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Review Paper

Mathematical Formulation of Nonlinear Free Vibration Analysis of Curved Panels

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ABSTRACT
The curved shell panels made of laminated carbon/epoxy composite have been numerically studied for their nonlinear frequency responses and verified through in-house experiments. The direct iterative method and the current mathematical model were used to numerically compute the nonlinear responses using MATLAB environment-specific computer code. The two higher-order theories and different shear deformable kinematic models, together with Green-Lagrange nonlinear strains, were used to develop the mathematical model of the layered composite structure. In order to achieve generalizability, the current model incorporates all of the nonlinear higher-order strain factors in the formulation. In addition, the rapid Fourier transform technique was used to obtain the appropriate frequency values from altered signals, and an experimental evaluation of the modal test was carried out. Another step is the batch input technique, which is used to calculate the results using the simulation model that was constructed in the commercial finite element software (ANSYS). The discussion concludes with numerical examples solved for various geometrical configurations, and a thorough examination of the influence of additional design parameters (thickness ratio, curvature ratio, and constraint condition) on the basic linear and nonlinear frequency responses.
KEYWORDS: *Free vibration, Curved Panels, ANSYS, MATLAB, Mathematical Model.*

I. INTRODUCTION AND LITERATURE

Advanced fiber reinforced laminated composite structures have become more popular in weight-sensitive and high-performance engineering due to recent developments in design and production methods. Accurate characterization of the reactions of laminated composite shells is now necessary due to their increasing use in mechanical, civil, aircraft, aerospace, automobile, biomedical, nuclear, petrochemical, and marine engineering. Large amplitude vibration is something that these constructions are known to be subjected to frequently throughout their service life. Due to the impossibility of utilizing a linear strain displacement relation, precise mathematical modeling that incorporates geometrical nonlinearity is required to establish the state variables. The reason behind this is that the overall deformation of the layered structures is much greater than the strains acting linearly. Furthermore, laminated composites have a high shear failure rate, and the overall structure's qualities are heavily influenced by the geometrical and material features that contribute to shear deformation. Numerical investigations of the nonlinear vibration behavior of laminated structures have been conducted utilizing mathematical models grounded in a variety of classical, shear deformation, and revised theories in order to get the precise response with minimal computational effort1. In addition, the analysis of complicated laminated structures and structural components can be effectively done using the finite element method (FEM).

There is a large body of literature on the topic of free vibration analysis of laminated composite flat/curved panel structures utilizing geometrically nonlinear models and finite element methods. The vibration analysis of fabric composite structures has also been the subject of numerous experimental investigations2–8. Additionally, there have been studies that point to higher-order shear deformation theory (HSDT) being a good fit for analyzing laminated structures. This is because HSDT provides accurate approximations of transverse shear stress and strains, and it also eliminates the need for the shear correction factor, which means that errors in the final responses are reduced. For the first time, Reddy and Liu10 investigated the vibration and bending responses of spherical and cylindrical shells using a model for laminated composite shell panels based on the HSDT mid-plane kinematics. Using the HSDT and von-Karman strain, Reddy11 developed a laminated composite plate mathematical model that outperforms both classical plate theory (CPT) and first-order shear deformation theory (FSDT) in terms of stress predictions, frequency predictions, and deflection predictions. Using von-Karman type geometrical nonlinearity within the framework of the FSDT kinematics, Shin12 examined the large amplitude vibration behavior of laminated composite doubly curved shells. By including nonlinear kinematics through the von-Karman sense into the framework of the FSDT, Liu and Huang13 documented the laminated plate structure's nonlinear free vibration responses under unlike temperature loads. In addition, the HSDT kinematic model proposed by Kant and Swaminathan14 is used to examine the analytical solutions of the free vibration responses of the sandwich plate and laminated composite structure. By creating a model utilizing the FSDT kinematics and von-Karman nonlinear strain, Kerur and Ghosh15 were able to publish numerical solutions for the geometrically nonlinear transient responses of the laminated composite plate via the FEM approach. In their study, Kishore et al.16 detailed the smart composite plate structure's nonlinear static deflection behavior. They used the third-order shear deformation theory (TSDT) to define the mid-plane deformation and used the von-Karman type of geometrical nonlinear strain. In their study, Naidu and Sinha17 utilized finite element modeling (FEM) to examine the free vibration responses of composite shell panels in hygrothermal settings. The model was based on the FSDT kinematic model and Green-Lagrange nonlinear stresses. The numerical model created utilizing the FSDT kinematics and von-Karman type of geometrical nonlinear strain was provided by Nanda and Bandyopadhyay18. The structure in question is a laminated composite cylindrical shell panel. The frequency responses of this panel are nonlinear and free vibration. Using the FSDT mid-plane kinematics framework, Ngo-Cong19 et al. analyze the laminated composite plate's free vibration responses. Even more so, Pradyumna and Bandyopadhyay20 have created a C0 FE model that uses the HSDT mid-plane theory to forecast the laminated composite shell's static and dynamic deflections. In a similar vein, Tornabene21 et al. propose a generic theory of generalized higher-order equivalent single layers to examine the free vibration behavior of composite shell panels with two curved laminates. The nonlinear finite element method (FEM) procedures used to calculate the large amplitude free vibration frequency responses of the doubly curved laminated composite shell panels, taking into account the environmental influence, were developed by Singh and Panda22 and Mahapatra23, utilizing the HSDT kinematics model. Additionally, the variational asymptotic approach (VAM)24–31 has been published in a substantial number of research works concerning the dynamic behavior of laminated composite plate and shell structures. Research shows that VAM method can accurately analyze geometrical nonlinearity, including the effect of geometries.

II. MATHEMATICAL FORMULATION

Considered for this analysis are the single or doubly curved laminated composite shallow shell panels (cylindrical, hyperbolic, ellipsoidal, plate, or spherical) with dimensions a and b, made up of an infinite number of orthotropic layers with a uniform thickness h. The shallow shell panel's major radii of curvature along the x and y directions are Rx and Ry, respectively, with the twist radius of curvature $R_{xy} = \infty$. The current study made use of the following kinematic models.

At the outset, we build the laminated panel's kinematic model by integrating the cubic variation of in-plane displacement and the linear variation via thickness.

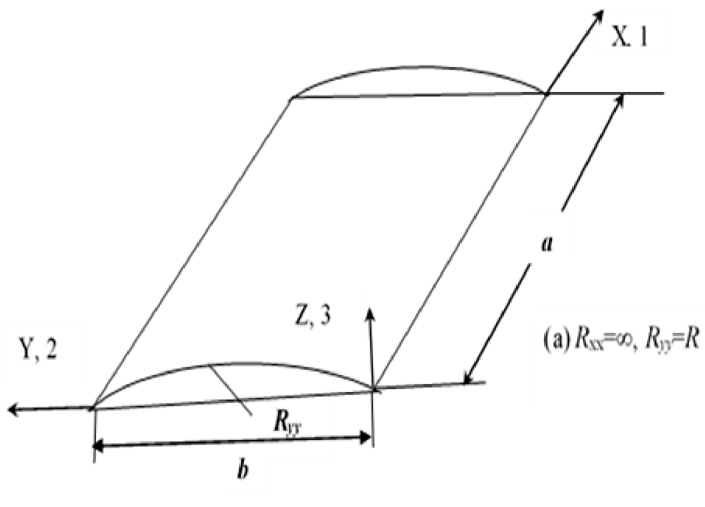

Figure 1. Laminated composite curved panels in Cylindrical shape.

$$
\begin{aligned}
u^{(k)}(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\
&\quad + z^2\phi_x(x,y,t) + z^3\lambda_x(x,y,t) \\
v^{(k)}(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\
&\quad + z^2\phi_y(x,y,,t) + z^3\lambda_y(x,y,t) \\
w^{(k)}(x,y,z,t) &= w_0(x,y,t) + z\theta_z(x,y,t)
\end{aligned}
\qquad (1)
$$

Every general material continuum has an expression for the nonlinear Green-Lagrange strain-displacement relation, which is

$$
\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ w_{,z} \\ v_{,z} + w_{,y} \\ u_{,z} + w_{,x} \\ u_{,y} + v_{,x} \end{Bmatrix} + 1/2 \begin{bmatrix} (u_{,x})^2 + (v_{,x})^2 + (w_{,x})^2 \\ (u_{,y})^2 + (v_{,y})^2 + (w_{,y})^2 \\ (u_{,z})^2 + (v_{,z})^2 + (w_{,z})^2 \\ 2\{(u_{,z})(u_{,y}) + (v_{,z})(v_{,y}) + (w_{,z})(w_{,y})\} \\ 2\{(u_{,z})(u_{,x}) + (v_{,z})(v_{,x}) + (w_{,z})(w_{,x})\} \\ 2\{(u_{,x})(u_{,y}) + (v_{,x})(v_{,y}) + (w_{,x})(w_{,y})\} \end{bmatrix}
$$

The total strain vector $\{\varepsilon\}$ is further modified as the linear $\{\varepsilon_L\}$εand nonlinear $\{\varepsilon_{NL}\}$ strains and expressed as: The in-plane strain displacement relations for laminated curved panels using figure are given by looking at the above Eqns.

$$
\{\varepsilon_L\} + \{\varepsilon_{NL}\} = \begin{Bmatrix} \varepsilon_1^{l_0} \\ \varepsilon_2^{l_0} \\ \varepsilon_3^{l_0} \\ \varepsilon_4^{l_0} \\ \varepsilon_5^{l_0} \\ \varepsilon_6^{l_0} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} \varepsilon_1^{nl_0} \\ \varepsilon_2^{nl_0} \\ \varepsilon_3^{nl_0} \\ 2\varepsilon_4^{nl_0} \\ 2\varepsilon_5^{nl_0} \\ 2\varepsilon_6^{nl_0} \end{Bmatrix} +
$$

$$
z \left[\begin{Bmatrix} k_1^{l_1} \\ k_2^{l_1} \\ 0 \\ k_4^{l_1} \\ k_5^{l_1} \\ k_6^{l_1} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} k_1^{nl_1} \\ k_2^{nl_1} \\ k_3^{nl_1} \\ 2k_4^{nl_1} \\ 2k_5^{nl_1} \\ 2k_6^{nl_1} \end{Bmatrix} \right] + z^2 \left[\begin{Bmatrix} k_1^{l_2} \\ k_2^{l_2} \\ 0 \\ k_4^{l_2} \\ k_5^{l_2} \\ k_6^{l_2} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} k_1^{nl_2} \\ k_2^{nl_2} \\ k_3^{nl_2} \\ 2k_4^{nl_2} \\ 2k_5^{nl_2} \\ 2k_6^{nl_2} \end{Bmatrix} \right]
$$

$$+z^{3}\left[\begin{Bmatrix} k_1^{l_3} \\ k_2^{l_3} \\ 0 \\ k_4^{l_3} \\ k_5^{l_3} \\ k_6^{l_3} \end{Bmatrix}+\frac{1}{2}\begin{Bmatrix} k_1^{nl_3} \\ k_2^{nl_3} \\ k_3^{nl_3} \\ 2k_4^{nl_3} \\ 2k_5^{nl_3} \\ 2k_6^{nl_3} \end{Bmatrix}\right]+z^{4}\frac{1}{2}\begin{Bmatrix} k_1^{nl_4} \\ k_2^{nl_4} \\ k_3^{nl_4} \\ k_4^{nl_4} \\ k_5^{nl_4} \\ k_6^{nl_4} \end{Bmatrix}+z^{5}\frac{1}{2}\begin{Bmatrix} k_1^{nl_5} \\ k_2^{nl_5} \\ k_3^{nl_5} \\ 2k_4^{nl_5} \\ 2k_5^{nl_5} \\ 2k_6^{nl_5} \end{Bmatrix}$$

$$+z^{6}\frac{1}{2}\begin{Bmatrix} k_1^{nl_6} \\ k_2^{nl_6} \\ 0 \\ 0 \\ 0 \\ 2k_6^{nl_6} \end{Bmatrix}$$

Here is a matrix representation of the strain-displacement relation:

$$\{\varepsilon_L\}+\{\varepsilon_{NL}\}=\left[T^{L}\right]\{\overline{\varepsilon_L}\}+\frac{1}{2}\left[T^{NL}\right]\{\overline{\varepsilon_{NL}}\}$$

$$\text{where } \{\overline{\varepsilon_L}\}=\{\varepsilon_1^{l_0}\,\varepsilon_2^{l_0}\,\varepsilon_3^{l_0}\,\varepsilon_4^{l_0}\,\varepsilon_5^{l_0}\,\varepsilon_6^{l_0}\,k_1^{l_1}\,k_2^{l_1}\,k_4^{l_1}\,k_5^{l_1}\,k_6^{l_1}\,k_1^{l_2}\,k_2^{l_2}\,k_4^{l_2}\,k_5^{l_2}$$

$$k_6^{l_2}\,k_1^{l_3}\,k_2^{l_3}\,k_4^{l_3}\,k_5^{l_3}\,k_6^{l_3}\}^{T} \text{ and } \{\overline{\varepsilon_{NL}}\}=\begin{Bmatrix} \varepsilon_1^{nl_0} & \varepsilon_2^{nl_0} & \varepsilon_3^{nl_0} & \varepsilon_4^{nl_0} \\ k_2^{nl_3} & k_3^{nl_3} & k_4^{nl_3} & k_5^{nl_3} \end{Bmatrix.$$

$$\begin{matrix} \varepsilon_5^{nl_0} & \varepsilon_6^{nl_0} & k_1^{nl_1} & k_2^{nl_1} & k_3^{nl_1} & k_4^{nl_1} & k_5^{nl_1} & k_6^{nl_1} & k_1^{nl_2} & k_2^{nl_2} \\ k_6^{nl_3} & k_1^{nl_4} & k_2^{nl_4} & k_3^{nl_4} & k_4^{nl_4} & k_5^{nl_4} & k_6^{nl_4} & k_1^{nl_5} & k_2^{nl_5} & k_4^{nl_5} \end{matrix}$$

$$\left.\begin{matrix} k_3^{nl_2} & k_4^{nl_2} & k_5^{nl_2} & k_6^{nl_2} & k_1^{nl_3} \\ k_5^{nl_5} & k_6^{nl_5} & k_1^{nl_6} & k_2^{nl_6} & k_6^{nl_6} \end{matrix}\right\}$$

are the mid-plane linear and nonlinear strain terms. Similarly, $[T]^{L}$ and $[T]^{NL}$ are the linear and the nonlinear thickness coordinate matrices.

The terms containing superscripts

$$l_0,\ l_1,\ l_2\text{-}l_3 \text{ in } \{\varepsilon_L\} \text{ and } nl_0,\ nl_1,\ nl_{2-6} \text{ in } \{\overline{\varepsilon_{NL}}\}$$

are the membrane, curvature and higher-order strain terms, respectively

$$\left.\begin{aligned}\{\varepsilon_L\} &= [B_L]\{d\} \\ \{\varepsilon_{NL}\} &= \tfrac{1}{2}[B_{NL}]\{d\}\end{aligned}\right\}$$

or $\{\varepsilon_{NL}\} = \tfrac{1}{2}[B_{NL}]\{d\} = \tfrac{1}{2}[A][G]\{d\}$

The stress-strain relationship for the laminated shell panel can be expressed as

$$\{\sigma\} = \left[\overline{Q}\right]\{\varepsilon\}$$

The final form of the governing motion equation of the vibrated laminated curved shell panel is derived by solving the Lagrangian functional via Hamilton's principle and expressed as:

$$\delta\int_{t_1}^{t_2}(V-U)dt = 0$$

$$[M]\{\ddot{\delta}\} + \left([K]_L + \frac{1}{2}[KN1] + \frac{1}{3}[KN2]\right)\{\delta\} = 0$$

where

$$\{\delta\} = \{u_0 \quad v_0 \quad w_0 \quad \theta_x \quad \theta_y \quad \theta_z \quad \phi_x \quad \phi_y \quad \lambda_x \quad \lambda_y\}^T$$

θφλθis the displacement vector, [M] and $[K]^L$ are the global mass matrix and global linear stiffness matrix, respectively. $[KN_1]$ and $[KN_2]$ are the linear and quadratic nonlinear mixed stiffness matrices that depend on the displacement vector linearly and quadratically, respectively.

III. Conclusion

A popular approach, similar to that used in finite element modeling (FEM), is used to initialize the elemental stiffness and the mass matrices. The evaluation of linear and nonlinear global stiffness matrices, as well as the associated mass matrix, is carried out after the assembly procedure with the assistance of the elemental matrices. In addition, linear frequency responses can be achieved in the first step by imposing the necessary restriction and removing the nonlinear elements that are not acceptable. The procedures were extended further, which included the extraction of the eigen values and the accompanying eigenvectors by utilizing the frequently used algorithm for eigen value extraction. By increasing the amplitude ratio, the eigenvectors are scaled up in order to bring the nonlinearity into the solution. Taking the linear values as the starting input, the nonlinear stiffness matrices are updated at each step. This is done by using the linear values. After that, the iteration steps are carried out until the convergence requirement is met, and then the final frequency values are printed out.

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