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Review Paper

Importance of 9 in Digital Root.

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Abstract.
We know that if we multiply any number by 9 then it's digital root (sum of all the digits) will be a multiple of 9. Digital root of a number doesn't change if 9 is added to that number. If we divide any number by 9, the digital root of that number will be the remainder. This paper specifies that the difference of a positive number with its digital root is always the multiple of 9. The Digital Root of that difference is also a multiple of 9. Here, the difference cannot be smaller than 9. An Alternative way to find digital root of difference and understanding some equations/statements.

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1) What is the Digital Root of a number?
Ans: The sum of digits in a number is called the digital root of that number.
For example:
The digital root of 23 will be 2 + 3 = 5.
Similarly, the digital root of 563 will be 5 + 6 + 3 = 14.
2) What if we subtract the digital root from its number?
Let's try this with the number 48.
Digital Root of 48 = 4 + 8 = 12
48 – Its Digital Root = ?
48 – 12 = 36.(Multiple of 9)
Now, let us find out the digital root of 36
Digital Root of 36 = 3 + 6 = 9.(Multiple of 9)

Let's try this with the number of 87.
Digital Root of 87 = 8 + 7 = 15
87 – Its Digital Root
87 – 15 = 72.(Multiple of 9)
Now, let us find out the digital root of 72
Digital root of 72 = 7 + 2 = 9.(Multiple of 9)

From the above examples we come to know that-

The difference of a positive number with its digital root is always the multiple of 9. The Digital Root of that difference is also a multiple of 9. Here, the difference cannot be smaller than 9.

Now, let us denote this difference with 'dX'.
If, dX = 55344744, then let the digital root of difference be dXr.
Let d^n be the number of digits.

Let us find the digital root of the above difference = 55344744.
Therefore, digital root of dX will be-

dXr = 5 + 5 + 3 + 4 + 4 + 7 + 4 + 4 = 36

An alternative and easy method would be this-

$$dXr = \frac{D^n}{2} \times 9$$

Note: This rule is for difficult or complicated numbers hence it is not applicable for easy numbers whose digital root is less than 9 or other easy numbers.

Steps to understand this method:

Step 1: Write the total number of digits in place on d^n, in case the digits are odd make them even by adding a 1 to $d^n = d^n + 1$
Step 2: If the difference contains 9 then add a 1 for every 9 in the number. For example, if a number contain two 9s then in case of even digits = $d^n + 2$ and in case of odd digits = $d^n + 1 + 2$
Step 3: If the digits repeat in the difference then add 1 for every one pair of repetition (Only for digits greater that 5). For example: If the number is 56778945389 then $d^n = 11 + 1 + 2 + 3 = 17$. As 17 is an odd number we will add 1 = 17 + 1 = 18. Now

$$dXr = \frac{18}{2} \times 9$$

$dXr = 9 \times 9 = 81$

The digital root of 56778945389 is 81

Let us consider one more number.
Let the difference (dX) be 45682537977. It's digital root (dXr) will be-

$$dXr = \frac{d^n}{2} \times 9$$

$$dXr = \frac{11 + 1 + 1 + 2}{2} \times 9 = 54.$$

$$dXr = \frac{15 + 1}{2} \times 9 = 72$$

Thus, 72 is the Digital Root of the Difference dX.

From the above information an equation/statement can be formed.
"If $(10x + y) – (x + y) = 10a + b$, then $(a + b) = 9$"
$(a + b) = 9$ can be written as
$(9 – a = b)$ and $(9 – b = a)$
$a = (9 – b)$ and $b = (9 – a)$
Let us substitute the values of a and b in $(a + b) = 9$
$(a + b) = 9$
$[(9 – b) + (9 – a)] = 9$
In this way we can make the equation more interesting.

"If (10x + y) – (x + y) = 10a + b, then (a + b) = 9."

For example :
Let x be 7 and y be 4
Now, If
(10x + y) - (x + y) = 10a + b
(70 + 4) - (7 + 4) = 10a + b
74 - 11 = 10a + b
63 = 10a + b
10a + b = 60 + 3
Here, b = 3,
10a = 60
a = 6
Then, (a + b) = 9
(6 + 3) = 9. Hence Verified.

Similarly
"If (100x + 10y + z) – (x + y + z) = 100a + 10b + c, then (a + b + c) = Multiple of 9".
Let x be 4, y be 8 and z be 3.
Now, If
(100x + 10y + z) – (x + y + z) = 100a + 10b + c,
(400 + 80 + 3) – (4 + 8 + 3) = 100a + 10b + c,
483 – 15 = 100a + 10b + c
468 = 100a + 10b + c
100a + 10b + c = 400 + 60 + 8
100a = 400
a = 4
10b = 60
b = 6
c = 8
then (a + b + c) = Multiple of 9
(4 + 6 + 8) = 18 (Multiple of 9).
Hence Verified.

And
"If (1000x + 100y + 10z + w) – (x + y + z + w) = 1000a + 100b + 10c + d, then (a + b + c + d) = Multiple of 9".
Let x be 5, y be 9, z be 1 and w be 6.
Now, If
(1000x + 100y + 10z + w) – (x + y + z + w) = 1000a + 100b + 10c + d,
(5000 + 900 + 10 + 6) – (5 + 9 + 1 + 6) = 1000a + 100b + 10c + d,
5916 – 21 = 1000a + 100b + 10c + d,
5895 = 1000a + 100b + 10c + d,
1000a + 100b + 10c + d = 5000 + 800 + 90 + 5
1000a = 5000
a = 5
100b = 800
b = 8
10c = 90
c = 9
d = 5
then (a + b + c + d) = Multiple of 9
(5 + 8 + 9 + 5) = 27 (Multiple of 9).
Hence Verified.

In this manner you can form many such infinite equations/statements.
Fun Fact: 45 digital theory.
The digital root of the sum of 45 with any single digit number is the single digit number itself.

For example:
Let's take 1 as the single digit number.

45
\+ 1
\------
46 Digital root
4 + 6 = 10 Digital root
1 + 0 = 1(Here, we got the number)

Let's take 2

45
\+ 2
\------
47 Digital root
4 + 7 = 11 Digital root
1 + 1 = 2 (Here, we got the number)

Let’s Take 5

45
\+ 5
\------
50 Digital root
5 + 0 = 5 (Here, we got the number)

Let’s Take 9

45
\+ 9
\------
54 Digital root
5 + 4 = 9 (Here, we got the number)

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