



Research Paper

## Block-wise Density Distribution of Primes less than a Trillion in Arithmetical Progressions $10n + k$

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Received 05 June, 2016; Accepted 20 June, 2016 © The author(s) 2016. Published with open access at [www.questjournals.org](http://www.questjournals.org)

**ABSTRACT :** All prime numbers, except 2 and 5, fit in one of the forms of arithmetical progressions  $10n + 1$ ,  $10n + 3$ ,  $10n + 7$  and  $10n + 9$ . Each of them contains infinitely many primes. Here a comparison of abundance of primes in them in ranges  $1 - 10^n$  for  $n = 1, 2, \dots, 12$  is done. In all blocks of these sizes covered till 1 trillion, first and last primes of these forms are determined. 10 power blocks containing minimum number of primes of these forms, first & last blocks of such minimum occurrences of primes, and number of such blocks till 1 trillion are determined. All this analysis is also done for maximum number primes within blocks.

**Keywords:-** Arithmetical progressions, block-wise distribution, prime, prime density

### I. INTRODUCTION

Fundamental Theorem of Arithmetic highlights the role of prime numbers as multiplicative building units of all integers. Primes are divisors of other numbers but themselves have no non-trivial divisors. They have been proven to be infinite long ago [1].

### II. PRIMES NUMBERS AND ARITHMETICAL PROGRESSIONS

It is said that the simplest example of integer sequences are arithmetical progressions as the successive terms in them are constructed just by adding a fixed number. The other way round, an arithmetical progression can be seen to be the list of those numbers that when divided by a fixed number give a fixed remainder. If  $an + b$  is an arithmetical progression, it contains all those numbers which when divided by  $a$  give remainder  $b$ . Since there are  $a$  different possible remainders  $b$  in the division by  $a$ , viz.,  $b = 0, 1, \dots, a - 1$ , for any positive integer  $a$ , there are  $a$  number of different arithmetical progressions :  $an + 0, an + 1, \dots, an + (a - 1)$ .

We consider prime numbers and arithmetical progressions together. Dirichlet, in his famous result [2], has proved that an arithmetical progression  $an + b$  contains infinite number of primes if, and only if,  $a$  and  $b$  are relatively prime, i.e., have greatest common divisor 1. For all single digit  $a$ 's from 2 to 9, the detailed analysis of primes in all such possible arithmetical progressions  $an + b$  is recently done [3]-[12]. For the first time now a two digit  $a = 10$  is considered here.

The symbol  $\pi_{a,b}(x)$  introduced in previous works which means the number of primes of form  $an + b$  that are less than or equal to  $x$  is used here also.

### III. PRIMES DISTRIBUTIONS IN ARITHMETICAL PROGRESSIONS $10n + k$

By property stated earlier, if the dividing number is chosen to be 10, then there will be 10 possible remainders in the process of division, viz., 0, 1, 2,  $\dots$ , 9. They give rise to overall 10 arithmetical progressions  $10n + k$  for these 10 values of  $k$ . Of these, by Dirichlet's property only 4 will contain infinite number of primes, viz.,  $10n + 1, 10n + 3, 10n + 7, 10n + 9$ .

Apart from these four, there are two more progressions from these categories which contain primes. They are  $10n + 2$  and  $10n + 5$ . But it is so that they contain only one prime each.  $10n + 2$  contains 2 only and  $10n + 5$  contains 5 only. Hence in our main analysis of prime density here, we have omitted them.

### IV. PRIMES NUMBER RACE

Granville and Martin coined the term prime number race [13] for dominance of primes in the arithmetical progressions of same family  $an + b$ . Due to the reasons stated earlier, ignoring  $10n + 2$  and  $10n + 5$ , the number of primes till 1,000,000,000,000, i.e.,  $10^{12}$  in  $10n + 1, 10n + 3, 10n + 7$  and  $10n + 9$  are determined. This required large prime data that could be generated by choosing most efficient algorithm resulting out of

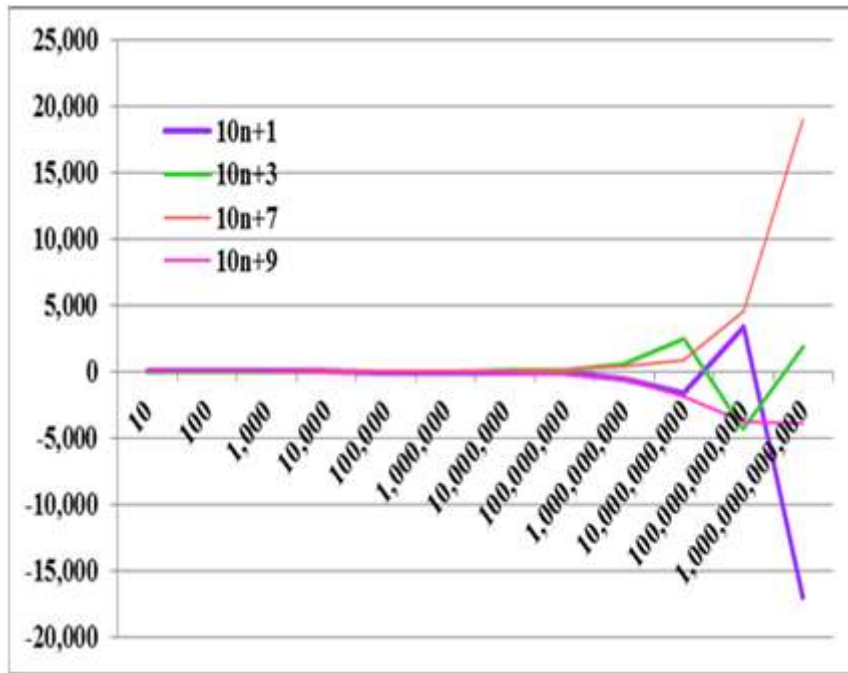
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comparisons in [14]-[20]. Java programming language made handy by excellent explanation in [21] was used for implementations of these.

**Table 1 :** Number of Primes of form  $10n + k$  in First Blocks of 10 Powers

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | Number of Primes of Form       |                                |                                |                                |
|---------|----------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
|         |                            | $10n + 1$<br>$(\pi_{10,1}(x))$ | $10n + 3$<br>$(\pi_{10,3}(x))$ | $10n + 7$<br>$(\pi_{10,7}(x))$ | $10n + 9$<br>$(\pi_{10,9}(x))$ |
| 1.      | 1-10                       | 0                              | 1                              | 1                              | 0                              |
| 2.      | 1-100                      | 5                              | 7                              | 6                              | 5                              |
| 3.      | 1-1,000                    | 40                             | 42                             | 46                             | 38                             |
| 4.      | 1-10,000                   | 306                            | 310                            | 308                            | 303                            |
| 5.      | 1-100,000                  | 2,387                          | 2,402                          | 2,411                          | 2,390                          |
| 6.      | 1-1,000,000                | 19,617                         | 19,665                         | 19,621                         | 19,593                         |
| 7.      | 1-10,000,000               | 166,104                        | 166,230                        | 166,211                        | 166,032                        |
| 8.      | 1-100,000,000              | 1,440,298                      | 1,440,474                      | 1,440,495                      | 1,440,186                      |
| 9.      | 1-1,000,000,000            | 12,711,386                     | 12,712,499                     | 12,712,314                     | 12,711,333                     |
| 10.     | 1-10,000,000,000           | 113,761,519                    | 113,765,625                    | 113,764,039                    | 113,761,326                    |
| 11.     | 1-100,000,000,000          | 1,029,517,130                  | 1,029,509,448                  | 1,029,518,337                  | 1,029,509,896                  |
| 12.     | 1-1,000,000,000,000        | 9,401,960,980                  | 9,401,979,904                  | 9,401,997,000                  | 9,401,974,132                  |

Since all primes, except 2 and 5, are of only of one of these forms, their quantity is expected to be averagely distributed. The deviation from respective averages is plotted separately.



**Figure 1 :** Deviation of  $\pi_{10,k}(x)$  from Average

The number of primes of the form  $10n + 7$  and  $10n + 3$  seem most of the times ahead of the average, while those of the form  $10n + 9$  always lag behind, up to  $10^{12}$  in discrete blocks of 10 powers. This trend is a subject matter of confirmation in intermediate and higher ranges.

### V. BLOCK-WISE DISTRIBUTION OF PRIMES

Because primes cannot be generalized by any formula, nor their distribution is uniform, we continue with the approach of following them in blocks of powers of 10 as

- 1-10, 11-20, 21-30, 31-40, . . .
- 1-100, 101-200, 201-300, 301-400, . . .
- 1-1000, 1001-2000, 2001-3000, 3001-4000, . . .
- ⋮

For total range of  $1-10^{12}$ , there will be  $10^{12-n}$  number of blocks of size  $10^n$  for every  $1 \leq n \leq 12$ .

**A. THE FIRST AND THE LAST PRIMES IN THE FIRST BLOCKS OF 10 POWERS**

Within our range of 1 to 1 trillion, for aforementioned initial blocks of all sizes, the smallest and the largest primes haven determined to be as follows.

**Table 2 : First Primes of form  $10n + k$  First Blocks of 10 Powers**

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | First Prime in the First Block |                |                |                |
|---------|----------------------------|--------------------------------|----------------|----------------|----------------|
|         |                            | Form $10n + 1$                 | Form $10n + 3$ | Form $10n + 7$ | Form $10n + 9$ |
| 1.      | 1-10                       | Not Found                      | 3              | 7              | Not Found      |
| 2.      | 1-100                      | 11                             | 3              | 7              | 19             |
| 3.      | 1-1,000                    | 11                             | 3              | 7              | 19             |
| 4.      | 1-10,000                   | 11                             | 3              | 7              | 19             |
| 5.      | 1-100,000                  | 11                             | 3              | 7              | 19             |
| 6.      | 1-1,000,000                | 11                             | 3              | 7              | 19             |
| 7.      | 1-10,000,000               | 11                             | 3              | 7              | 19             |
| 8.      | 1-100,000,000              | 11                             | 3              | 7              | 19             |
| 9.      | 1-1,000,000,000            | 11                             | 3              | 7              | 19             |
| 10.     | 1-10,000,000,000           | 11                             | 3              | 7              | 19             |
| 11.     | 1-100,000,000,000          | 11                             | 3              | 7              | 19             |
| 12.     | 1-1,000,000,000,000        | 11                             | 3              | 7              | 19             |

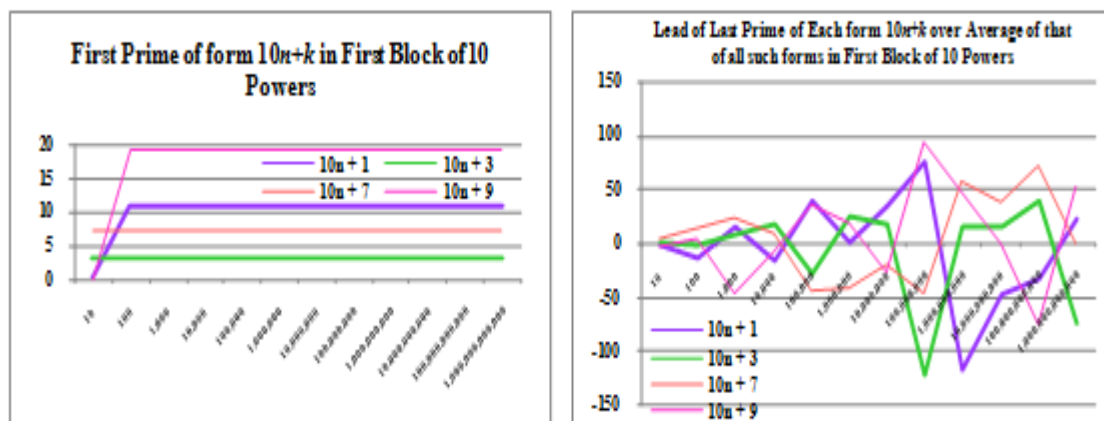
The last primes within these blocks go on increasing with increasing block sizes.

**Table 3 : Last Primes of form  $10n + k$  First Blocks of 10 Powers**

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | Last Prime in the First Block |                 |                 |                 |
|---------|----------------------------|-------------------------------|-----------------|-----------------|-----------------|
|         |                            | Form $10n + 1$                | Form $10n + 3$  | Form $10n + 7$  | Form $10n + 9$  |
| 1.      | 1-10                       | Not Found                     | 3               | 7               | Not Found       |
| 2.      | 1-100                      | 71                            | 83              | 97              | 89              |
| 3.      | 1-1,000                    | 991                           | 983             | 997             | 929             |
| 4.      | 1-10,000                   | 9,941                         | 9,973           | 9,967           | 9,949           |
| 5.      | 1-100,000                  | 99,991                        | 99,923          | 99,907          | 99,989          |
| 6.      | 1-1,000,000                | 999,961                       | 999,983         | 999,917         | 999,979         |
| 7.      | 1-10,000,000               | 9,999,991                     | 9,999,973       | 9,999,937       | 9,999,929       |
| 8.      | 1-100,000,000              | 99,999,971                    | 99,999,773      | 99,999,847      | 99,999,989      |
| 9.      | 1-1,000,000,000            | 999,999,761                   | 999,999,893     | 999,999,937     | 999,999,929     |
| 10.     | 1-10,000,000,000           | 9,999,999,881                 | 9,999,999,943   | 9,999,999,967   | 9,999,999,929   |
| 11.     | 1-100,000,000,000          | 99,999,999,871                | 99,999,999,943  | 99,999,999,977  | 99,999,999,829  |
| 12.     | 1-1,000,000,000,000        | 999,999,999,961               | 999,999,999,863 | 999,999,999,937 | 999,999,999,989 |

So long as the omitted forms  $10n + 2$  and  $10n + 5$  are concerned, since they contain unique primes 2 and 5, respectively, the same happen to be the first and last primes of these forms in every block of 10 power starting with  $10^1$  going virtually till  $\infty$ .

Now follows the graphical representations of first and last primes of the important 4 forms of type  $10n + k$ .



**Figure 2: First & Last Primes of form  $10n + k$  in First Blocks of 10 Powers.**

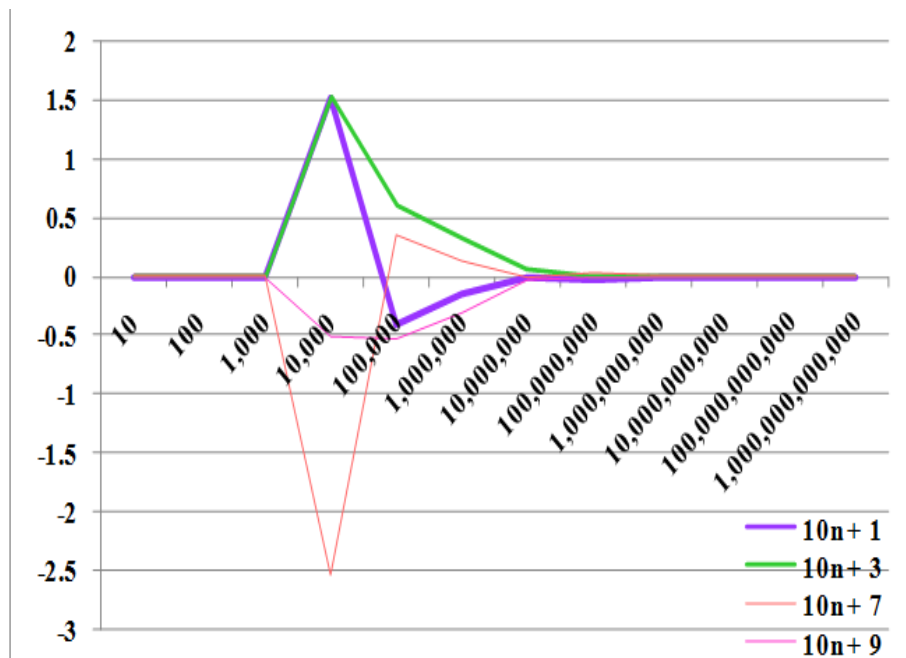
**B. Minimum Number Of Primes In Blocks Of 10 Powers**

For the same sized blocks, the minimum number of primes of each form found in each of them is next point analysis.

**Table 4 :** Minimum Number of Primes of form  $10n + k$  in Blocks of 10 Powers

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | Minimum Number of Primes in Blocks |                |                |                |
|---------|----------------------------|------------------------------------|----------------|----------------|----------------|
|         |                            | Form $10n + 1$                     | Form $10n + 3$ | Form $10n + 7$ | Form $10n + 9$ |
| 1.      | 1-10                       | 0                                  | 0              | 0              | 0              |
| 2.      | 1-100                      | 0                                  | 0              | 0              | 0              |
| 3.      | 1-1,000                    | 0                                  | 0              | 0              | 0              |
| 4.      | 1-10,000                   | 50                                 | 50             | 48             | 49             |
| 5.      | 1-100,000                  | 795                                | 803            | 801            | 794            |
| 6.      | 1-1,000,000                | 8,748                              | 8,789          | 8,772          | 8,734          |
| 7.      | 1-10,000,000               | 89,851                             | 89,904         | 89,846         | 89,846         |
| 8.      | 1-100,000,000              | 903,380                            | 903,467        | 903,712        | 903,526        |
| 9.      | 1-1,000,000,000            | 9,046,766                          | 9,046,777      | 9,046,962      | 9,046,857      |
| 10.     | 1-10,000,000,000           | 90,495,945                         | 90,493,544     | 90,493,875     | 90,494,057     |
| 11.     | 1-100,000,000,000          | 906,486,613                        | 906,472,632    | 906,481,722    | 906,483,465    |
| 12.     | 1-1,000,000,000,000        | 9,401,960,980                      | 9,401,979,904  | 9,401,997,000  | 9,401,974,132  |

The block-wise percentage deviation of minimum number of primes found there from respective averages is given in the figure next.



**Figure 3 :** % Deviation in Minimum Number of Primes of form  $10n + k$  in Blocks of  $10^n$  from Average

The first blocks each of these sizes in our range of  $10^{12}$  containing minimum number of primes of these forms in them are found be as follows.

**Table 5 :** First Blocks of 10 Powers with Minimum Number of Primes of form  $10n + k$  in Them

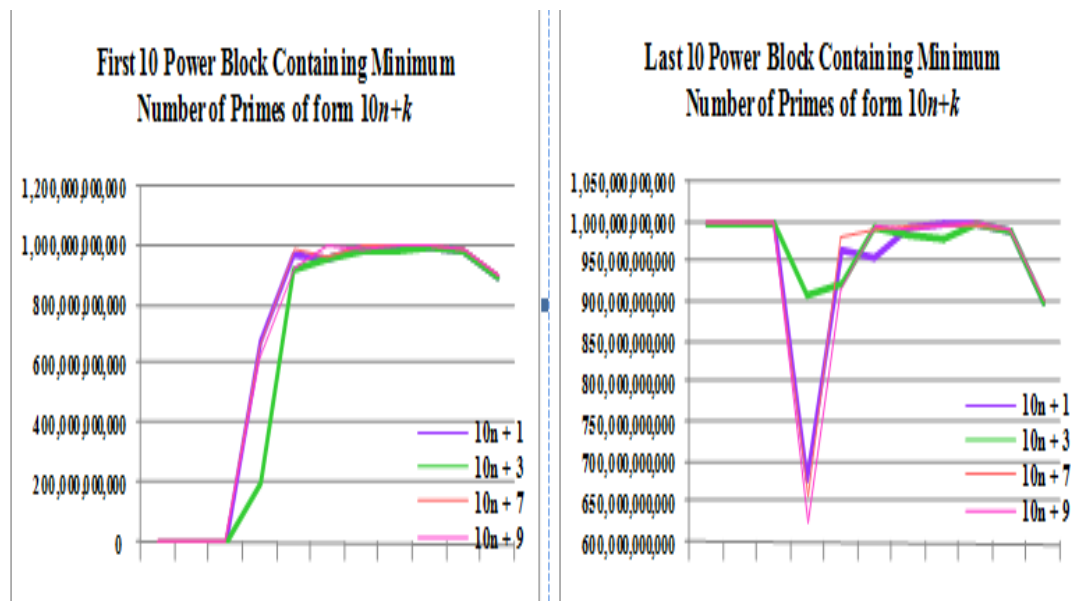
| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | First Block with Minimum Number of Primes |                 |                 |                 |
|---------|----------------------------|---|-----------------|-----------------|-----------------|
|         |                            | Form $10n + 1$                            | Form $10n + 3$  | Form $10n + 7$  | Form $10n + 9$  |
| 1.      | 1-10                       | 0   | 30              | 20              | 0               |
| 2.      | 1-100                      | 10,400                                    | 13,200          | 8,900           | 13,500          |
| 3.      | 1-1,000                    | 1,992,636,000                             | 1,054,256,000   | 2,174,469,000   | 1,036,101,000   |
| 4.      | 1-10,000                   | 681,769,270,000                           | 200,077,450,000 | 657,874,630,000 | 625,725,710,000 |
| 5.      | 1-100,000                  | 967,423,100,000                           | 924,727,600,000 | 979,846,600,000 | 918,734,500,000 |
| 6.      | 1-1,000,000                | 957,750,000,000                           | 956,012,000,000 | 957,617,000,000 | 995,465,000,000 |
| 7.      | 1-10,000,000               | 994,560,000,000                           | 985,230,000,000 | 994,120,000,000 | 989,830,000,000 |
| 8.      | 1-100,000,000              | 997,800,000,000                           | 981,100,000,000 | 996,300,000,000 | 997,000,000,000 |
| 9.      | 1-1,000,000,000            | 997,000,000,000                           | 998,000,000,000 | 998,000,000,000 | 999,000,000,000 |
| 10.     | 1-10,000,000,000           | 990,000,000,000                           | 990,000,000,000 | 990,000,000,000 | 990,000,000,000 |
| 11.     | 1-100,000,000,000          | 900,000,000,000                           | 900,000,000,000 | 900,000,000,000 | 900,000,000,000 |

And the last such blocks till  $10^{12}$  are also found.

**Table 6 :** Last Blocks of 10 Powers with Minimum Number of Primes of form  $10n + k$  in Them

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | Last Block with Minimum Number of Primes |                 |                 |                 |
|---------|----------------------------|--|-----------------|-----------------|-----------------|
|         |                            | Form $10n + 1$                           | Form $10n + 3$  | Form $10n + 7$  | Form $10n + 9$  |
| 1.      | 1-10                       | 999,999,999,990                          | 999,999,999,990 | 999,999,999,990 | 999,999,999,990 |
| 2.      | 1-100                      | 999,999,999,800                          | 999,999,999,900 | 999,999,999,300 | 999,999,999,700 |
| 3.      | 1-1,000                    | 999,945,413,000                          | 999,969,741,000 | 999,936,675,000 | 999,928,156,000 |
| 4.      | 1-10,000                   | 681,769,270,000                          | 909,482,100,000 | 657,874,630,000 | 625,725,710,000 |
| 5.      | 1-100,000                  | 967,423,100,000                          | 924,727,600,000 | 979,846,600,000 | 918,734,500,000 |
| 6.      | 1-1,000,000                | 957,750,000,000                          | 994,187,000,000 | 993,599,000,000 | 995,465,000,000 |
| 7.      | 1-10,000,000               | 994,560,000,000                          | 985,230,000,000 | 994,120,000,000 | 989,830,000,000 |
| 8.      | 1-100,000,000              | 997,800,000,000                          | 981,100,000,000 | 996,300,000,000 | 997,000,000,000 |
| 9.      | 1-1,000,000,000            | 997,000,000,000                          | 998,000,000,000 | 998,000,000,000 | 999,000,000,000 |
| 10.     | 1-10,000,000,000           | 990,000,000,000                          | 990,000,000,000 | 990,000,000,000 | 990,000,000,000 |
| 11.     | 1-100,000,000,000          | 900,000,000,000                          | 900,000,000,000 | 900,000,000,000 | 900,000,000,000 |

Here comes their graphical comparison.



**Figure 4 :** First & Last Blocks of 10 Powers with Minimum Number of Primes of form  $10n + k$ .

The values for forms  $10n + 2$  and  $10n + 5$  are missing here. They are parallel to corresponding values for arithmetical progression  $8n + 2$  given in [9].

It was also necessary to find out the as to how many times do these minimum number of primes of different forms in various  $10^n$  sized blocks occur till  $10^{12}$ .

**Table 7 :** Frequency of 10 Power Blocks with Minimum Number of Primes of form  $10n + k$  in them

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | No. of Times Minimum No. of Primes Occurring in Blocks |                |                |                |
|---------|----------------------------|--|----------------|----------------|----------------|
|         |                            | Form $10n + 1$   | Form $10n + 3$ | Form $10n + 7$ | Form $10n + 9$ |
| 1.      | 1-10                       | 90,598,039,020   | 90,598,020,096 | 90,598,003,000 | 90,598,025,868 |
| 2.      | 1-100                      | 3,549,112,098  | 3,549,128,343  | 3,549,105,296  | 3,549,101,467  |
| 3.      | 1-1,000                    | 18,529   | 18,764         | 18,534         | 18,709         |
| 4.      | 1-10,000                   | 1  | 3              | 1              | 1              |
| 5.      | 1-100,000                  | 1  | 1              | 1              | 1              |
| 6.      | 1-1,000,000                | 1  | 2              | 2              | 1              |
| 7.      | 1-10,000,000               | 1  | 1              | 1              | 1              |
| 8.      | 1-100,000,000              | 1  | 1              | 1              | 1              |
| 9.      | 1-1,000,000,000            | 1  | 1              | 1              | 1              |
| 10.     | 1-10,000,000,000           | 1  | 1              | 1              | 1              |
| 11.     | 1-100,000,000,000          | 1  | 1              | 1              | 1              |
| 12.     | 1-1,000,000,000,000        | 1  | 1              | 1              | 1              |

Graphical representation of the block-wise percentage deviation of frequency of occurrence of minimum number of primes from respective averages is due.

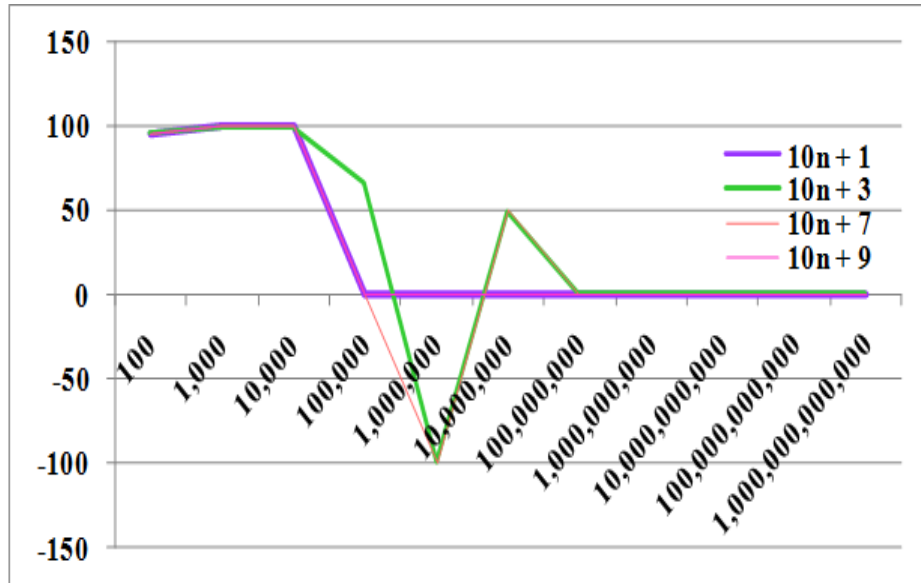


Figure 5 : % Decrease in Occurrences of Minimum Number of Primes of form  $10n + k$  in Blocks of  $10^n$ .

C. Maximum Number Of Primes In Blocks Of 10 Powers

After investigation for the minimum number of primes in  $10^n$  sized blocks, now maximality is under critical analysis.

Table 8 : Maximum Number of Primes of form  $10n + k$  in Blocks of 10 Powers

| Sr. No. | Range<br>1-x (1 to x) | Maximum Number of Primes in Blocks |                |                |                |
|---------|-----------------------|------------------------------------|----------------|----------------|----------------|
|         |                       | Form $10n + 1$                     | Form $10n + 3$ | Form $10n + 7$ | Form $10n + 9$ |
| 1.      | 1-10                  | 1                                  | 1              | 1              | 1              |
| 2.      | 1-100                 | 7                                  | 7              | 7              | 7              |
| 3.      | 1-1,000               | 40                                 | 42             | 46             | 38             |
| 4.      | 1-10,000              | 306                                | 310            | 308            | 303            |
| 5.      | 1-100,000             | 2,387                              | 2,402          | 2,411          | 2,390          |
| 6.      | 1-1,000,000           | 19,617                             | 19,665         | 19,621         | 19,593         |
| 7.      | 1-10,000,000          | 166,104                            | 166,230        | 166,211        | 166,032        |
| 8.      | 1-100,000,000         | 1,440,298                          | 1,440,474      | 1,440,495      | 1,440,186      |
| 9.      | 1-1,000,000,000       | 12,711,386                         | 12,712,499     | 12,712,314     | 12,711,333     |
| 10.     | 1-10,000,000,000      | 113,761,519                        | 113,765,625    | 113,764,039    | 113,761,326    |
| 11.     | 1-100,000,000,000     | 1,029,517,130                      | 1,029,509,448  | 1,029,518,337  | 1,029,509,896  |
| 12.     | 1-1,000,000,000,000   | 9,401,960,980                      | 9,401,979,904  | 9,401,997,000  | 9,401,974,132  |

Average deviation analysis is presented graphically.

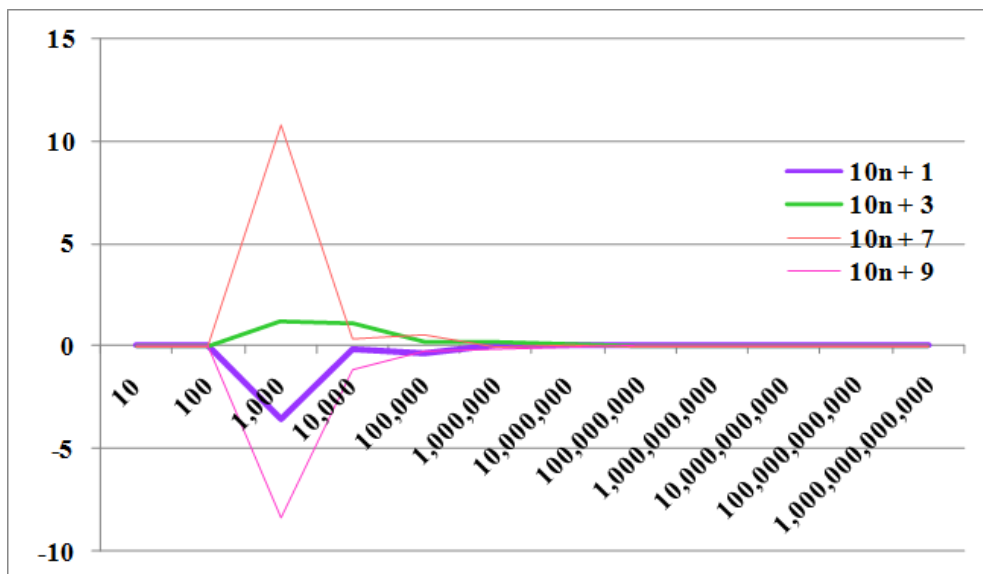


Figure 6 : % Deviation in Maximum Number of Primes of form  $10n + k$  in Blocks of  $10^n$  from Average.

The First & last blocks till  $10^{12}$  with max number of primes of these four forms in them are determined.

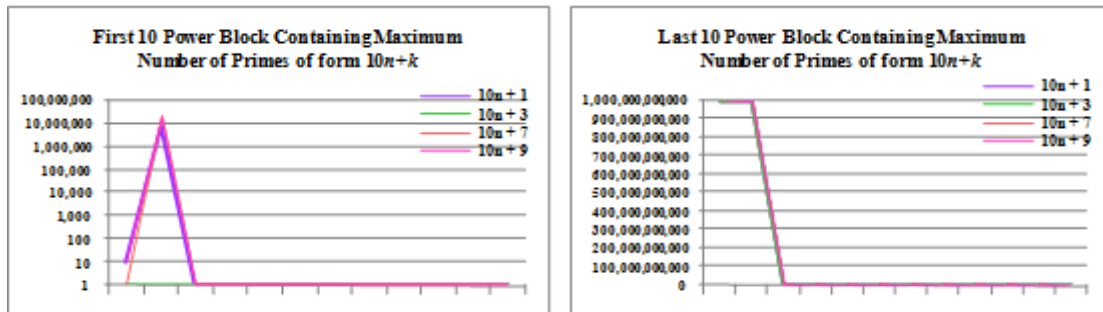
**Table 9 :** First Blocks of 10 Powers with Maximum Number of Primes of form  $10n + k$  in Them

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | First Block with Maximum Number of Primes |                |                |                |
|---------|----------------------------|---|----------------|----------------|----------------|
|         |                            | Form $10n + 1$                            | Form $10n + 3$ | Form $10n + 7$ | Form $10n + 9$ |
| 1.      | 1-10                       | 10  | 0              | 0              | 10             |
| 2.      | 1-100                      | 8,056,200                                 | 0              | 22,424,100     | 21,169,600     |
| 3.      | 1-1,000                    | 0   | 0              | 0              | 0              |
| 4.      | 1-10,000                   | 0   | 0              | 0              | 0              |
| 5.      | 1-100,000                  | 0   | 0              | 0              | 0              |
| 6.      | 1-1,000,000                | 0   | 0              | 0              | 0              |
| 7.      | 1-10,000,000               | 0   | 0              | 0              | 0              |
| 8.      | 1-100,000,000              | 0   | 0              | 0              | 0              |
| 9.      | 1-1,000,000,000            | 0   | 0              | 0              | 0              |
| 10.     | 1-10,000,000,000           | 0   | 0              | 0              | 0              |
| 11.     | 1-100,000,000,000          | 0   | 0              | 0              | 0              |
| 12.     | 1-1,000,000,000,000        | 0   | 0              | 0              | 0              |

**Table 10 :** Last Blocks of 10 Powers with Maximum Number of Primes of form  $10n + k$  in Them

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | Last Block with Maximum Number of Primes |                 |                 |                 |
|---------|----------------------------|--|-----------------|-----------------|-----------------|
|         |                            | Form $10n + 1$                           | Form $10n + 3$  | Form $10n + 7$  | Form $10n + 9$  |
| 1.      | 1-10                       | 999,999,999,960                          | 999,999,999,860 | 999,999,999,930 | 999,999,999,980 |
| 2.      | 1-100                      | 996,503,865,600                          | 999,318,647,900 | 998,658,215,200 | 998,726,687,000 |
| 3.      | 1-1,000                    | 0  | 0               | 0               | 0               |
| 4.      | 1-10,000                   | 0  | 0               | 0               | 0               |
| 5.      | 1-100,000                  | 0  | 0               | 0               | 0               |
| 6.      | 1-1,000,000                | 0  | 0               | 0               | 0               |
| 7.      | 1-10,000,000               | 0  | 0               | 0               | 0               |
| 8.      | 1-100,000,000              | 0  | 0               | 0               | 0               |
| 9.      | 1-1,000,000,000            | 0  | 0               | 0               | 0               |
| 10.     | 1-10,000,000,000           | 0  | 0               | 0               | 0               |
| 11.     | 1-100,000,000,000          | 0  | 0               | 0               | 0               |
| 12.     | 1-1,000,000,000,000        | 0  | 0               | 0               | 0               |

As the prime density has a decreasing trend for higher range of numbers, for larger block sizes, the first and the last occurrences of maximum number of primes in them starts in the very first block of 0.



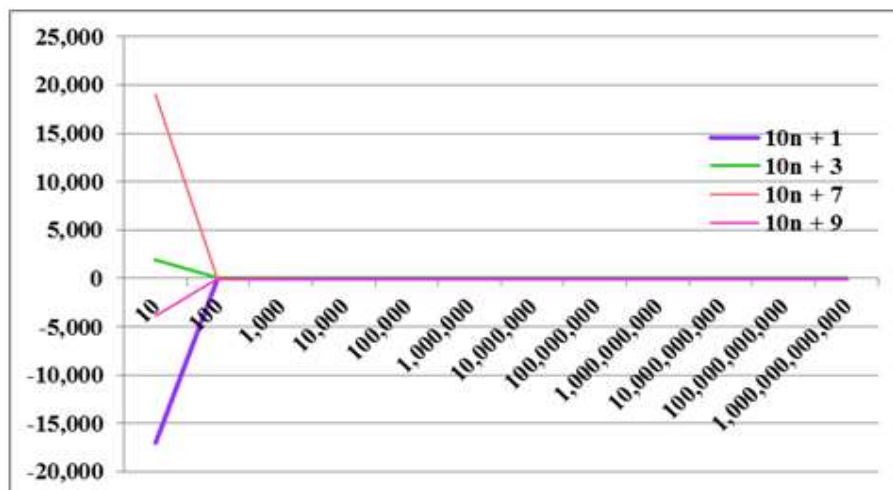
**Figure 7 :** First & Last Blocks of 10 Powers with Maximum Number of Primes of form  $10n + k$ .

The same decreasing density of primes leads to lesser frequencies of maximality occurrences for higher sized blocks.

**Table 11 :** Frequency of 10 Power Blocks with Maximum Number of Primes of form  $10n + k$  in them

| Sr. No. | Range<br>$1-x$ (1 to $x$ ) | No. of Times Maximum No. of Primes Occurring in Blocks |                |                |                |
|---------|----------------------------|--|----------------|----------------|----------------|
|         |                            | Form $10n + 1$   | Form $10n + 3$ | Form $10n + 7$ | Form $10n + 9$ |
| 1.      | 1-10                       | 9,401,960,980  | 9,401,979,904  | 9,401,997,000  | 9,401,974,132  |
| 2.      | 1-100                      | 1,158  | 1,266          | 1,138          | 1,241          |
| 3.      | 1-1,000                    | 1  | 1              | 1              | 1              |
| 4.      | 1-10,000                   | 1  | 1              | 1              | 1              |
| 5.      | 1-100,000                  | 1  | 1              | 1              | 1              |
| 6.      | 1-1,000,000                | 1  | 1              | 1              | 1              |
| 7.      | 1-10,000,000               | 1  | 1              | 1              | 1              |
| 8.      | 1-100,000,000              | 1  | 1              | 1              | 1              |
| 9.      | 1-1,000,000,000            | 1  | 1              | 1              | 1              |
| 10.     | 1-10,000,000,000           | 1  | 1              | 1              | 1              |
| 11.     | 1-100,000,000,000          | 1  | 1              | 1              | 1              |
| 12.     | 1-1,000,000,000,000        | 1  | 1              | 1              | 1              |

Finally their graphical representation remains.



**Figure 8 :** Deviation in Frequency of Maximum Number of Primes in Blocks from Average.

The values for forms  $10n + 2$  and  $10n + 5$  are dropped here also. They are same as corresponding values for arithmetical progression  $8n + 2$  in [9].

### ACKNOWLEDGEMENTS

The author acknowledges extensive use of Java Programming Language, NetBeans IDE & Microsoft Office Excel for which their Development Teams must be thanked.

The computer systems of the Department of Mathematics & Statistics of the author's own institution and the uninterrupted power supply facility by the Department of Electronics both were instrumental in getting this heavy work done.

Special thanks are due to the University Grants Commission (U.G.C.), New Delhi of the Government of India which funded this work under a Research Project (F.No. 47-748/13(WRO)).

The anonymous referee doing his/her painstaking work is also acknowledged.

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