



Low Density Distribution of 2-Primefactors Numbers Till 1 Trillion

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ABSTRACT: Positive integer greater than 1 having no positive divisors other than 1 and itself is a prime number. Author coins a new term 'k-PrimeFactors Number' for a positive integer having exactly k prime divisors (not necessarily distinct). In this work, the number of 2-PrimeFactors numbers in ranges of increasing powers of 10, like ten, hundred, thousand, ten thousand, lakh, million, ten million, hundred million, billion, ten billion, hundred billion and trillion are presented along with first and last 2-PrimeFactors numbers within them. To inspect their density distribution, minimum number of 2-PrimeFactors numbers in these ranges is determined. For all blocks of these sizes till 1 trillion, minimum number of 2-PrimeFactors numbers in each sized block, first and last blocks of minimum number of 2-PrimeFactors numbers and number of blocks with minimum number of 2-PrimeFactors numbers are determined within increasing ranges.

Keywords - Prime number, k-PrimeFactors number, 2-PrimeFactors number, Low density distribution

Mathematics Subject Classification (2010) - 11A51, 11N05, 11N80

I. INTRODUCTION

The primes

2, 3, 5, 7, 11, 13, 17, ...

have been the core of number theory study. Each prime number has only 2 positive divisors, viz., 1 & itself and no other [1]. The rigorous analysis of primes and their distribution in higher ranges has been done in recent past [3]. Their special types like twin primes are also under study [4]. These endeavors could be possible owing to wise choice of fast & efficient algorithms that generate all primes in given ranges [2].

II. K-PRIMEFACTORS NUMBERS

The author coins a new term here.

Definition (k-PrimeFactors Number) : For any integer $k \geq 0$, a positive integer greater than 1 having k number of prime factors, which need not be necessarily distinct, is called as k-PrimeFactors number.

Since there are infinite number of primes, there are infinite types of k-PrimeFactors numbers : one type for each positive integral value of k, viz., 1, 2, 3, ...

Naturally enough, 1-PrimeFactor numbers are nothing else but usual primes. So, k-PrimeFactors numbers are generalization of prime numbers.

Each positive integer, including 1, is of one and only one type of k-PrimeFactors number for a unique k. Very first positive integer 1 has no prime factors and hence is a 0-PrimeFactors number. In fact, 1 is the only 0-PrimeFactors number.

III. PRIMEFACTORS NUMBERS

Taking specific value of k to be equal to 2, we undertake study of 2-PrimeFactor numbers here.

Definition (2-PrimeFactors Number) : A positive integer having 2 prime factors, which need not be necessarily distinct, is called as 2-PrimeFactors number.

List of first few 2-PrimeFactors numbers is as follows :

4, 6, 9, 10, 14, 21, ...

Of these, $4 = 2^2$, $6 = 2 \times 3$, $9 = 3 \times 3$, $10 = 2 \times 5$, $14 = 2 \times 7$, $21 = 3 \times 7$, and so on.

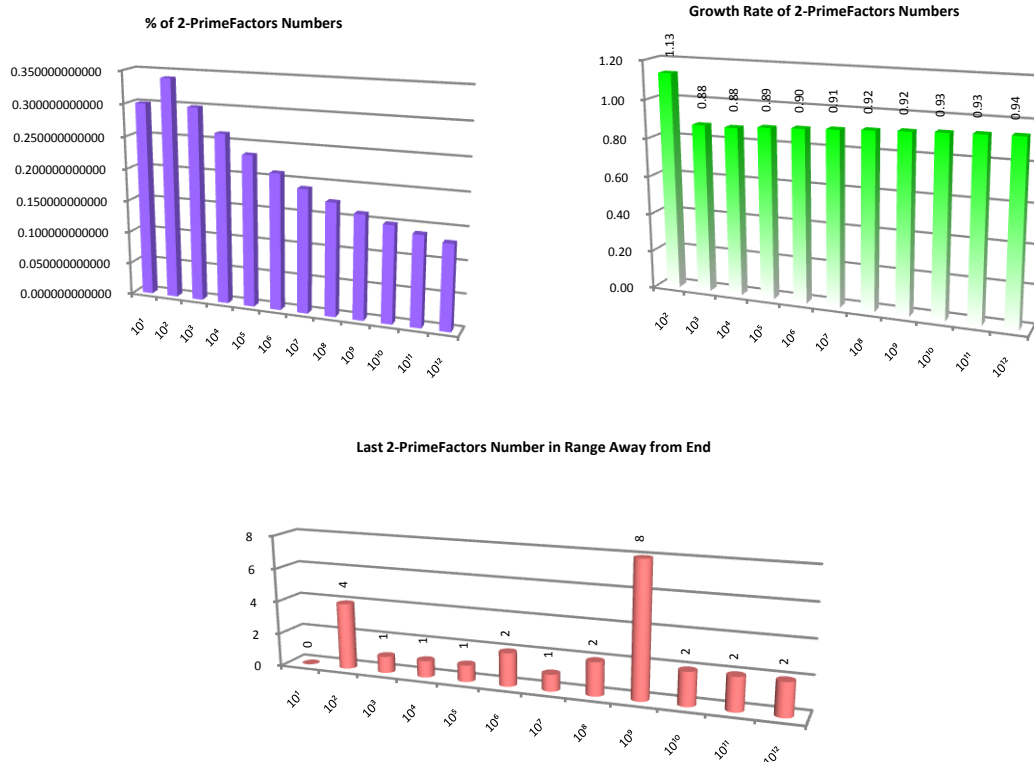
Determination of 2-PrimeFactors numbers first requires determination of usual primes, i.e., 1-PrimeFactor numbers. Out of many methods available for the same, more effective method is selected [2]. Because prime numbers are infinite, 2-PrimeFactors numbers are also infinite. The sequence of 2-PrimeFactors numbers is neither arithmetic nor geometric nor harmonic. As sequence of prime numbers seems random, sequence of 2-PrimeFactors numbers also doesn't fit in standard classified types.

IV. NUMBER OF 2-PRIMEFACTORS NUMBERS TILL 1 TRILLION

Using modern electronic computers and high level programming languages like Java [5], all 2-PrimeFactors numbers till 1 trillion are determined. Their distribution is also analyzed in this work.

Sr. No.	Range	Number of 2-PrimeFactors Numbers in Range	First 2-PrimeFactors Number in Range	Last 2-PrimeFactors Number in Range
1	$<10^1$	3	4	9
2	$<10^2$	34	4	95
3	$<10^3$	299	4	998
4	$<10^4$	2,625	4	9,998
5	$<10^5$	23,378	4	99,998
6	$<10^6$	210,035	4	999,997
7	$<10^7$	1,904,324	4	9,999,998
8	$<10^8$	17,427,258	4	99,999,997
9	$<10^9$	160,788,536	4	999,999,991
10	$<10^{10}$	1,493,776,443	4	9,999,999,997
11	$<10^{11}$	13,959,990,342	4	99,999,999,997
12	$<10^{12}$	131,126,017,178	4	999,999,999,997

Naturally, their count increases with the increasing range. Their number keeps growing till 1 trillion at a slower rate. The first 2-PrimeFactors number is 4, for 10 power blocks. The last 2-PrimeFactors number in each increasing 10 power range keeps distances from the corresponding end values as shown in last graph.

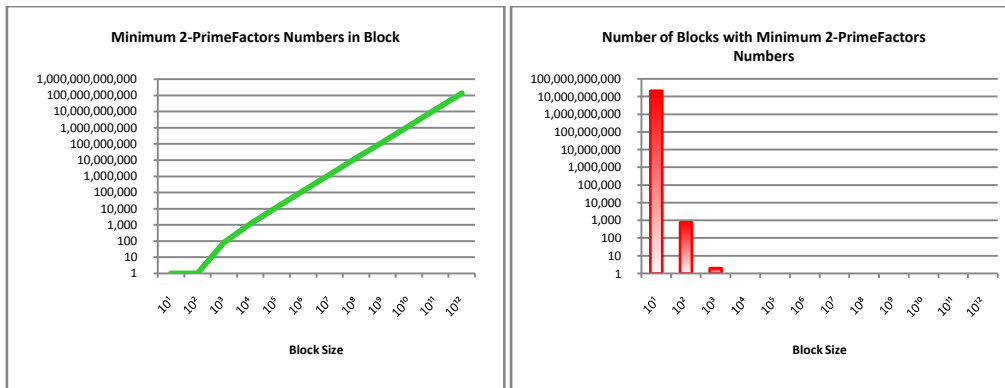


V. MINIMUM NUMBER OF 2-PRIMEFACTORS NUMBERS IN BLOCKS OF SIZES 10^N

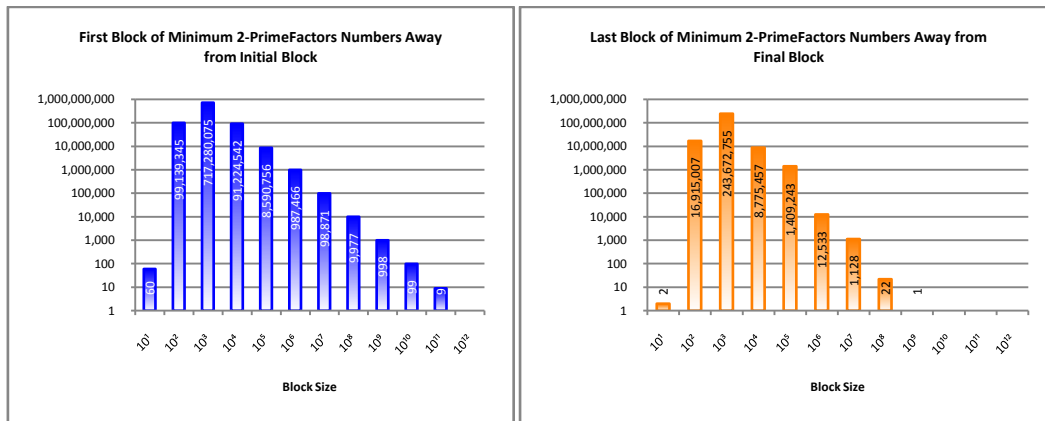
Encompassing the range of all number till 1 trillion and selecting blocks of sizes of 10 powers like, 10, 100, 1000 and so on till 1000000000000 itself, the minimum number of 2-PrimeFactors numbers in these blocks, first and last blocks containing such minimum 2-PrimeFactors numbers and number of such blocks are determined to be as follows.

Sr. No.	Block-Size	Minimum 2-PrimeFactors Numbers in Block	First Block of Minimum 2-PrimeFactors Numbers	Last Block of Minimum 2-PrimeFactors Numbers	Number of Blocks with Minimum 2-PrimeFactors Numbers
1	10^1	0	600	999,999,999,970	21,182,218,841
2	10^2	0	9,913,934,500	998,308,499,200	770
3	10^3	79	717,280,075,000	756,327,244,000	2
4	10^4	1,138	912,245,420,000	912,245,420,000	1
5	10^5	12,380	859,075,600,000	859,075,600,000	1
6	10^6	126,678	987,466,000,000	987,466,000,000	1
7	10^7	1,274,694	988,710,000,000	988,710,000,000	1
8	10^8	12,762,762	997,700,000,000	997,700,000,000	1
9	10^9	127,689,890	998,000,000,000	998,000,000,000	1
10	10^{10}	1,277,071,583	990,000,000,000	990,000,000,000	1
11	10^{11}	12,785,851,504	900,000,000,000	900,000,000,000	1
12	10^{12}	131,126,017,178	0	0	1

As expected, the minimum number of 2-PrimeFactors numbers in blocks increases with increasing block size. But there is steep decrease in number of increasing sized blocks containing respective minimum number of 2-PrimeFactors numbers in them.



Till 1 trillion, the block distances of various-sized first and last blocks of minimum numbers from starting and ending blocks respectively are given below.



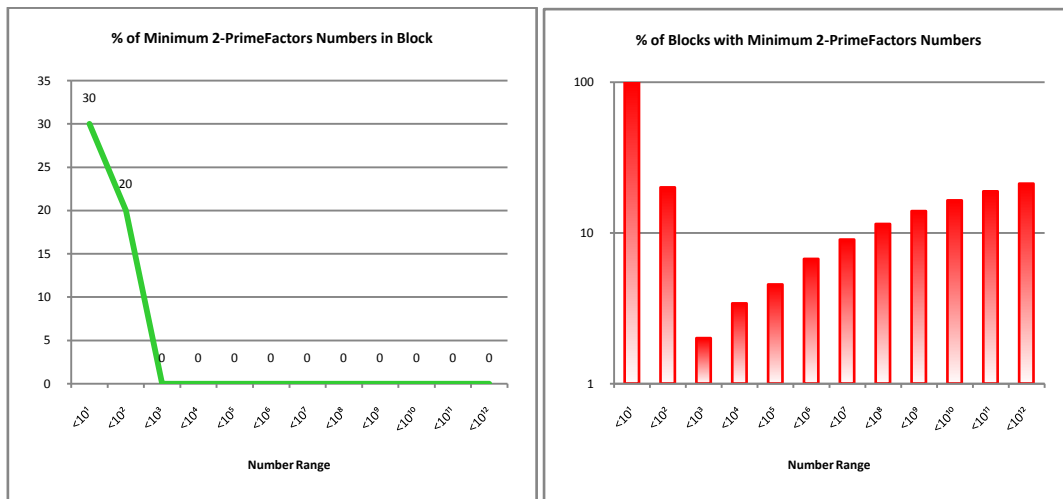
Next we take up the block-wise analysis within increasing ranges till 1 trillion.

V-1. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10

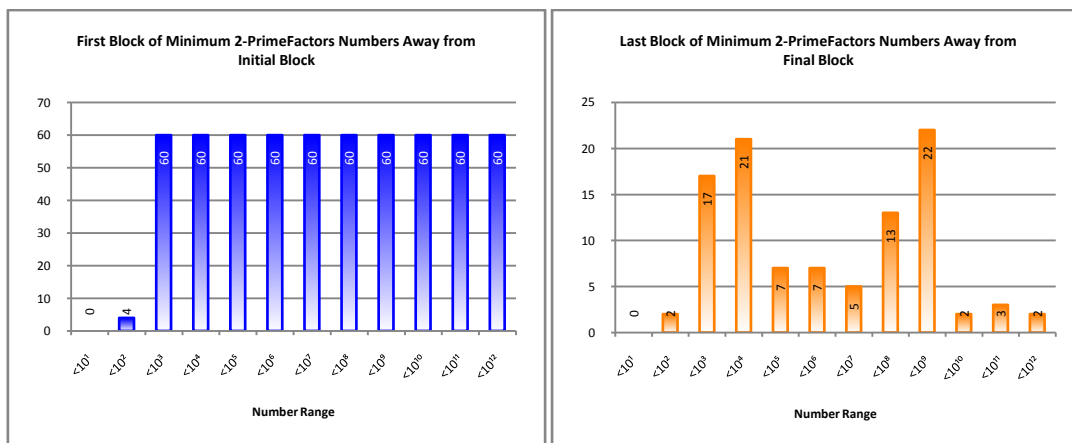
In earlier section, for fixed range of 1 trillion, analysis in different blocks was done. Now, we fix the block size 10 and inspect different ranges of increasing 10 power sizes for occurrence of minimum number of 2-PrimeFactors numbers in blocks of size 10. For this work, block 0 means number range 0 to 9, block 10 means number range 10 to 19 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10-Size Block	First 10-Size Block of Minimum 2-PrimeFactors Numbers	Last 10-Size Block of Minimum 2-PrimeFactors Numbers	Number of 10-Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^1$	3	0	0	1
2	$<10^2$	2	40	70	2
3	$<10^3$	0	600	820	2
4	$<10^4$	0	600	9,780	34
5	$<10^5$	0	600	99,920	454
6	$<10^6$	0	600	999,920	6,706
7	$<10^7$	0	600	9,999,940	90,130
8	$<10^8$	0	600	99,999,860	1,143,651
9	$<10^9$	0	600	999,999,770	13,922,476
10	$<10^{10}$	0	600	9,999,999,970	164,009,184
11	$<10^{11}$	0	600	99,999,999,960	1,883,088,867
12	$<10^{12}$	0	600	999,999,999,970	21,182,218,841

Except first two ranges of single and double digit numbers, the minimum number of 2-PrimeFactors numbers in blocks of 10 is 0. After initial decrease, the percentage of number of blocks with minimum number of 2-PrimeFactors numbers in them steadily increases.



Beginner minimality block at distance of 60 blocks from initial block for all ranges except first two. The block distance of last such block from corresponding final block fluctuates in increasing ranges.

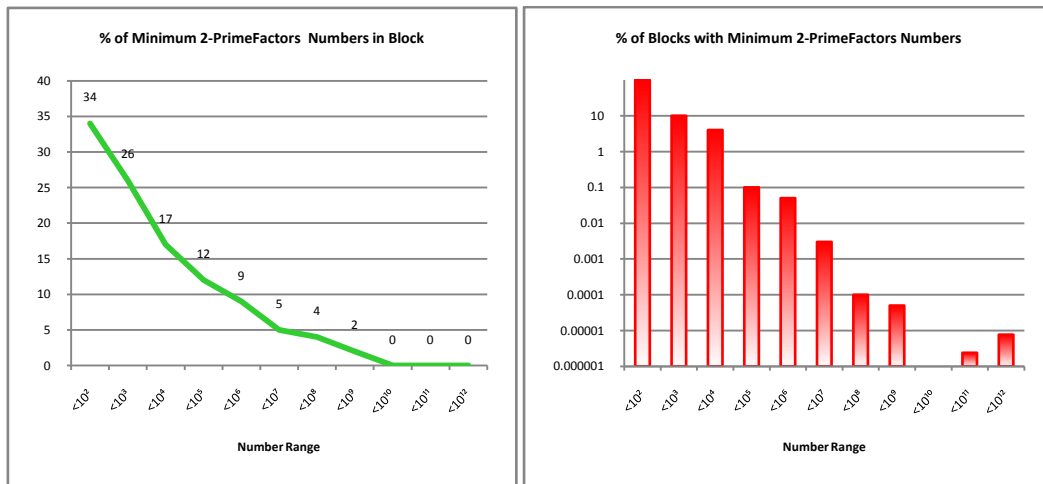


V-2. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^2

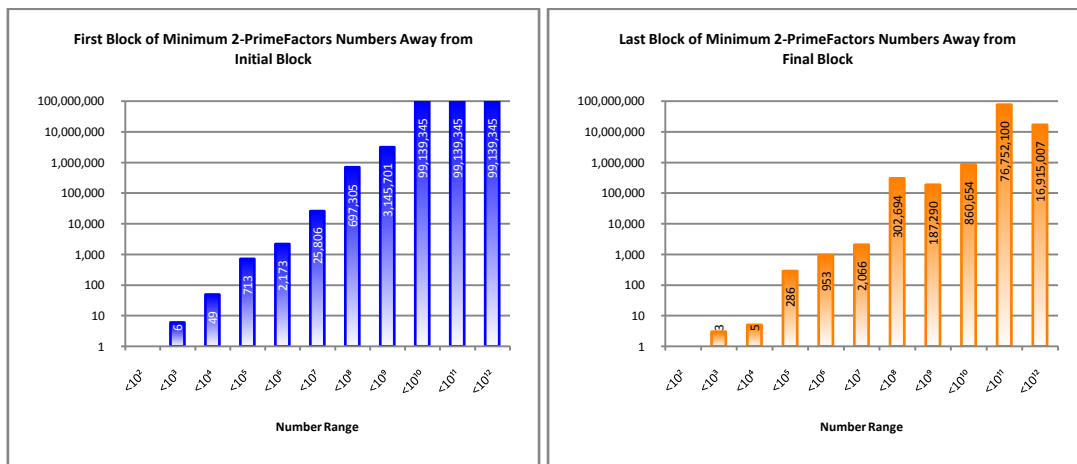
Next we pick up block size of 10^2 , i.e., 100. So, here block 0 means number range 0 to 99, block 100 means number range 100 to 199 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^2 -Size Block	First 10^2 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^2 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^2 -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^2$	34	0	0	1
2	$<10^3$	26	600	600	1
3	$<10^4$	17	4,900	9,400	4
4	$<10^5$	12	71,300	71,300	1
5	$<10^6$	9	217,300	904,600	5
6	$<10^7$	5	2,580,600	9,793,300	3
7	$<10^8$	4	69,730,500	69,730,500	1
8	$<10^9$	2	314,570,100	981,270,900	5
9	$<10^{10}$	0	9,913,934,500	9,913,934,500	1
10	$<10^{11}$	0	9,913,934,500	92,324,789,900	24
11	$<10^{12}$	0	9,913,934,500	998,308,499,200	770

The minimum number of 2-PrimeFactors numbers in blocks of size 10^2 steadily decreases till it settles down to 0 for range till 10^{10} . The percentage of number of minimality blocks also goes down till range of 10^{10} after which it starts increasing.



First and last blocks of minimum 2-PrimeFactors numbers keep increasing block distance from respective initial and final blocks.

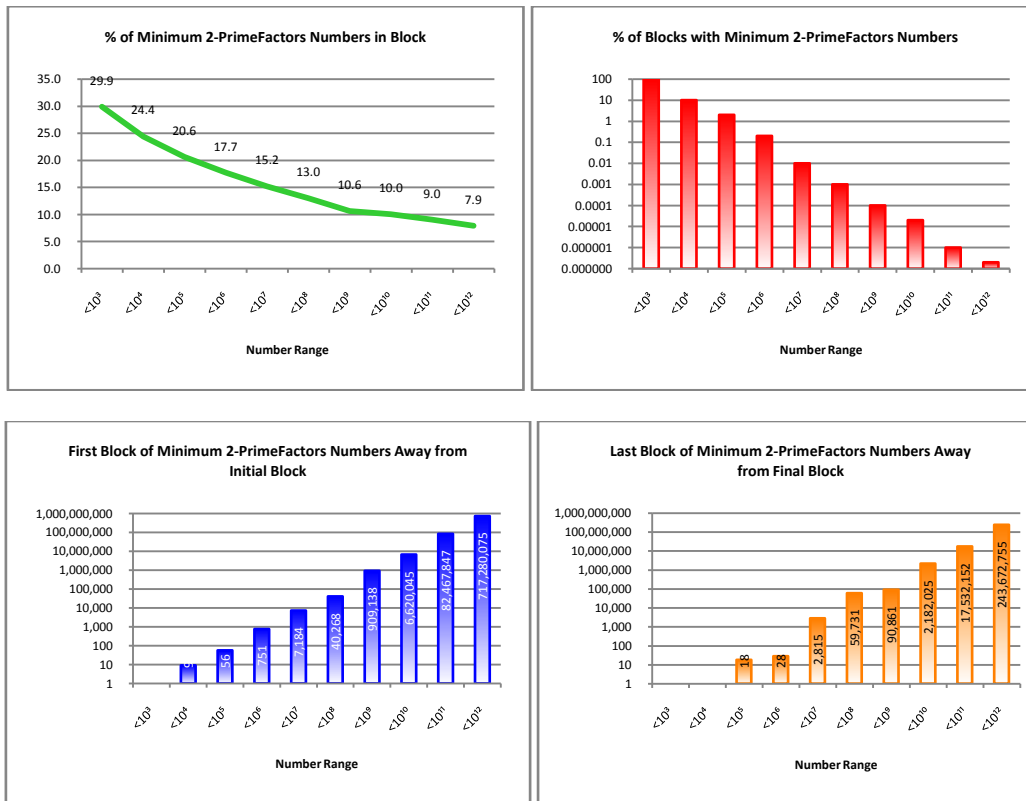


V-3. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^3

Now chosen block size is 10^3 , i.e., 1000, where block 0 means number range 0 to 999, block 1000 means number range 1000 to 1999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^3 -Size Block	First 10^3 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^3 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^3 -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^3$	299	0	0	1
2	$<10^4$	244	9,000	9,000	1
3	$<10^5$	206	56,000	81,000	2
4	$<10^6$	177	751,000	971,000	2
5	$<10^7$	152	7,184,000	7,184,000	1
6	$<10^8$	130	40,268,000	40,268,000	1
7	$<10^9$	106	909,138,000	909,138,000	1
8	$<10^{10}$	100	6,620,045,000	7,817,974,000	2
9	$<10^{11}$	90	82,467,847,000	82,467,847,000	1
10	$<10^{12}$	79	717,280,075,000	756,327,244,000	2

Pattern little similar to previous sized blocks is seen here also.



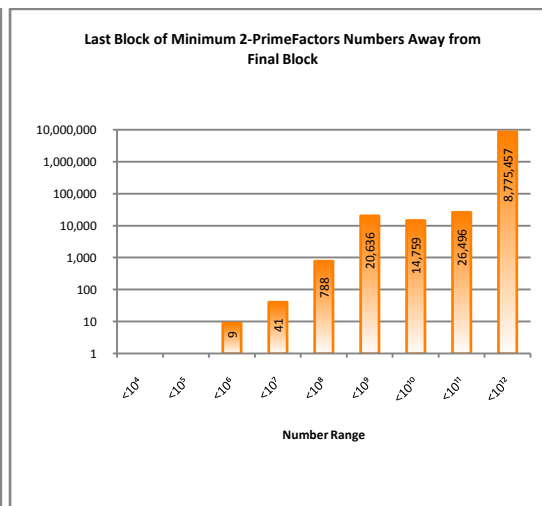
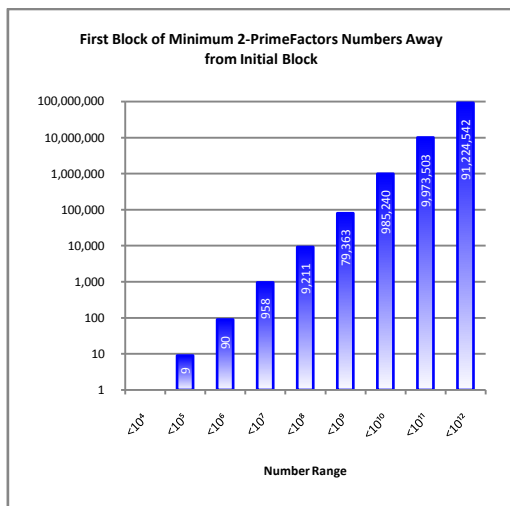
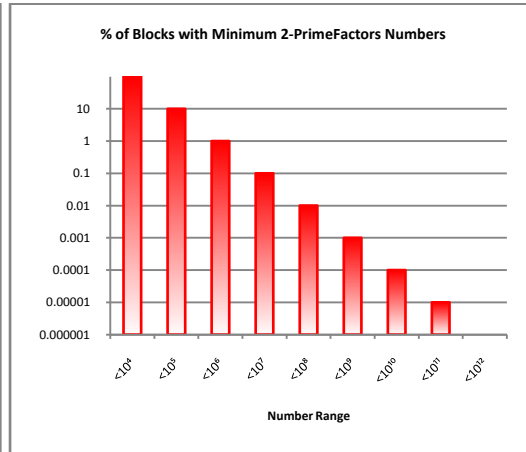
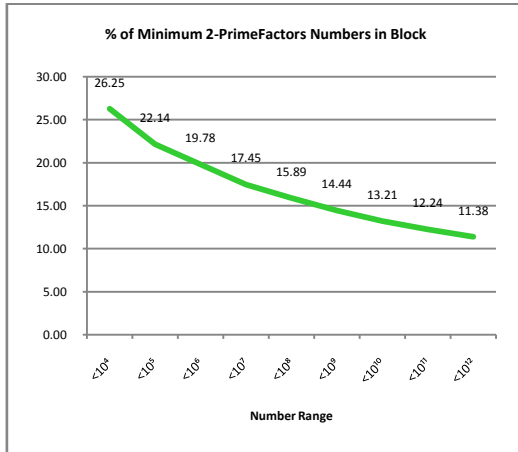
V-4. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^4

Higher block size is 10^4 , i.e., 10000, block 0 meaning number range 0 to 9999, block 10000 meaning number range 10000 to 19999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^4 -Size Block	First 10^4 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^4 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^4 -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^4$	2,625	0	0	1
2	$<10^5$	2,214	90,000	90,000	1
3	$<10^6$	1,978	900,000	900,000	1
4	$<10^7$	1,745	9,580,000	9,580,000	1
5	$<10^8$	1,589	92,110,000	92,110,000	1
6	$<10^9$	1,444	793,630,000	793,630,000	1

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^4 -Size Block	First 10^4 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^4 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^4 -Size Blocks with Minimum 2-PrimeFactors Numbers
7	$<10^{10}$	1,321	9,852,400,000	9,852,400,000	1
8	$<10^{11}$	1,224	99,735,030,000	99,735,030,000	1
9	$<10^{12}$	1,138	912,245,420,000	912,245,420,000	1

Some trends continue more or less.



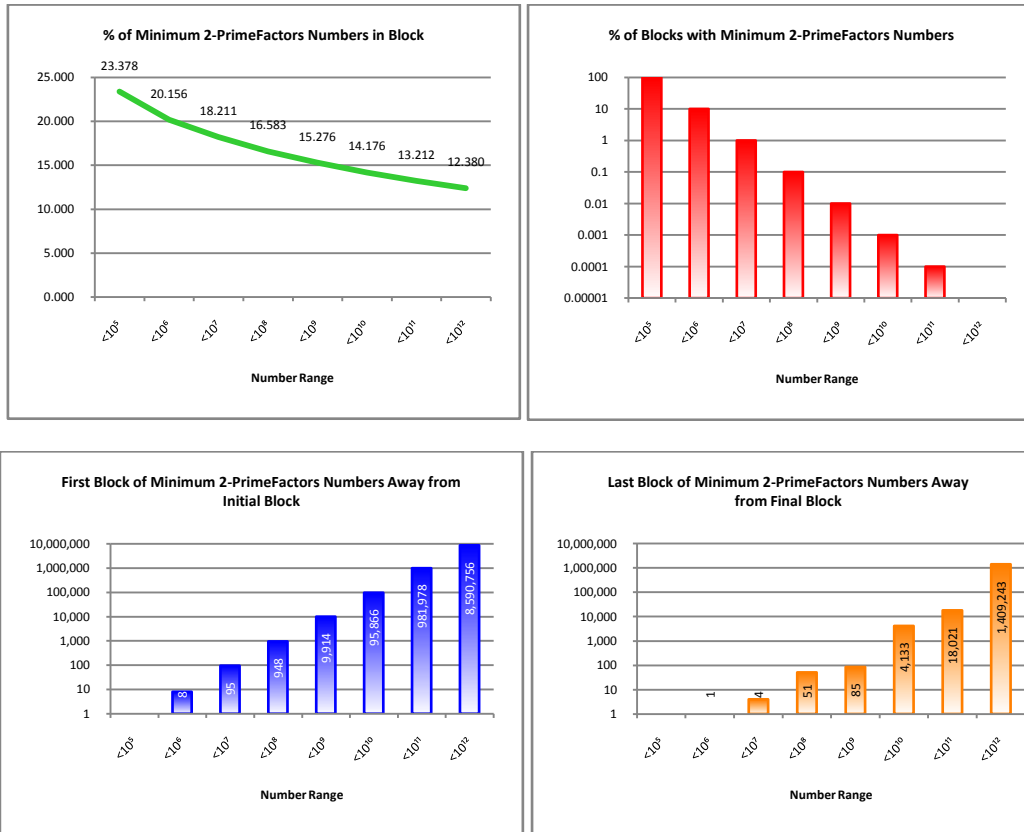
V-5. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^5

Then we consider block size 10^5 , i.e., 100000, block 0 being number range 0 to 99999, block 100000 being number range 100000 to 199999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^5 -Size Block	First 10^5 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^5 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^5 -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^5$	23,378	0	0	1
2	$<10^6$	20,156	800,000	800,000	1
3	$<10^7$	18,211	9,500,000	9,500,000	1
4	$<10^8$	16,583	94,800,000	94,800,000	1
5	$<10^9$	15,276	991,400,000	991,400,000	1
6	$<10^{10}$	14,176	9,586,600,000	9,586,600,000	1
7	$<10^{11}$	13,212	98,197,800,000	98,197,800,000	1
8	$<10^{12}$	12,380	859,075,600,000	859,075,600,000	1

Attributes under our analysis show following patterns.

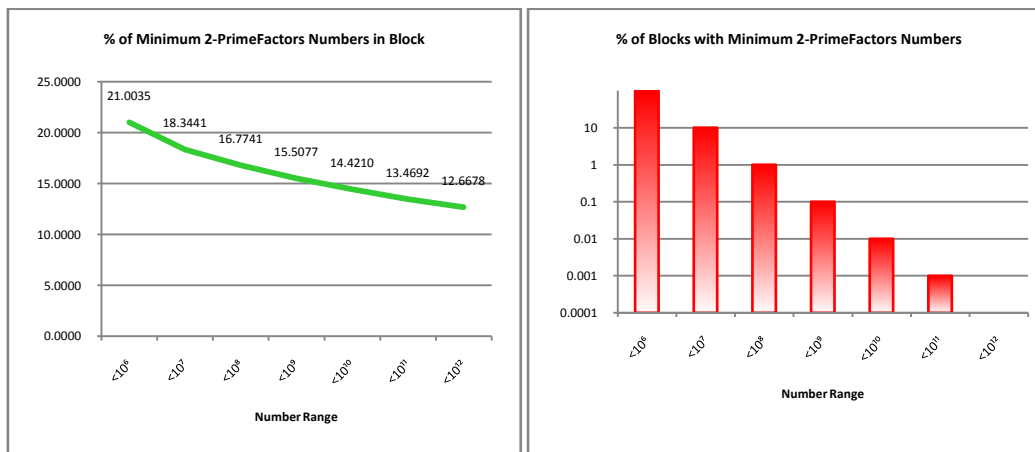
Low Density Distribution of 2-PrimeFactors Numbers till 1 Trillion

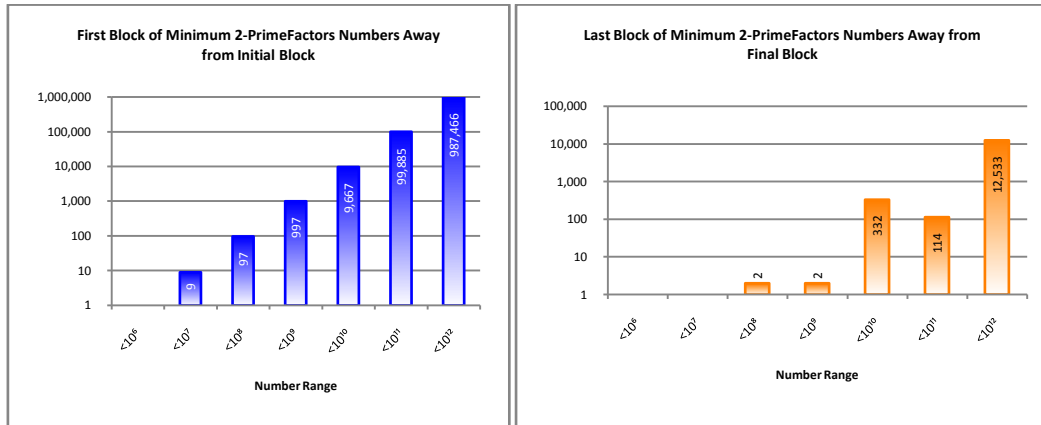


V-6. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10⁶

Further we take up block size 10⁶, i.e., 1000000, block 0 being number range 0 to 999999, block 1000000 being number range 1000000 to 1999999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10 ⁶ -Size Block	First 10 ⁶ -Size Block of Minimum 2-PrimeFactors Numbers	Last 10 ⁶ -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10 ⁶ -Size Blocks with Minimum 2-PrimeFactors Numbers
1	<10 ⁶	210,035	0	0	1
2	<10 ⁷	183,441	9,000,000	9,000,000	1
3	<10 ⁸	167,741	97,000,000	97,000,000	1
4	<10 ⁹	155,077	997,000,000	997,000,000	1
5	<10 ¹⁰	144,210	9,667,000,000	9,667,000,000	1
6	<10 ¹¹	134,692	99,885,000,000	99,885,000,000	1
7	<10 ¹²	126,678	987,466,000,000	987,466,000,000	1

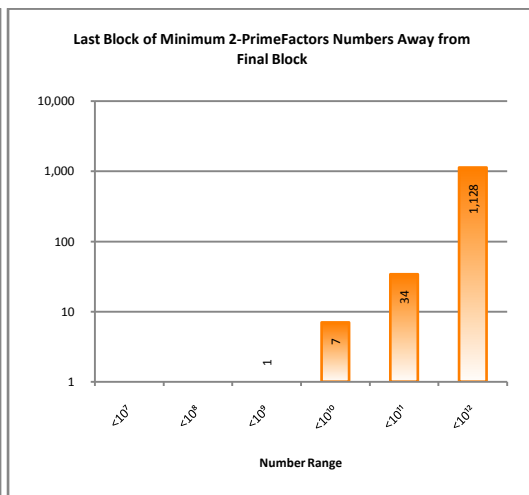
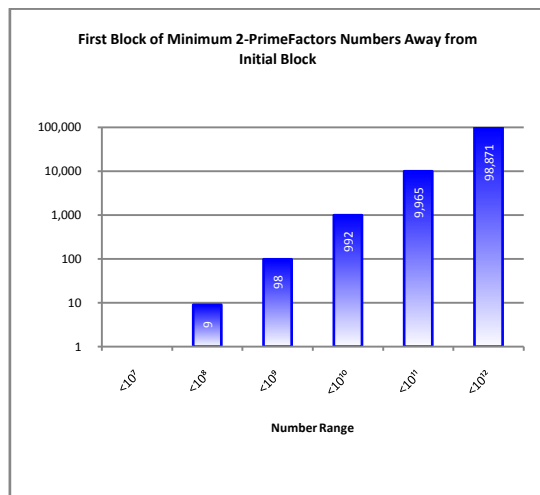
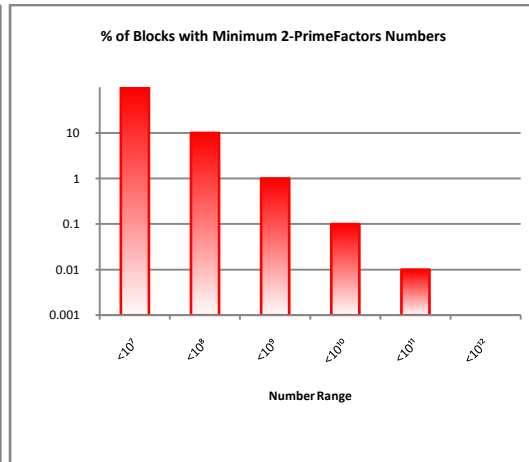
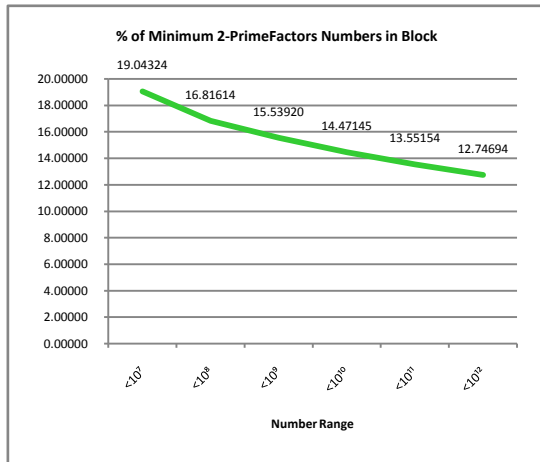




V-7. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^7

Now taking block size 10^7 , i.e., 10000000, block 0 is number range 0 to 9999999, block 10000000 is number range 10000000 to 19999999 and so on.

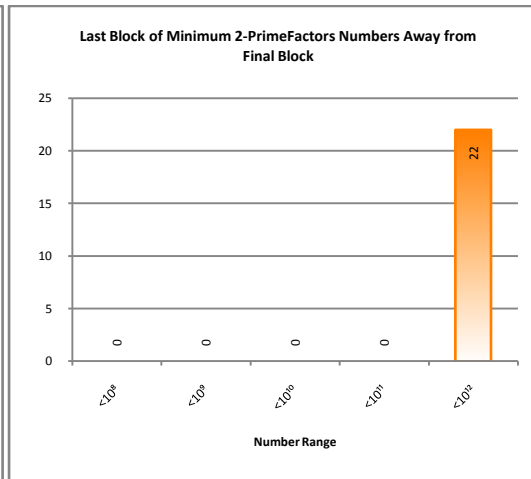
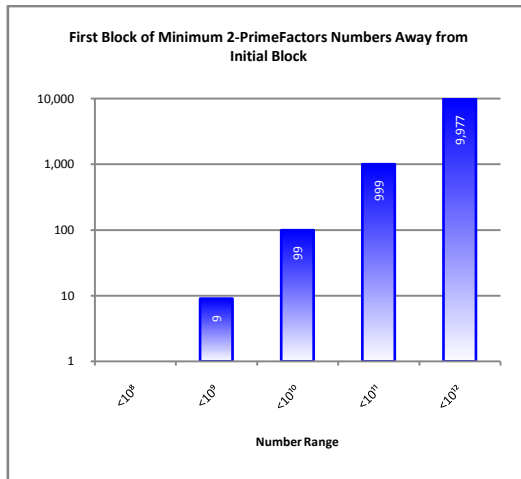
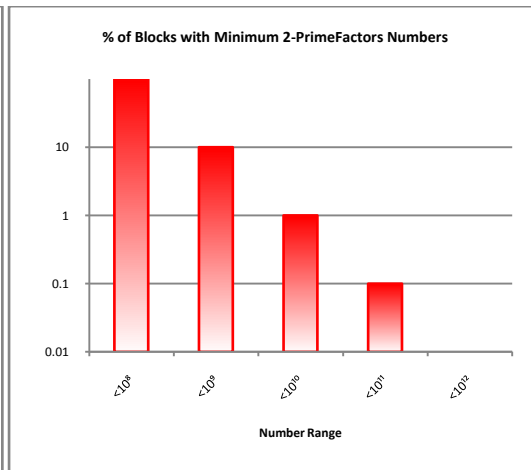
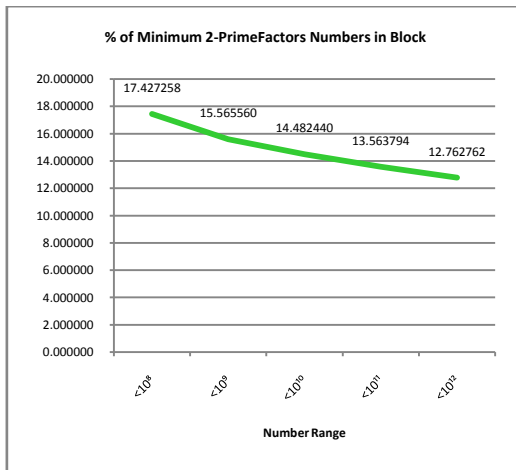
Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^7 -Size Block	First 10^7 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^7 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^7 -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^7$	1,904,324	0	0	1
2	$<10^8$	1,681,614	90,000,000	90,000,000	1
3	$<10^9$	1,553,920	980,000,000	980,000,000	1
4	$<10^{10}$	1,447,145	9,920,000,000	9,920,000,000	1
5	$<10^{11}$	1,355,154	99,650,000,000	99,650,000,000	1
6	$<10^{12}$	1,274,694	988,710,000,000	988,710,000,000	1



V-8. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^8

Higher block size is of 10^8 , i.e., 100000000, block 0 referring to range 0 to 99999999, block 100000000 to range 100000000 to 199999999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^8 -Size Block	First 10^8 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^8 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^8 -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^8$	17,427,258	0	0	1
2	$<10^9$	15,565,560	900,000,000	900,000,000	1
3	$<10^{10}$	14,482,440	9,900,000,000	9,900,000,000	1
4	$<10^{11}$	13,563,794	99,900,000,000	99,900,000,000	1
5	$<10^{12}$	12,762,762	997,700,000,000	997,700,000,000	1

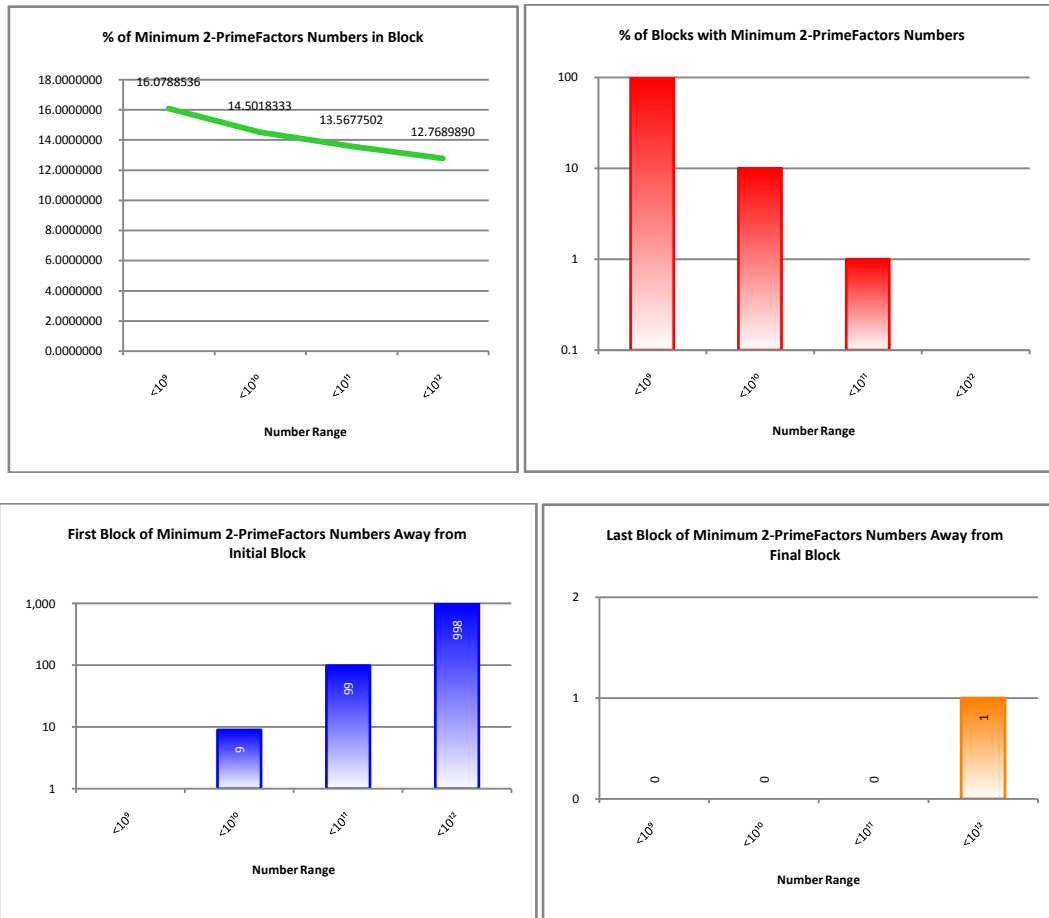


V-9. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^9

Next turn is of block size of 10^9 , i.e., 1000000000, block 0 denoting range 0 to 999999999, block 1000000000 denoting 1000000000 to 1999999999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^9 -Size Block	First 10^9 -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^9 -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^9 -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^9$	160,788,536	0	0	1
2	$<10^{10}$	145,018,333	9,000,000,000	9,000,000,000	1
3	$<10^{11}$	135,677,502	99,000,000,000	99,000,000,000	1
4	$<10^{12}$	127,689,890	998,000,000,000	998,000,000,000	1

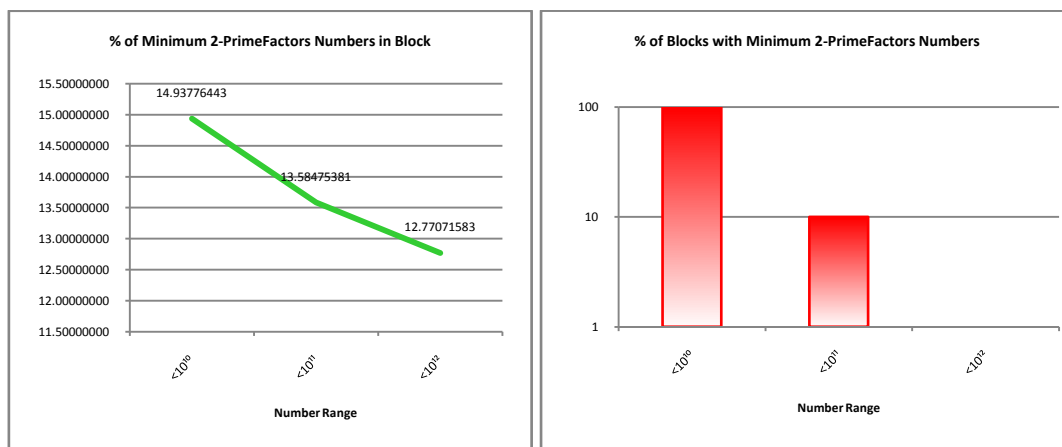
Low Density Distribution of 2-PrimeFactors Numbers till 1 Trillion

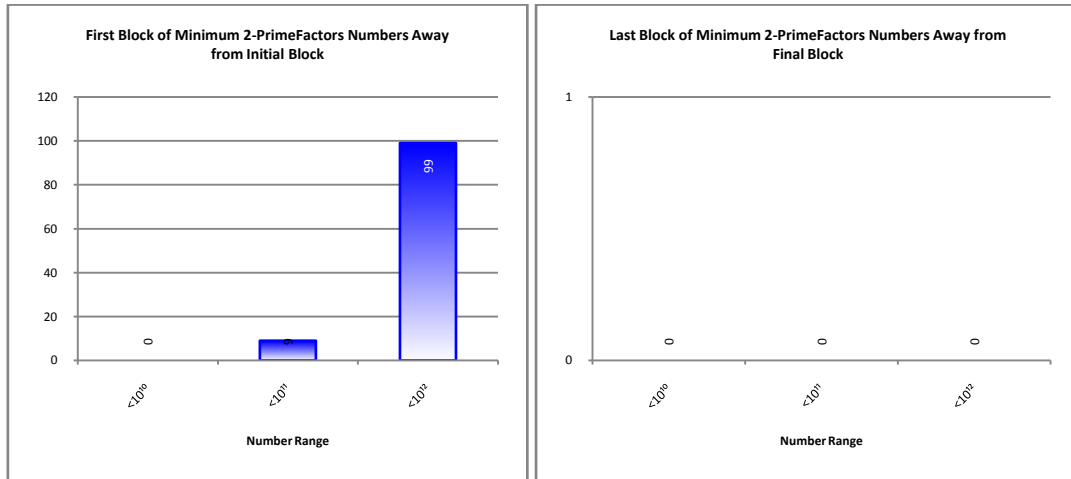


V-10. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^{10}

Next higher block size under consideration is of 10^{10} , i.e., 10000000000, block 0 meaning range 0 to 9999999999, block 10000000000 meaning 10000000000 to 19999999999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^{10} -Size Block	First 10^{10} -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^{10} -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^{10} -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^{10}$	1,493,776,443	0	0	1
2	$<10^{11}$	1,358,475,381	90,000,000,000	90,000,000,000	1
3	$<10^{12}$	1,277,071,583	990,000,000,000	990,000,000,000	1

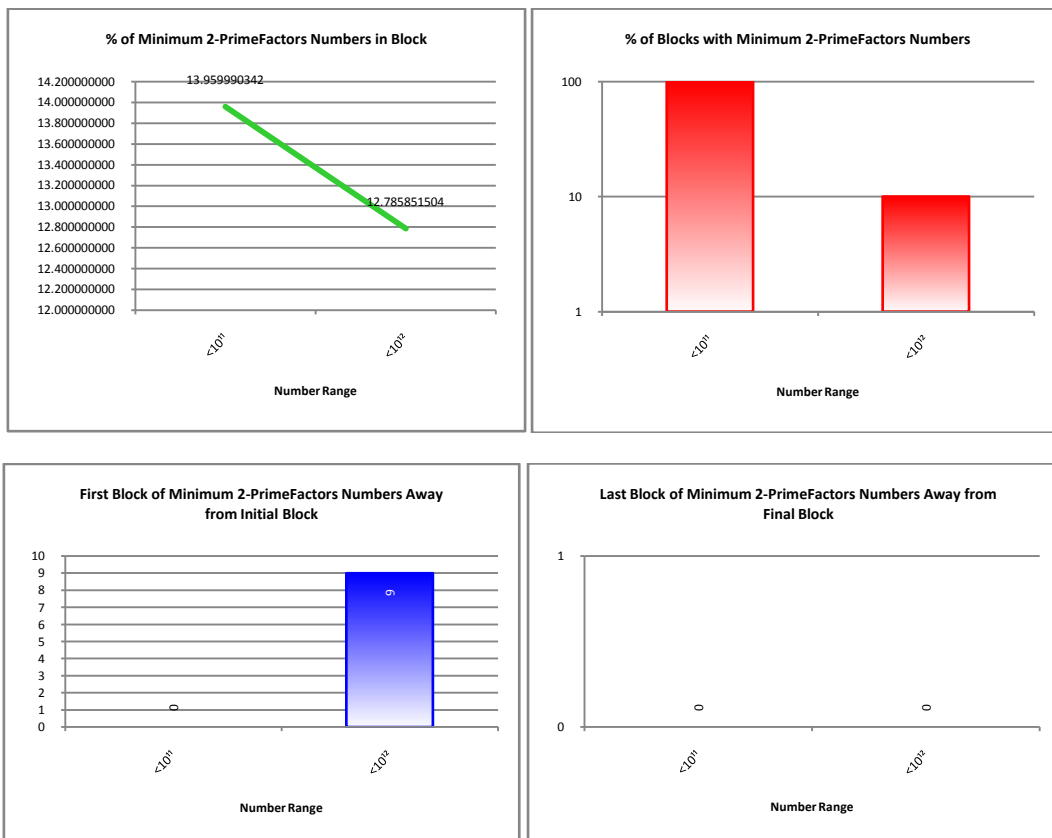




V-11. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^{11}

Next block size is 10^{11} , i.e., 100000000000, where block 0 gives range 0 to 99999999999, block 100000000000 gives range 100000000000 to 199999999999 and so on.

Sr. No.	Range	Minimum 2-PrimeFactors Numbers in 10^{11} -Size Block	First 10^{11} -Size Block of Minimum 2-PrimeFactors Numbers	Last 10^{11} -Size Block of Minimum 2-PrimeFactors Numbers	Number of 10^{11} -Size Blocks with Minimum 2-PrimeFactors Numbers
1	$<10^{11}$	13,959,990,342	0	0	1
2	$<10^{12}$	12,785,851,504	900,000,000,000	900,000,000,000	1



V-12. Minimum Number of 2-PrimeFactors Numbers in Blocks of Size 10^{12}

Our study range is till 1 trillion only. So next block size of 10^{12} , i.e., 1000000000000, which is 1 trillion itself, is the only block of its size under consideration. It happens to be first as well as last block of minimum number of 2-PrimeFactors numbers in it with whatever number of these numbers are in it, viz., 131126017178, to be minimum also!

This study reveals that in-block minimality of 2-PrimeFactors numbers does come in late in almost all sized blocks. Their frequency is also whirls around unity. The implication is that there tend to be lesser blocks with minimum number of 2-PrimeFactors numbers and that too they appear quite late.

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