modulo 8).
φ (w_i) = 3 if i≡0, 3 (modulo 8).
φ (w_i) = 4 if i≡5, 6 (modulo 8).
Subcase 4.2: When $m \equiv 1$ (modulo 8).
For $2 \leq i \leq nm - 2$and $i \neq nm - n, nm - (n + 1)$.
φ (w_i) = 1 if i≡ 0, 3 (modulo 8).
φ (w_i) = 2 if i≡ 5, 6(modulo 8).
φ (w_i) = 3 if i≡ 4, 7 (modulo 8).
φ (w_i) = 4 if i≡ 1, 2 (modulo 8).
φ (w_1) = 1, φ (w_{nm}) = φ (w_{nm-1}) = 3, φ (w_{nm-n}) = 4, φ ($w_{nm-(n+1)}$) = 2.
Subcase 4.3: When $m \equiv 4$ (modulo 8).
For $3 \leq i \leq nm - 3$.
φ (w_i) = 1 if i≡ 2, 7 (modulo 8).
φ (w_i) = 2 if i≡ 4, 5 (modulo 8).
φ (w_i) = 3 if i≡ 3, 6 (modulo 8).
φ (w_i) = 4 if i≡ 0, 1 (modulo 8).
φ (w_1) = 1, φ (w_2) = 3, φ (w_{nm}) = φ (w_{nm-2}) = 4, φ (w_{nm-1}) = 2.
Subcase 4.4: When $m \equiv 5$ (modulo 8).
For $1 \leq i \leq nm - 6$
φ (w_i) = 1 if i≡ 2, 5 (modulo 8).
φ (w_i) = 2 if i≡ 0, 7 (modulo 8).
φ (w_i) = 3 if i≡ 3, 4 (modulo 8).
φ (w_i) = 4 if i≡ 1, 6 (modulo 8).
φ (w_{nm}) = φ (w_{nm-1}) = 2, φ (w_{nm-2}) = φ (w_{nm-3}) = 3, φ (w_{nm-4}) = 1, φ (w_{nm-5}) = 4.
Subcase 4.5: When $m \equiv 6$ (modulo 8).
For $5 \leq i \leq nm$, the labeling of $w_i$values are same as subcase 3.1.
φ (w_1) = φ (w_2) = 4, φ (w_3) = 1, φ (w_4) = 3.
Subcase 4.6: When $m \equiv 7$ (modulo 8).
For $3 \leq i \leq nm - 3$, the labeling of $w_i$values are same as subcase 4.3.
φ (w_1) = 4, φ (w_2) = 1, φ (w_{nm}) = 4, φ (w_{nm-1}) = 2, φ (w_{nm-2}) = 3.
Case 5: When $n \equiv 4$ (modulo 8).
Subcase 5.1: When $m \equiv 0{,}2{,}4{,}6$ (modulo 8).
For $1 \leq i \leq nm$.
φ (w_i) = 1 if i≡1, 6, 9, 12 (modulo 16).
φ (w_i) = 2 if i≡3, 4, 14, 15 (modulo 16).
φ (w_i) = 3 if i≡7, 8, 10, 11 (modulo 16).
φ (w_i) = 4 if i≡0, 2, 5, 13 (modulo 16).
Subcase 5.2: When $m \equiv 1$ (modulo 8).

For $3 \leq i \leq nm - 3$,the labeling of $w_i$values are same as subcase 5.1.
φ (w_1) = φ (w_{nm}) = 4, φ (w_2) = 1, φ (w_{nm-1}) = 2, φ (w_{nm-2}) = 3.
Subcase 5.3: When $m \equiv 3$ (modulo 8) and $n \equiv 4$ (modulo 16),
$m \equiv 5$ (modulo 8) and $n \equiv 12$ (modulo 16),
$m \equiv 7$ (modulo 8) and $n \equiv 4$ (modulo 16).
For $3 \leq i \leq nm - 2$, the labeling of $w_i$values are same as subcase 5.1.
φ (w_1) = φ (w_{nm-1}) = 4, φ (w_2) = 2, φ (w_{nm}) = 1.
Subcase 5.4: When $m \equiv 3$ (modulo 8) and $n \equiv 12$ (modulo 16).
For $1 \leq i \leq nm - 5$, the labeling of $w_i$values are same as subcase 5.1.
φ (w_{nm}) = φ (w_{nm-1}) = 4, φ (w_{nm-2}) = 2, φ (w_{nm-3}) = 3, φ (w_{nm-4}) = 1.
Subcase 5.5: When $m \equiv 5$ (modulo 8) and $n \equiv 4$ (modulo 16).
For $3 \leq i \leq nm - 2$, the labeling of $w_i$values are same as subcase 5.1.
φ (w_1) = 4, φ (w_2) = 2, φ (w_{nm}) = 1 φ (w_{nm-1}) = 3.
Subcase 5.6: When $m \equiv 7$ (modulo 8) and $n \equiv 12$ (modulo 16).
For $1 \leq i \leq nm - 5$, the labeling of w_i values are same as subcase 5.1.
φ (w_{nm}) = φ (w_{nm-1}) = 4, φ (w_{nm-2}) = 2, φ (w_{nm-3}) = 1, φ (w_{nm-4}) = 3.
Case 6: When $n \equiv 5$ (modulo 8).
Subcase 6.1: When $m \equiv 0$ (modulo 8).
For $1 \leq i \leq nm$.
φ (w_i) = 1 if i≡ 1, 4 (modulo 8).
φ (w_i) = 2 if i≡ 6, 7 (modulo 8).
φ (w_i) = 3 if i≡ 0, 5 (modulo 8).
φ (w_i) = 4 if i≡ 2, 3 (modulo 8).
Subcase 6.2: When $m \equiv 1$ (modulo 8).
For $1 \leq i \leq nm - 4$, the labeling of w_i values are same as subcase 6.1.
φ (w_{nm}) = 4,φ (w_{nm-1}) = φ (w_{nm-2}) = 2, φ (w_{nm-3}) = 3.
Subcase 6.3: When $m \equiv 2$ (modulo 8).
For $3 \leq i \leq nm - 3$, the labeling of w_i values are same as subcase 6.1.
φ (w_1) = 4, φ (w_2) = 1, φ (w_{nm}) = φ (w_{nm-1}) = 3, φ (w_{nm-2}) = 2.
Subcase 6.4: When $m \equiv 3$ (modulo 8).
For $1 \leq i \leq nm - 4$, the labeling of w_i values are same as subcase 6.1.
φ (w_{nm}) = φ (w_{nm-3}) = 3, φ (w_{nm-1}) = φ (w_{nm-2}) = 2.
Subcase 6.5: When $m \equiv 4$ (modulo 8).
For $1 \leq i \leq nm - 5$, the labeling of w_i values are same as subcase 6.1.
φ (w_{nm}) = φ (w_{nm-1}) = 3, φ (w_{nm-2}) = 4, φ (w_{nm-3}) = 1, φ (w_{nm-4}) = 2.
Subcase 6.6: When $m \equiv 5$ (modulo 8).
For $1 \leq i \leq nm - 2$, the labeling of w_i values are same as subcase 6.1.
φ (w_{nm}) = 3, φ (w_{nm-1}) = 4.
Subcase 6.7: When $m \equiv 6$ (modulo 8).
For $2 \leq i \leq nm$.
φ (w_i) = 1 if i≡ 0, 5 (modulo 8).
φ (w_i) = 2 if i≡ 2, 3 (modulo 8).
φ (w_i) = 3 if i≡ 1, 4 (modulo 8).
φ (w_i) = 4 if i≡ 6, 7 (modulo 8).
φ (w_1) = 4.
Subcase 6.8: When $m \equiv 7$ (modulo 8).
For $2 \leq i \leq nm - 3$, the labeling of w_i values are same as subcase 6.7.
φ (w_1) = φ (w_{nm}) = 3, φ (w_{nm-1}) = 2, φ (w_{nm-2}) = 4.
Case 7: When $n \equiv 6$ (modulo 8).
Subcase 7.1: When $m \equiv 0{,}4$ (modulo 8).
For $1 \leq i \leq nm$, the labeling of w_i values are same as subcase 6.1.
Subcase 7.2: When $m \equiv 1{,}5$ (modulo 8).
For $1 \leq i \leq nm - 3$, the labeling of w_i values are same as subcase 6.1. φ (w_{nm}) = 3, φ (w_{nm-1}) = φ (w_{nm-2}) = 2.
Subcase 7.3: When $m \equiv 2$ (modulo 8).
For $1 \leq i \leq nm - 5$, the labeling of w_i values are same as subcase 6.1.
φ (w_{nm}) = φ (w_{nm-3}) = 3, φ (w_{nm-1}) = 4, φ (w_{nm-2}) = 1, φ (w_{nm-4}) = 2.
Subcase 7.4: When $m \equiv 3, 7$ (modulo 8).

For $2 \leq i \leq nm - 2$, the labeling of w_i values are same as subcase 6.7.
$\varphi(w_1) = 3$, $\varphi(w_{nm}) = 3$, $\varphi(w_{nm-1}) = 4$.
Subcase 7.5: When $m \equiv 6$ (modulo 8).
For $2 \leq i \leq nm - 4$, the labeling of w_i values are same as subcase 6.7.
$\varphi(w_1) = 3$, $\varphi(w_{nm}) = 3$, $\varphi(w_{nm-1}) = 1$, $\varphi(w_{nm-2}) = 2$, $\varphi(w_{nm-3}) = 4$.
Case 8: When $n \equiv 7$ (modulo 8).
Subcase 8.1: When $m \equiv 0$ (modulo 8).
For $1 \leq i \leq nm$.
$\varphi(w_i) = 1$ if $i \equiv 1, 6$ (modulo 8).
$\varphi(w_i) = 2$ if $i \equiv 3, 4$ (modulo 8).
$\varphi(w_i) = 3$ if $i \equiv 2, 5$ (modulo 8).
$\varphi(w_i) = 4$ if $i \equiv 0, 7$ (modulo 8).
Subcase 8.2: When $m \equiv 1$ (modulo 8).
For $1 \leq i \leq nm - 4$ and $i \neq nm - n$.
$\varphi(w_i) = 1$ if $i \equiv 2, 5$ (modulo 8).
$\varphi(w_i) = 2$ if $i \equiv 0, 7$ (modulo 8).
$\varphi(w_i) = 3$ if $i \equiv 1, 6$ (modulo 8).
$\varphi(w_i) = 4$ if $i \equiv 3, 4$ (modulo 8).
$\varphi(w_{nm}) = 4$, $\varphi(w_{nm-1}) = \varphi(w_{nm-2}) = \varphi(w_{nm-3}) = 2$, $\varphi(w_{nm-n}) = 3$.
Subcase 8.3: When $m \equiv 2$ (modulo 8).
For $1 \leq i \leq nm - 5$, the labeling of w_i values are same as subcase 8.1.
$\varphi(w_{nm}) = \varphi(w_{nm-1}) = 3$, $\varphi(w_{nm-2}) = \varphi(w_{nm-3}) = 2$, $\varphi(w_{nm-4}) = 4$.
Subcase 8.4: When $m \equiv 3$ (modulo 8).
For $1 \leq i \leq nm - 4$, the labeling of w_i values are same as subcase 8.1.
$\varphi(w_{nm}) = 3$, $\varphi(w_{nm-1}) = \varphi(w_{nm-2}) = 2$, $\varphi(w_{nm-3}) = 4$.
Subcase 8.5: When $m \equiv 4$ (modulo 8).
For $1 \leq i \leq nm - 6$, the labeling of w_i values are same as subcase 8.1.
$\varphi(w_{nm}) = \varphi(w_{nm-1}) = \varphi(w_{nm-2}) = 4$, $\varphi(w_{nm-3}) = 1$, $\varphi(w_{nm-4}) = 2$, $\varphi(w_{nm-5}) = 3$.
Subcase 8.6: When $m \equiv 5$ (modulo 8).
For $1 \leq i \leq nm - 4$, the labeling of w_i values are same as subcase 7.4.
$\varphi(w_{nm}) = 3$, $\varphi(w_{nm-1}) = 1$, $\varphi(w_{nm-2}) = 4$, $\varphi(w_{nm-3}) = 2$.
Subcase 8.7: When $m \equiv 6$ (modulo 8).
For $1 \leq i \leq nm - 2$.
$\varphi(w_i) = 1$ if $i \equiv 2, 5$ (modulo 8).
$\varphi(w_i) = 2$ if $i \equiv 0, 7$ (modulo 8).
$\varphi(w_i) = 3$ if $i \equiv 1, 6$ (modulo 8).
$\varphi(w_i) = 4$ if $i \equiv 3, 4$ (modulo 8).
$\varphi(w_{nm}) = 3$, $\varphi(w_{nm-1}) = 4$.
Subcase 8.8: When $m \equiv 7$ (modulo 8).
For $1 \leq i \leq nm - 1$, the labeling of w_i values are same as subcase 8.1.
$\varphi(w_{nm}) = 4$.

The following table concurrence is realized with modulo value 8.

Nature of n and m	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0,4$ $m \equiv 0,1,2,3,4,5,6,7$	$\frac{nm}{4}+1$	$\frac{nm}{4}$	$\frac{nm}{4}$	$\frac{nm}{4}$
$n \equiv 1,5,7$ $m \equiv 0,4$	$\frac{nm}{4}+1$	$\frac{nm}{4}$	$\frac{nm}{4}$	$\frac{nm}{4}$
$n \equiv 1$ $m \equiv 1,5$	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}$	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}$
$n \equiv 1$ $m \equiv 2,6$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$
$n \equiv 1,5$ $m \equiv 3,7$	$\frac{nm+1}{4}$	$\frac{nm+1}{4}$	$\frac{nm+1}{4}$	$\frac{nm+1}{4}$

n ≡ 2,6 m ≡ 0,2,4,6	$\frac{nm}{4}+1$	$\frac{nm}{4}$	$\frac{nm}{4}$	$\frac{nm}{4}$
n ≡ 2 m ≡ 1,5	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$
n ≡ 2 m ≡ 3,7	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$
n ≡ 3 m ≡ 0	$\frac{nm}{4}+1$	$\frac{nm}{4}$	$\frac{nm}{4}$	$\frac{nm}{4}$
n ≡ 3,7 m ≡ 1,5	$\frac{nm+1}{4}$	$\frac{nm+1}{4}$	$\frac{nm+1}{4}$	$\frac{nm+1}{4}$
n ≡ 3 m ≡ 2	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$	$\frac{nm+2}{4}$
n ≡ 3,7 m ≡ 3	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}$	$\frac{nm-1}{4}$
n ≡ 3 m ≡ 4	$\frac{nm}{4}$	$\frac{nm}{4}$	$\frac{nm}{4}$	$\frac{nm}{4}+1$
n ≡ 3,7 m ≡ 6	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$
n ≡ 3,7 m ≡ 7	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}$	$\frac{nm-1}{4}$	$\frac{nm-1}{4}+1$
n ≡ 5 m ≡ 1	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}$	$\frac{nm-1}{4}$
n ≡ 5,7 m ≡ 2	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$
n ≡ 5 m ≡ 5	$\frac{nm-1}{4}+1$	$\frac{nm-1}{4}$	$\frac{nm-1}{4}$	$\frac{nm-1}{4}+1$
n ≡ 5 m ≡ 6	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$	$\frac{nm+2}{4}$
n ≡ 6 m ≡ 1,5	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$	$\frac{nm+2}{4}$
n ≡ 6 m ≡ 3,7	$\frac{nm+2}{4}$	$\frac{nm+2}{4}-1$	$\frac{nm+2}{4}$	$\frac{nm+2}{4}$

Table 1: Vertex labeling of $J_{n,m}$ graph.

The following table concurrence is realized with modulo value 8.

Nature of n and m	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
n ≡ 0,2,4,6 m ≡ 0,4	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$
n ≡ 0 m ≡ 1	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}+1$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$
n ≡ 0 m ≡ 2	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}-1$	$\frac{m(n+1)+2}{4}-1$
n ≡ 0 m ≡ 3	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}-1$
n ≡ 0 m ≡ 5	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}+1$
n ≡ 0 m ≡ 6	$\frac{m(n+1)+2}{4}-1$	$\frac{m(n+1)+2}{4}-1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$

$n \equiv 0$ $m \equiv 7$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4} - 1$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$
$n \equiv 1,5$ $m \equiv 0,2,4,6$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$
$n \equiv 1$ $m \equiv 1$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$
$n \equiv 1$ $m \equiv 3,7$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$
$n \equiv 1$ $m \equiv 5$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$
$n \equiv 2$ $m \equiv 1,5$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4} - 1$
$n \equiv 2$ $m \equiv 2,6$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$
$n \equiv 2$ $m \equiv 3$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4} + 1$
$n \equiv 2$ $m \equiv 7$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4} + 1$	$\frac{m(n+1)-1}{4}$
$n \equiv 3,7$ $m \equiv 0,1,2,3,4,5,6,7$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$	$\frac{m(n+1)}{4}$
$n \equiv 4$ $m \equiv 1$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4} + 1$	$\frac{m(n+1)-1}{4}$
$\frac{n}{2} \equiv 2$ $m \equiv 2,6$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$
$\frac{n}{2} \equiv 6$ $m \equiv 2,6$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$
$n \equiv 4$ $m \equiv 3,7$	$\frac{m(n+1)+1}{4} - 1$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$
$n \equiv 4$ $m \equiv 5$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4} + 1$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$
$n \equiv 5$ $m \equiv 1,7$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$
$n \equiv 5$ $m \equiv 3,5$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$
$n \equiv 6$ $m \equiv 1,5$	$\frac{m(n+1)+1}{4} - 1$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$	$\frac{m(n+1)+1}{4}$
$n \equiv 6$ $m \equiv 2$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4}$
$n \equiv 6$ $m \equiv 3,7$	$\frac{m(n+1)-1}{4} + 1$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$	$\frac{m(n+1)-1}{4}$
$n \equiv 6$ $m \equiv 6$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$	$\frac{m(n+1)+2}{4}$	$\frac{m(n+1)+2}{4} - 1$

Table 2: Edge labeling of $J_{n,m}$ graph.

The above tables 1 and 2 we find that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the graph $J_{n,m}$ is quotient-4 cordial labeling.

IV.CONCLUSION

In this paper, it is proved that the Jahangir graph $J_{n,m}$ which admits quotient-4 cordial. The existence of quotient-4 cordial labeling of different families of graphs will be the future work.

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REFERENCES

[1]. Albert William, IndraRajasingh and S Roy, Mean Cordial Labeling of Certain graphs, J.Comp.& Math. Sci. Vol.4 (4),274-281 (2013).
[2]. Ashaq Ali, Imran Hashim, WaquasNazeer and Shin Min Kang, ON MORE TOPOLOGICAL INDICES OF JAHANGIR GRAPHS, International Journal of Pure and Applied Mathematics, Volume 119 No. 1 2018, 1-8.
[3]. I. Cahit and R. Yilmaz, E3-cordial graphs, ArsCombin., 54 (2000) 119-127.
[4]. S. Freeda and R. S. Chellathurai, H- and H2-cordial labeling of some graphs Open J. Discrete Math., 2 (2012) 149-155.
[5]. F. Harary, Graph Theory. Narosa Publishing House Reading, New Delhi, (1988).
[6]. Joseph A. Gallian, A Dynamic survey of Graph Labeling , Twenty-first edition, December 21, 2018.
[7]. S.Kavitha,P.Sumathi Quotient-4 Cordial Labeling of Some Unicyclic Graphs and Some Corona of Ladder Graphs, Journal of Survey in Fisheries Sciences, Volume 10(2S), 2023, p.1815-1831.
[8]. P. Sumathi, S.Kavitha, Quotient-4 cordial labeling for path related graphs, The International Journal of Analytical and Experimental Modal analysis, Volume XII, Issue I, January – 2020, pp. 2983-2991.
[9]. P.Sumathi, S.Kavitha, Quotient-4 Cordial Labeling of Some Ladder Graphs, Journal of Algebraic Statistics, Volume 13, No.2, 2022, p.3243-3264.
[10]. P. Sumathi, and A. Rathi, Some results on quotient labeling of special graphs, Global Journal of Pure and Applied Mathematics, vol. 13, no. 1 (2017) 176-178.
[11]. P.Sumathi, S.Kavitha, Quotient-4 cordial labeling of some unicyclic graphs-paper-I, AIP Conference Proceedings, 2718, 020003 (2023).