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Research Paper

Before Time Runs Out: Modeling Machines That Cannot Wait for Repairs

K.P.S. Baghel
Govt. Degree College Manikpur, Chitrakoot (U.P.)

Abstract
Some machines can't wait. A stamping press feeding a just-in-time assembly line, a furnace that damages its lining if it sits cold, a robot cell whose output is booked to the hour: when one of these fails, the clock starts. A long queue for the repair crew is more than an inconvenience. It forces a decision, which is whether to stop waiting and send the unit to an outside vendor. This article models that decision. I build a finite-source Markovian machine repair model with M required machines, S cold spares, R repairmen and exponential patience, so that each failed unit waiting for service leaves the queue at rate ξ. The system is a birth–death chain with a product-form solution. For one repairman the weights reduce to a simple product over (μ + kξ). I derive availability, the outsourcing flow and the Little's-law sojourn time. I also show that a failed unit with k units ahead of it is repaired in-house with probability μ/(μ + kξ), and that the derivative of expected shortage with respect to ξ is a negative covariance, so impatience never hurts availability. Whether it helps cost depends on the price of the vendor. In a worked example computed from the model (M = 2, S = 1, R = 1, λ = 1, μ = 4), raising ξ from 0 to 2 lifts availability from 0.897 to 0.938 while 13.2 percent of failures leave for the vendor. The total cost curve is nearly flat between ξ = 4 and ξ = 8. I then extend the model to N-policy start-up, two failure modes and general repair times, following the machine repair literature of Jain and co-authors. The numerical results are computed illustrations, not plant data.

Keywords: *machine repair problem, reneging, finite-source queue, spare provisioning, availability, N-policy*

I. Introduction

1.1 The Cost of Waiting

Most maintenance models quietly assume that a broken machine can wait. It joins the queue, the crew works through it, and the only damage is lost production time. That's fine for a lathe in a job shop with plenty of slack. It's a poor description of a line where the downstream station starves in twenty minutes.

When waiting is expensive, people act. The maintenance supervisor calls a vendor. A technician is pulled off another job. A machine is cannibalized for parts. In queueing language, the failed unit has reneged: it gave up on the in-house queue before service started. Queueing theorists have studied this behavior for sixty years, first for customers who lose patience (Ancker &Gafarian, 1963a, 1963b) and later for call centers (Garnett et al., 2002; Zeltyn& Mandelbaum, 2005). Applying the idea to a machine population is less common, and I think it deserves more attention.

There's a modeling reason to care. If reneging is ignored, the model overstates how long failed units sit in the repair queue. It also gives the wrong answer to a practical question, namely how many spares and repairmen are enough. Impatience acts as a free extra repair channel, but a channel that costs money every time it's used.

1.2 Contribution and Outline

I treat the problem as a finite-source queue with M machines required, S spares, R repairmen, failure rate λ per operating machine, repair rate μ per repairman and reneging rate ξ per waiting unit. The finite-source structure matters because failures slow down as machines stop running. That feedback is the whole reason machine interference differs from an ordinary queue (Haque & Armstrong, 2007).

Section 2 places the model in the literature. Section 3 states it. Section 4 derives the steady state and the performance measures. Section 5 proves two results about patience: a closed form for the chance of in-house repair, and a covariance expression for the effect of ξ on shortage. Section 6 joins spares, repair capacity and

patience into one cost function and works a numerical example. Section 7 covers extensions, and Section 8 lists limits. Section 9 concludes.

II. Related Work

2.1 Machine Repair Models

The machine repair problem has a long history, and Gross et al. (2008) give the standard treatment of the birth–death formulation. Haque and Armstrong (2007) survey the machine interference literature and point out how many variants still lack closed forms.

A substantial strand comes from India. Jain, Singh and Baghel (2000) analyzed an M/M/c/K/N system with balking, reneging, spares and an additional repairman. Jain and Baghel (2001) handled a multi-component system with state-dependent rates. Jain (2001) added two failure modes to a machine repair model with spares and reneging. Jain, Sharma and Baghel (2002) predicted the performance of a flexible manufacturing system, and Jain, Sharma and Baghel (2004) moved to M/G/1 repair with mixed standbys, degraded failure, Bernoulli feedback and an N-policy. Chikara, Jain and Baghel (2006) let arrival rates be controllable.

2.2 Impatience and Vacation Policies

Reneging theory starts with Ancker and Gafarian (1963a, 1963b) and Baccelli and Hébuterne (1981). Zeltyn and Mandelbaum (2005) treat general patience distributions with many servers. For N-policies, the origin is Yadin and Naor (1963), and Heyman (1968) proved optimality results for M/G/1 control policies. Takács (1962) and Kleinrock (1975) supply the underlying single-server machinery. Little's (1961) law ties everything together.

Table 1 lists the sources and what each one contributes to the model that follows.

Table 1: Published Models Used as Building Blocks in This Article

Source	Model class	Feature contributed	Used in section
Gross et al. (2008)	Finite-source birth–death	Product-form solution	3, 4
Ancker &Gafarian (1963a, 1963b)	M/M/1 with balking and reneging	Impatience mechanism	3, 5
Jain et al. (2000)	M/M/c/K/N	Balking, reneging, spares, extra repairman	3, 6
Jain & Baghel (2001)	Multi-component	State-dependent rates	7
Jain (2001)	Finite source	Two failure modes	7
Jain et al. (2004)	M/G/1	N-policy, mixed standbys, feedback	7
Yadin & Naor (1963)	Removable server	Start-up threshold	7
Zeltyn& Mandelbaum (2005)	M/M/n+G	General patience	5, 8

Reading across the table, I see one gap. The reneging models treat the source as infinite, and the finite-source models mostly treat reneging as a side feature. The quantities in Sections 4 and 5 are meant to fill part of that gap.

III. Model Formulation

3.1 Assumptions

A line needs M machines running. The plant holds S identical cold spares, so the total population is N = M + S. A cold spare doesn't fail while idle. Each operating machine fails after an exponential time with mean 1/λ. There are R repairmen, each repairing at rate μ, and failed units are served first come, first served.

Each failed unit waiting for a repairman has an independent exponential patience time with mean 1/ξ. If service hasn't started by then, the unit is shipped to an outside vendor, repaired there, and returns to the pool as a working unit. The sojourn at the vendor isn't modeled in detail. I treat it as instantaneous return, which keeps the population fixed at N and is a simplification I return to in Section 8. A failure that arrives in state n enters the queue with probability b_n, and otherwise balks straight to the vendor. For the numerical work I set $b_n = 1$.

3.2 Rates

Let n be the number of failed units in the in-house system, counting those in service and those waiting. Then n runs from 0 to N, and the number of machines actually operating is $min(M, N - n)$. The birth and death rates are

$$\lambda_n = b_n, \lambda, \min(M, , N - n)\,, \qquad \mu_n = \mu, \min(n, R) + \xi, (n - R)^+.$$

Here $(x)^+ = \max(x, 0)$. The death rate has two parts: genuine repair completions and reneging departures from the waiting line. This is the rate structure of Jain et al. (2000), specialized to the case without the additional repairman.

IV. Steady-State Analysis

4.1 Product Form

The chain is a finite birth–death process, so detailed balance holds across every cut: $\lambda_{n-1}p_{n-1} = \mu_n p_n$. Iterating gives

$$p_n = p_0, w_n, \qquad w_n = \prod_{k=1}^{n} \frac{\lambda_{k-1}}{\mu_k}, \qquad p_0 = \left[\sum_{n=0}^{N} w_n\right]^{-1}.$$

As shown in Figure 1, the chain has rates that change twice along its length. Failures slow down once the spares are gone (n > S), and the death rate gains a ξ-term once all repairmen are busy (n > R). Those two kinks are where the decisions about S and R enter.

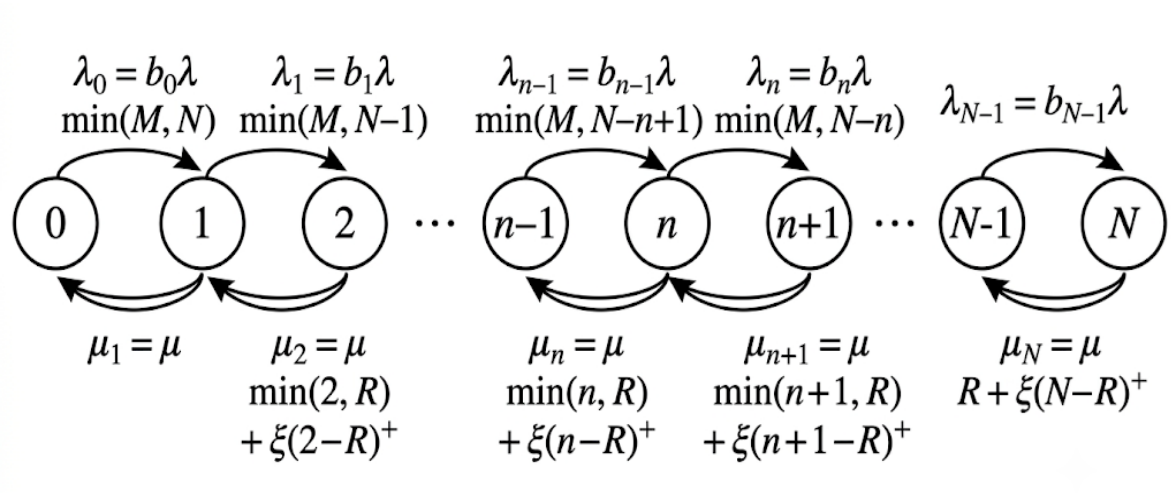


Figure 1: State-Transition Diagram of the Finite-Source Machine Repair Chain with Spares and Reneging

States 0 through N sit in a row, where state n means n failed units are in the in-house repair system. Right-pointing arrows carry the failure rate λ_n = b_n λ min(M, N − n), and left-pointing arrows carry μ_n = μ min(n, R) + ξ(n − R)⁺. The key feature is that the leftward arrows grow faster than linearly once n exceeds R, because reneging adds to repair. The structure follows the birth–death formulation in Gross et al. (2008) and the rate structure in Jain et al. (2000).

4.2 Closed Forms for One Repairman

When R = 1, the death rate is $\mu_k = \mu + (k-1)\xi for k \geq 1$. The weights become

$$w_n = \prod_{k=0}^{n-1} \frac{\lambda_k}{\mu + k\xi}.$$

If there are no spares (S = 0, so M = N) and no balking, $\lambda_k = \lambda(N-k)$, and the product can be written with gamma functions:

$$w_n = \left(\frac{\lambda}{\xi}\right)^n, \frac{N!}{(N-n)!}, \frac{\Gamma(\mu/\xi)}{\Gamma(\mu/\xi + n)}.$$

As $\xi \to 0$ the gamma ratio tends to $(\xi/\mu)^n$ and the expression collapses to the classical $(\lambda/\mu)^n N!/(N-n)!$.That's a good sanity check. Larger ξ shrinks the denominator's growth and pulls probability mass toward lower states.

4.3 Performance Measures

The expected number of operating machines and the line availability are

$$E[O] = \sum_{n=0}^{N} \min(M, N-n), p_n, \qquad A = \frac{E[O]}{M}.$$

The expected shortage, in machines below the target, is

$$E[B] = \sum_{n=S+1}^{N} (n-S), p_n = M(1-A) \quad \text{when } b_n \equiv 1.$$

Three flows matter. The effective failure rate, the in-house repair throughput and the reneging flow are

$$\lambda_{\text{eff}} = \sum_n \lambda_n p_n, \qquad \theta = \mu \sum_n \min(n, R), p_n, \qquad r = \xi \sum_n (n-R)^+ p_n.$$

In steady state they satisfy $\lambda_e ff = \theta + r$, which is a flow-conservation check any implementation should pass. The fraction of failures that leave for the vendor is $P_r en = r/\lambda_e ff$. With L = Σ n p_n, Little's (1961) law gives the mean time a failed unit spends in the in-house system, including time before it reneges:

$$W = \frac{L}{\lambda_{\text{eff}}}.$$

Failures don't arrive at a constant rate, so an arriving failure doesn't see the time-average distribution. It sees the arrival-point distribution

$$\pi_n = \frac{\lambda_n p_n}{\lambda_{\text{eff}}},$$

and that distribution is the one that matters in Section 5.

V. Patience, Deadlines and the Chance of In-House Repair

5.1 A Tagged-Unit Argument

Pick one failed unit and follow it. Suppose it arrives and finds k units ahead. With R = 1, one of them is in service and k − 1 are waiting. While the tagged unit waits, the count ahead drops at rate μ + (k − 1)ξ, from a service completion or from a waiting unit ahead losing patience. The tagged unit itself reneges at rate ξ. The count falls before the tagged unit gives up with probability $(\mu + (k-1)\xi)/(\mu + k\xi)$.

Chain those steps from k down to zero and the product telescopes:

$$P_k = \prod_{j=1}^{k} \frac{\mu + (j-1)\xi}{\mu + j\xi} = \frac{\mu}{\mu + k\xi}.$$

For general R and k ≥ R, the same argument gives

$$P_k = \frac{\mu R}{\mu R + (k - R + 1), \xi},$$

and $P_k = 1$ when k < R. I like this result because it says plainly what impatience does. The chance of in-house repair doesn't decay geometrically with queue length. It decays like a hyperbola, and the ratio ξ/μ sets the scale.

5.2 Consistency with the Flow Balance

For R = 1 the reneging fraction must satisfy

$$P_{\text{ren}} = \sum_n \pi_n , (1 - P_n) = \sum_n \pi_n , \frac{n\xi}{\mu + n\xi}.$$

Check it against the flow r. Detailed balance gives $\lambda_n p_n = (\mu + n\xi) p_{n+1}$. Multiply the n-th term of the sum by λ_eff and substitute:

$$\lambda_{\text{eff}} P_{\text{ren}} = \sum_n \lambda_n \, p_n, \frac{n\xi}{\mu + n\xi} = \xi \sum_n n \, , p_{n+1} = \xi \sum_{m \geq 1} (m-1) \, , p_m = r.$$

Both routes land on the same quantity. When I first coded this model, the identity caught an off-by-one in my queue-length indexing, so it's more than decoration.

5.3 The Marginal Value of Patience

How does shortage respond to ξ? Take R fixed and define

$$g_n = \sum_{k=1}^{n} \frac{(k - R)^+}{\mu_k}.$$

Since $\partial \ln w_n / \partial \xi = -g_n$, the normalized probabilities satisfy

$$\frac{\partial p_n}{\partial \xi} = -, p_n \left(g_n - \sum_j p_j \, g_j \right).$$

Let b(n) = (n − S)⁺. Then

$$\frac{\partial E[B]}{\partial \xi} = -, \mathrm{Cov}\big(b(N), , g(N)\big) \leq 0,$$

where N is distributed according to the steady state. Both b(n) and g_n are nondecreasing in n, so the covariance is nonnegative and shortage can only fall as patience shortens. That's the mathematical form of the statement that outsourcing helps the line.

It doesn't follow that outsourcing helps the budget. Every unit that reneges costs c_o at the vendor. The derivative of the outsourcing cost with respect to ξ has no sign in general, and the total can rise or fall. That's why the cost function in the next section carries both terms.

VI. Joint Decisions on Spares, Repair Crew and Deadline

6.1 The Cost Rate

Let $c_s$be the holding cost of a spare per unit time, $c_r$the cost of a repairman, $c_d$the cost of one machine-unit of shortage per unit time and c_o the cost of one outsourced repair. The long-run cost rate is

$$C(S, R, \xi) = c_s S + c_r R + c_d, EB + c_o, r(S, R, \xi).$$

The variable ξ = 1/d is the inverse of the mean time the plant is willing to wait. Unlike S and R, it costs nothing to set. A supervisor decides it by policy. The search space is small: S and R are single-digit integers and ξ can be scanned over a grid. The first-order condition in ξ is dC/dξ = 0, which can be solved numerically using the derivative from Section 5.3 for the shortage term.

6.2 A Worked Example

The figures below aren't plant records. I computed them from the model to show how the pieces behave. Take M = 2, S = 1, R = 1, λ = 1 and μ = 4, with b_n = 1. Then N = 3, $\lambda_0 = \lambda_1 = 2$, $\lambda_2 = 1$, $\mu_1 = 4$, $\mu_2 = 4 + \xi$ and $\mu_3 = 4 + 2\xi$.

Set ξ = 2. The weights are 1, 1/2, 1/6 and 1/48, which sum to 27/16. The steady-state probabilities are therefore

$$p = \left(\frac{16}{27}, \frac{8}{27}, \frac{8}{81}, \frac{1}{81}\right) \approx (0.5926,\ 0.2963,\ 0.0988,\ 0.0123).$$

Then E[O] = 152/81 ≈ 1.8765 and A = 76/81 ≈ 0.9383. The reneging flow is r = 20/81 ≈ 0.2469, and $\lambda_e ff = 152/81$. So P_ren = 5/38 ≈ 0.1316. The in-house repair throughput is θ = 44/27 ≈ 1.6296, and θ + r = 152/81 = $\lambda_e ff$, as it should. Finally, L = 43/81 and W = 43/152 ≈ 0.283.

As a second check, use Section 5. The arrival-point distribution is π = (96, 48, 8)/152 for n = 0, 1, 2. The reneging fraction is (48/152)(1/3) + (8/152)(1/2) = 5/38. It matches.

Figure 2 shows how A and $P_r en$ move together as ξ rises.

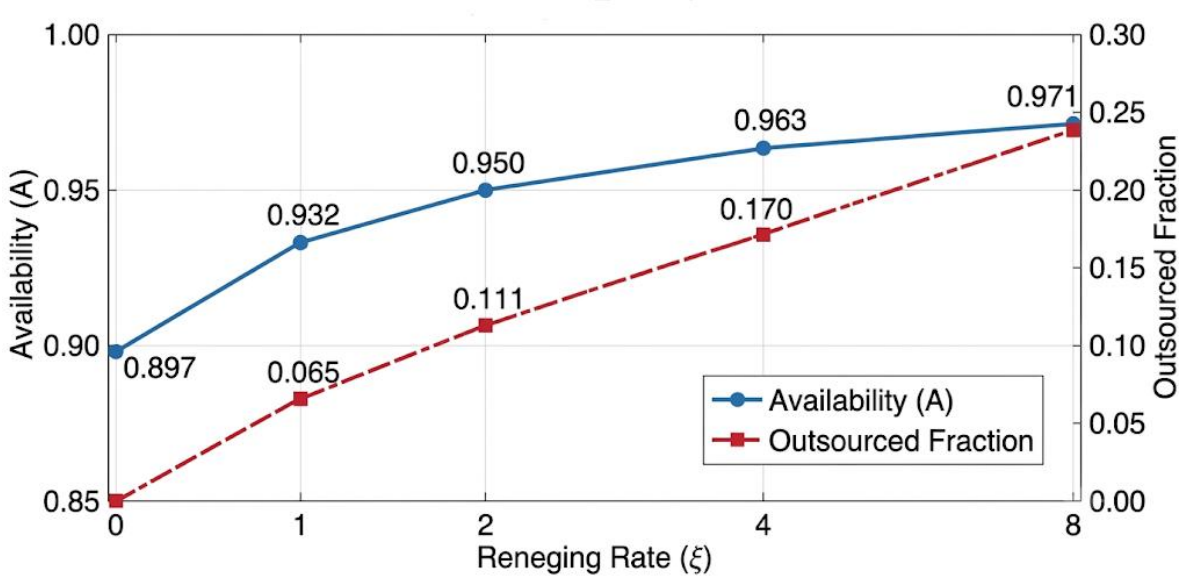


Figure 2: Computed Availability and Outsourced Fraction versus Reneging Rate (M = 2, S = 1, R = 1, λ = 1, μ = 4)

A line chart with the reneging rate ξ on the horizontal axis at values 0, 1, 2, 4 and 8. Two curves share the plot: availability A rising from 0.897 to 0.971, and the fraction of failures outsourced rising from 0 to about 0.238. All values were computed by the author from the birth–death model in Gross et al. (2008) and the rate structure of Jain et al. (2000), and aren't measured plant data. The takeaway is that the gain in availability slows while the share of work sent outside keeps climbing.

The full set of computed values is: A = 0.897, 0.923, 0.938, 0.955, 0.971 and $P_r en = 0{,}0.083$, 0.132, 0.187, 0.238 at ξ = 0, 1, 2, 4, 8. Mean time in the system W falls from 0.365 at ξ = 0 to 0.283 at ξ = 2.

6.3 Reading the Cost Curve

Now add costs. Let $c_d = 10$ and $c_o = 3$ per outsourced repair, and leave aside $c_s S + c_r R$, which doesn't change with ξ. The variable part of the cost rate is then 2.069, 2.000, 1.975, 1.962 and 1.963 at ξ = 0, 1, 2, 4 and 8. These are illustrative unit costs. Going from no reneging to ξ = 4 saves about 5 percent. The curve flattens near ξ = 4 to 8, and it would probably turn upward beyond that, because each extra point of patience loss buys less shortage reduction (Section 5.3) while the vendor bill keeps growing.

I'd draw two lessons. The first is that the optimum is flat, so a rough deadline policy is nearly as good as a precise one. The second is that the answer depends on the ratio c_o/c_d. If the vendor charges ten times what I assumed, the optimum slides toward ξ = 0.

There's also a point about fluid thinking. The traffic intensity here is $M\lambda/(\mu R) = 0.5$. A deterministic fluid model says the crew is never swamped, so nothing should renege. The exact model says 13 percent of failures do. With only three units, the randomness of a few overlapping failures matters more than the average load. This is the usual warning that small populations behave differently from large ones.

VII. Extensions

7.1 N-Policy Start-Up and Impatience

Jain et al. (2004) studied an M/G/1 repair model with an N-policy: the crew stays away until N failures have accumulated, then works until the system empties. The idea goes back to Yadin and Naor (1963), and Heyman (1968) proved optimality results for such control policies. With a start-up cost K per busy cycle, a holding cost h per failed unit per unit time, and Poisson failures at rate λ_f, the N-dependent part of the cost is

$$f(N) = h,\frac{N-1}{2} + \frac{K\lambda_f}{N}, \qquad N^* = \sqrt{\frac{2K\lambda_f}{h}}.$$

Impatience changes this. A unit that's waiting while the crew is away reneges at rate ξ. Take the j-th unit in a batch of N. It must survive N − j further interarrival times, and it survives each with probability $q = \lambda_f/(\lambda_f + \xi)$. Averaging over j = 1, …, N gives the mean survival probability $(1 - q^N)/(N(1-q))$, so the fraction lost before the crew returns is

$$\ell(N) = 1 - \frac{(\lambda_f + \xi),(1 - q^N)}{N,\xi}.$$

At N = 1 this is 0, as it should be. For $\lambda_f = 1$, ξ = 2 and N = 2, we get q = 1/3 and ℓ = 1/3. The augmented cost is $f(N) + c_o\lambda_f\ell(N)$, and the optimal N is now smaller than N*. The reason is simple: waiting units leak out, so the crew should come sooner. I treat arrivals as Poisson here, which is an approximation for a finite source.

7.2 Two Failure Modes and General Repair Times

Jain (2001) allowed two failure modes with rates λ_1 and λ_2 and repair rates μ_1 and μ_2. A failed unit is of type 1 with probability $q = \lambda_1/(\lambda_1 + \lambda_2)$, so the repair time has moments

$$E[T] = \frac{q}{\mu_1} + \frac{1-q}{\mu_2}, \qquad E[T^2] = \frac{2q}{\mu_1^2} + \frac{2(1-q)}{\mu_2^2}.$$

The squared coefficient of variation is $c^2 = E[T^2]/E[T]^2 - 1$. If $\mu_1 = 10\mu_2$ and q = 0.5, the coefficient is well above 1, and an exponential assumption understates waiting. For a single repairman and Poisson arrivals, the Pollaczek–Khinchine formula (Takács, 1962; Kleinrock, 1975) gives

$$W_q = \frac{\lambda,E[T^2]}{2(1-\rho)}, \qquad \rho = \lambda E[T].$$

Variability pushes more failed units past their patience limit. A shop with mixed quick and slow repairs should expect more reneging than the exponential model predicts, even when the mean repair time is the same.

7.3 State-Dependent Failure Rates

The assumption of constant λ is convenient. Barlow and Proschan (1965) describe wear-out through an increasing hazard rate, for example the Weibull form

$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}, \qquad \beta > 1.$$

Jardine et al. (2006) review how condition-monitoring systems estimate where a machine sits on such a curve. If machines are grouped into degradation classes with failure rates $\lambda^{(1)} < \lambda^{(2)} < \ldots$, the state becomes the vector of class counts, and the balance equations keep the same form on a larger state space. Jain and Baghel (2001) already treat state-dependent rates for a multi-component system, so the machinery exists. A machine in a high-wear class is more likely to fail soon and, in a time-critical line, more likely to trigger outsourcing (Baghel, 2013; 2014; 2016; 2017).

VIII. Limits of the Analysis

I'd rather be direct about what the model leaves out.

Exponential patience is the first compromise. A real deadline is closer to a fixed number than to an exponential variable. An exponential variable has a constant hazard rate, so a unit that has already waited a long time is as likely to leave as a fresh arrival. A deterministic deadline would cause no reneging before d and certain reneging at d. Zeltyn and Mandelbaum (2005) handle general patience for large systems, and a small finite-source version with Erlang patience is a natural next step.

The second compromise is the vendor. I let reneged units return instantly, which keeps the algebra clean but hides the vendor's turnaround time. A longer round trip would remove the unit from the population for a while and lower availability. It would be easy to add by making the vendor an infinite-server stage.

Third, the parameters have to come from data. Estimating λ and μ from work-order logs is easy to say and hard to do, because logs often record when a ticket closed and not when the repair ended. Patience is even harder, since nobody records it directly. I'd estimate ξ from the observed rate at which tickets are diverted, using $r = \xi\, E[(n - R)^+]$ to back it out.

Fourth, the cost coefficients drive the result. The optimal ξ in Section 6.3 moved with the ratio c_o/c_d, and in practice c_d is the least known of the numbers. Managers should spend more effort on that figure than on refining the queueing model.

IX. Conclusion

I started with a simple question: what changes when a failed machine can't wait for the crew? The model's answer is that it gains an exit, and the exit has a price. A finite-source birth–death chain with reneging keeps the product-form structure, and for one repairman the weights reduce to a product over $(\mu + k\xi)$. Availability, reneging flow and sojourn time follow as sums over the steady-state probabilities.

Two results seem worth keeping. The first is that a unit with k others ahead of it is repaired in-house with probability $\mu/(\mu + k\xi)$, a hyperbolic decay that ties the queue length directly to the outsourcing share. The second is that $\partial E[B]/\partial\xi$ equals minus a covariance, so shortening patience always helps availability and the real question is cost.

The worked example (M = 2, S = 1, R = 1, λ = 1, μ = 4) makes the trade concrete. Moving from ξ = 0 to ξ = 2 raises availability from 0.897 to 0.938, with 13.2 percent of failures going to the vendor. Mean time in the system drops from 0.365 to 0.283. With illustrative costs of $c_d = 10$ and $c_o = 3$, the cost curve flattens between ξ = 4 and ξ = 8, so a rough deadline policy gets most of the savings. These numbers are computed from the model and aren't plant measurements.

The extensions show that the structure holds. An N-policy with impatience calls for a smaller batch than the classical square-root rule. Two failure modes inflate repair-time variability and push more units past their limit. State-dependent failure rates connect the model to condition monitoring.

What I'd want next is an empirical test. Fit λ, μ and ξ from a real plant's tickets, solve for (S, R, ξ), and compare the result with current practice. The mathematics is ready for that. The data usually isn't.

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