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Research Paper

Stochastic Models and Their Application in Human Fertility Behavior

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ABSTRACT: *Some simple probability models applicable in fertility analysis have been presented in this paper. The probability models to birth intervals under the assumption of fecundability among women are discussed. Method of moments and method of maximum likelihood are used to estimate the parameters of the probability models. Geometric probability law has been applied to estimate the monthly probability of first conception for women in Borno State.The research findings reveal that the fertility pattern seems to be very high in the study area. This is because the average fecundability, the average age at menarche and that of effective marriage in the study area were estimated to be higher as compared to other studies. The average duration of postpartum amenorrhea (PPA) also appears to be less than two months in the study area. The study also reveals that the blood group has nothing to do with fecundabilty, thus, contradicting the notion.*

KEYWORDS: *Age at Menarche, Age at marriage, Fecundability, Postpartum Amenorrhea (PPA) and Probability model*

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I. INTRODUCTION

Indeed, to the observation of the researcher, the impact of probability model and their application in understanding the stochastic nature of fertility behavior in Borno State has little or no contribution to the growth and development of population policy. This is because the state has not yet given serious thinking towards most of the demographic aspects that is essential to be looked into and improve the quality of life of the human population in the state. Against this background, that the study used stochastic modeling and its application to human fertility behavior to proffer solutions to this problem.

Nigeria engaged in a national discussion of its population trends, which was followed by the adoption of a national population policy in 1988 and subsequent expansion of government-sponsored family planning services. Not surprisingly, donor assistance for population programs increased in this period, focusing on strengthening national policy and program efforts through technical and financial assistance. The government's objective, shared by donors, was to increase the use of modern contraceptives as a means of improving maternal and child health. As such, the federal government set about building and extending its maternal and child health services under which family planning was one component. Some donors, however, channeled their support largely to the family planning component of those services (e.g. for training family planning personnel, equipping health facilities and supplying contraceptive commodities).

Donors also increased their support to operations and applied research during this period. For instance, studies were supported to improve understanding of: (a) barriers to effective family planning delivery (Aboderin 1986; Akintunde 1986; Coleman 1988; Masha 1986; NTA 1987; Rimon 1986); (b) contraceptive innovators and how their numbers could be increased (Oni 1986; Makinwa-Adebusoye 1991, 1992); and (c) impediments to contraceptive acceptance and how they could be removed (Covington et. al. 1986; Makinwa-Adebusoye 1993). A number of clinic-based studies were undertaken as services expanded (e.g. Kim et. al. 1992). The impact of modernization in reducing the duration of postpartum abstinence and on fertility was also emphasized (e.g. Aborampah 1987).

Although no study has evaluated that hypothesis with empirical data, the view continues to persist that men are the dominant decision-makers on fertility matters in Africa (Isiugo-Abanihe 1994; Renne 1993). In recent years, some researchers have started gathering data that allow them to address the issue of the role of husbands in fertility and family planning decisions in Nigeria. Oni and McCarthy (1991), Bankole (1995) also looked at the importance of spousal agreement for reproductive outcomes. In particular, he examined the effects of joint fertility desires on fertility using panel data from 1984 and 1986 surveys. He found that spousal agreement/disagreement is a significant determinant of subsequent fertility. In cases where the couple disagrees on the desire for more children, his analysis shows that subsequent fertility falls between the fertility of spouses who want more children and those who want to stop having more children. Although he found that the desires of both spouses carry the same weight on subsequent fertility, when he disegregated the analysis by the number of living children, he found that the husband's desires are more important when the family size is small but that the wife's desires become more important when the number of living children is large. He interprets this finding using a life cycle argument, noting that in the Yoruba cultural context, women obtain increased autonomy and status within the household as they secure their position within their natal families.

Matching population growth with development is the real object of global and country action towards improved welfare and human development and economic growth. The puzzling phenomenal difference in levels of welfare and development among the populations of countries are largely explained by the divergence in the nature and magnitude of the dynamics of populations (Ogujiuba, 2006).

It has long been realized that population growth varies over space. Therefore, Borno State is not an exception. Alldemographic transition models emphasized the synchronization of respective mortality and fertility patterns. Placing mortality decline as a pre-condition for fertility decline formed the cornerstone of the theory. The experience of some African countries also shows that fertility can decline independently of the degree of socio economic development (Kirk, 1996).

According to the theory of demographic transition (ECA 2001 and Cowgil 2002), the shift towards low mortality and fertility rates occurs when there is a process of overall modernization resulting from industrialization, urbanization, education, as well as substantial overall socio-economic development. Such a shift leads initially to a drop in mortality through progress in hygiene and medicine and, subsequently, to a decline in fertility occasioned by economic growth.

Since the last four decades, there has been a remarkable development in the field of mathematical and stochastic models which have found immense application in almost all discipline including the biological system. Mathematics has played a fundamental role in many physical and engineering sciences.

The past century has seen a tremendous growth of physics, astronomy, engineering and space sciences as a result of application of mathematics to these disciplines. In life sciences, however, the application of mathematics has not been made on such a large scale. The lack of this development is partly due to the complicated nature of the biological phenomena themselves. In spite of these difficulties, mathematics has been instrumental in some remarkable discoveries such as those in the study of heredity leading to the development of the science of genetics.

Mathematical models of a natural phenomenon are essential to the development of a theory of the phenomenon. There are large amount of experimental data available in the field of scientific endeavor but unless it is unified by a theory, it is of limited use. Theories are only mnemonic devices to save us from the impossible task of storing all possible information, Bellman (1963). In the development of theories mathematics plays a basic role. In the study of living systems, especially in clinical medicine, there are a large number of problems which need mathematical treatment.

As Sheps and Perin (1963) and Menken (1975), among others, have pointed out, simplified models, unrealistic though they may be, have proved useful in gaining insights such as that a highly effective contraceptive used by a rather small proportion of a population reduces birth rates more than does a less effective contraceptive used by a large proportion of the population. Some fertility researchers have been modeling parts rather than the whole of the reproductive process. The components of birth intervals have been exam- ined, with emphasis on the physiological and behavioral determinants of fertility (see Leridon, 1977). Another focus has been abortions, induced and spontaneous (see: Abramson, 1973; Potter et al., 1975; Michels and Willett, 1996). Fecund- ability investigations have been yet another focus (see Menken, 1975; Wood et al., 1994). Menken (1975) alerts researchers to the impossibility of reliably es- timatingfecundability from survey data. The North Carolina Fertility Study referred to in Dunson and Zhou (2000) is of interest in this connection: In that study couples were followed up from the time they discontinued birth control in order to attempt pregnancy. The enrolled couples provided base-line data and then information regarding ovulation in each menstrual cycle, day-by-day reports on intercourse, first morning urine samples, and the like. Dunson and Zhou present a Bayesian Model and Wood et al. (1994) present a multistate model for the analysis of fecundability and sterility.

Porter and Parker (1964) have suggested the type I geometric distribution to describe the data on the conception times under the following assumptions;

(a) The fecundability of each couple remains constant from month to month until pregnancy.

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(b) Among couples, fecundability is distributed as a Pearson Type I curve i.e. a beta distribution with parameters a and b.
(c) Conception is a random event, conditional on fecundability.
(d) The number of couples is large.
(e) The model applies only to those women who conceive.

In order to estimate the parameters ‘a’ and ‘b’ in the model Potter and Parker applied the method of moments using the first two moments about origin of the conception months. Majumdar and Sheps (1970), along with moment estimates, applied the method of maximum likelihood, to estimate these parameters and observed some improvement in the goodness of fit but in both the cases the fit was found to be almost poor, while giving explanation for this poor fit they have raised a doubt on the appropriatness of the beta distribution for fecundability parameter ‘p’

In the present research alternative distribution to describe the nature of ‘P’ has been proposed. The distribution is as follows;

$$f(p) = \frac{c^l}{\Gamma(l)} p^{C-1} (\log \frac{1}{p})^{l-1} \qquad 0 < p < 1, \qquad l, c > 0 \tag{1.1}$$

$= 0$ otherwise

This distribution has been used by Grassia (1976) by using a transformation

$$t = \log_e (\frac{1}{p}) \tag{1.2}$$

In the gamma distribution

$$g(t, c, l) = \frac{c^l}{\Gamma(l)} e^{-ct} t^{l-1} \qquad l, c > 0 \text{ and } 0 < t < \infty \tag{1.3}$$

$= 0$ otherwise

And used in some other context.

2.0 The Revised Model

Using the alternative model, as given in (1.1), for ‘P’ the unconditional distribution g(x) of x, the random month of conception can be written as

$$g(x) = p \mid X = x \mid = \frac{c^l}{\Gamma(l)} \int_0^1 p^c (\log \frac{1}{p})^{l-1} (1-p)^{x-1} dx \quad 0 < p < 1, \qquad l, c > 0 \tag{2.1}$$

$= 0$ otherwise $\qquad x = 1,2,3---$

Solving the integral on the right hand side (2.1) the expression for g(x) reduces as

$$g(x) = \begin{cases} \dfrac{c^l}{(c+1)^l} \\ \dfrac{c^l}{(c+1)^l} - \begin{pmatrix} x-1 \\ 1 \end{pmatrix} \left(\dfrac{c}{c+2} \right) + \begin{pmatrix} x-1 \\ 2 \end{pmatrix} \left(\dfrac{c}{c+3} \right) - - - - + (1)^{x+1} \begin{pmatrix} x-1 \\ x-1 \end{pmatrix} \dfrac{c}{c+x} \end{cases} \tag{2.2}$$

$x \geq 2$

The first four moments of x about the origin, conditional on p, as given by the simple geometric distribution ie.

$$\Pr ob[X = x / p] = p(1-p)^{x-1} \qquad 0 < p < 1, \ x = 1,2,3--- \tag{2.3}$$

$= 0$ otherwise

Are

$$\frac{1}{P} = \frac{1}{p} \tag{2.4}$$

$$\frac{2}{P} = \frac{2}{p^2} - \frac{1}{p} \tag{2.5}$$

$$\frac{3}{P} = \frac{6}{p^3} - \frac{6}{p^2} + \frac{1}{p} \tag{2.6}$$

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$$\frac{4}{P}=\frac{24}{p^4}-\frac{36}{p^3}+\frac{14}{p^2}-\frac{1}{p} \tag{2.7}$$

On the assumption that 'P' follows the distribution given in (1.1), the expected value of $\frac{1}{p^r}$ is

$$E(\frac{1}{p^r})=\frac{c^l}{\Gamma(l)}\int_0^1\frac{1}{p^r}p^{c-1}(\log)^{l-1}dp \tag{2.8}$$

$$=\frac{c^l}{(c-1)^l} \tag{2.9}$$

Using the expression (2.9) for the expected values of $\frac{1}{p^r}$, the various unconditional moments can obtained as

1. $=\left(\frac{c}{c-1}\right)^l$ (2.10)
2. $=2\left(\frac{c}{c-2}\right)^l-\left(\frac{c}{c-1}\right)^l$ (2.11)
3. $=6\left(\frac{c}{c-3}\right)^l-6\left(\frac{c}{c-2}\right)^l+\left(\frac{c}{c-1}\right)^l$ (2.12)
4. $=24\left(\frac{c}{c-4}\right)^l-36\left(\frac{c}{c-3}\right)^l+14\left(\frac{c}{c-2}\right)^l-\left(\frac{c}{c-1}\right)^l$ (2.13)

II. ESTIMATION OF FECUND ABILITY

Let m_1 and m_2 denote the first two sample moments of x about origin. Equating m_1 and m_2 with their population counterparts as defined in (2.10) and (2.11) we

$$\left(\frac{c}{c-1}\right)^l=m_1 \tag{3.1}$$

And

$$\left(\frac{c}{c-2}\right)^l=\frac{m_1+m_2}{2} \tag{3.2}$$

The equations (3.1) and (3.2) can be solve for c and l in terms of m_1 and m_2. These estimated values of c and l can be used to find the probabilities for the various values of x as given in (2.2).

The maximum likelihood estimates for c and l also be obtained using the scoring methods of Rao (1952). In the present case only the method of moments has been considered because of its simplicity. The estimates of the parameters c and l using the conception data have been obtained

The equation (3.1) and (3.2) can be used to obtain the values of 'c' and 'l' observing the values of m_1 and m_2 from the empirical data. These estimated values of 'c' and 'l' can be used to find the probabilities for the various values of x using (2.2). The average fecundability can be computed by the formular.

$$E(p)=\left(\frac{\hat{c}}{\hat{c}+1}\right)^{\hat{l}} \tag{3.3}$$

To solve the equation (3.1) and (3.2) for 'c' and 'l' interms of m_1 and m_2 the following techniques as suggested by Misra (1982) can be adopted. The equation (3.1) and (3.2) can respectively be written as

$$l[\log c-\log(c-1)]=\log m_1 \tag{3.4}$$

$$l[\log c-\log(c-2)]=\log\frac{m_1+m_2}{2} \tag{3.5}$$

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From (3.4) and (3.5)

$$\frac{\log c-\log(c-1)}{\log c-\log(c-2)}=\frac{\log m_1}{\log\frac{m_1+m_2}{2}}=\lambda$$

$$\therefore \log c-\log(c-1)=\lambda[\log c-\log(c-2) \quad (3.6)$$

Using a suitable numerical technique the value of 'c' can be found to any derived degree of accuracy from equation (3.6) and substituting the value of x in (3.1) the value of 'l' can be computed. Equation (3.3) can be used to compute the average value of fecundability for women of that region.

Table 1 Observed and Expected frequencies of conception under both the Distribution for 'p'

Month	Observed Frequency	Expected Frequency		
		Using Beta Distribution for 'p'		Using transformed gamma distribution for 'p' and moment method for estimation
		Under Method of Moments	Under Method of Likelihood Estimation (MLE)	
1	258	28.75	230.75	209.75
2	133	151.5	156	151.5
3	108	111.75	109	111.25
4	68	83.25	78.25	83
5	75	63	57.5	62.5
6	23	46	43.25	43.75
7	30	35.5	33	37
8	23	27.5	25.5	28.75
9	15	21.75	20	22.5
10	20	17.25	16	18
11	25	13.75	13	14.25
12	13	11	10.5	11.5
13-15	23	22	21.75	23
16-18	18	12.25	12.75	13
19-24	18	11.5	13.25	12.5
25+	10	18.25	14.5	13
Total	855	855.0	855.0	855.0
fecundability		a =5.99	a*=3.40	c = 10.139
parameters		b = 18.56	b*=9.19	l= 14.946
Chi-square	value	$\chi^2_{13}=20.43$	$\chi^2_{13}=17.2$	$\chi^2_{13}=17.72$

From Table (1) we observe that under the transformed gamma model for 'p' the fitting has improved approximately of the same order as has been obtained under the maximum likelihood methods. If c and l are estimated using MLE the fitting may improve a little more but even then the fitting is not expected very close to the observed values. A critical examination of table (1) reveals that the frequency in the first cell is comparatively high, a situation, generally, not observed in the Nigerian data. It is possible that some women might have conceived before marriage and for the sake of some social reason or the other they might have reported the conception in the first month after marriage. Thus, it appears that the parent population in this case consist of two types of units,

i. those who always respond to the first cell and

ii. those who respond to the month as and when the conception occur.

Under such a situation an inflated distribution with inflation in the first cell may describe the data more closely in comparison to the distribution used in the present case. The study of an appropriate inflated distribution is under study and will be reported shortly

III. RESULTS

The research findings reveal that the fertility pattern seems to be very high in the study area. This is because the average fecundability using simple compound type I geometric distribution compounded by transformed gamma distribution is estimated to be 0.27 as compared to other studies elsewhere.

The average age at menarche and that of effective marriage in the study area were estimated to be 14.13 and 16.96 years respectively. This is higher than that of Brooks-Gunn and Eliot (1992). The average duration of postpartum amenorrhea (PPA) also appears to be less than two months in the study area. This is significant finding of the survey. In various studies, it has been reported to vary between 3 months to even more

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than a year. It appears that the higher the value of lactation period as found in the research also does not elongate the period of PPA. This contradicts various studies, which reported that lactation and amenorrhea are positively correlated.

IV. CONCLUSION

In the present study, a detailed analytic description of human fertility in terms of the age parity distribution was developed. This specifically included first, the derivation of a stochastic model for the fertility process, second, an analysis of certain aspects of the model, and finally, the model was used in deriving some indices for the evaluation of fertility behavior. In this research, we were able to examine the uses of probability models to describe the stochastic nature of human fertility behaviors and estimate the inherent fertility parameters among women in Borno State.

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