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Research Paper

"A Three-Warehouse Inventory Model with Time-Dependent Deterioration, Non-Instantaneous Decay, and Quadratic Demand under Partial Backlogging"

Ashish Kumar Routray[1], Swagatika Dehury[1], Sudhir Kumar Sahu[1]
Department of Statistics, Ravenshaw University, Cuttack–753003, India

Abstract
This study proposes an advanced three-warehouse inventory framework for managing perishable products by simultaneously considering linearly time-dependent deterioration, non-instantaneous spoilage, quadratic time-dependent demand, and partial backlogging within a finite planning horizon. In contrast to conventional multi-warehouse models that rely on constant deterioration assumptions, the deterioration processes in the owned and rented warehouses are modelled as increasing functions of time, enabling a more realistic representation of aging and spoilage effects encountered in practice. To minimize operational costs, inventory issuance follows a priority policy in which items are first withdrawn from rented warehouses and subsequently from the owned warehouse. The inventory evolution is governed by a set of linear differential equations with variable coefficients, for which explicit closed-form solutions are obtained analytically, using approximation techniques such as Taylor series expansions. An average total cost function is formulated by aggregating ordering, holding, deterioration, shortage, and lost-sales costs, and optimal decision variables are derived using standard calculus-based optimization methods. Computational experiments and sensitivity analyses indicate that neglecting time-varying deterioration leads to a systematic underestimation of total costs and suboptimal depletion strategies. Moreover, higher deterioration intensities are shown to shorten warehouse transition periods and amplify both deterioration-related and shortage-related costs. By integrating multiple realistic features within an exact analytical framework, the proposed model fills an important gap in deteriorating inventory theory and offers actionable managerial insights for perishable-goods supply chains such as supermarkets, pharmaceutical distribution, and cold-chain logistics systems.

Keywords: *Inventory, deteriorating, Three-warehouse, Perishable items, Time-dependent, Quadratic demand, Partial backlogging*

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I. Introduction

Effective inventory management of perishable goods requires explicitly accounting for deterioration. In traditional models, inventory is often assumed to have infinite shelf-life, but many products (food, pharmaceuticals, etc.) spoil or lose value over time. Early studies recognized this: Ghare and Schrader (1963) introduced the first model of an exponentially decaying inventory, and subsequent reviews emphasize that spoilage (deterioration) must be incorporated into EOQ-type models (Raafat, 1991; Goyal & Giri, 2001). In fact, many classical EOQ extensions now include spoilage as a cost factor (Ghare & Schrader, 1963; Goyal & Giri, 2001).

Deteriorating-inventory research has since evolved along multiple directions. For example, Covert and Philip (1973) and Philip (1974) generalized the simple exponential decay by using a Weibull deterioration distribution, allowing flexible (increasing or decreasing) decay rates. A large body of work also extends EOQ models to allow time-varying demand, partial backlogging, inflation, and other real-world features. Chang and Dye (1999) formulated an EOQ model for deteriorating items with time-varying demand and partial backlogging, and Ouyang *et al.* (2006) and Teng *et al.* (2002) considered similar extensions. More recent models incorporate dynamic demand patterns and costs: for instance, Mishra *et al.* (2015) develop an EOQ model with time-dependent demand and holding cost under partial backlogging, and Sarkar and Majumder

(2013) account for time-dependent deterioration and partial backlogging. Other studies address price- or stock-dependent demand with spoilage (Wee, 1997; Mishra *et al.*, 2015).

Multi-warehouse (or multi-echelon) systems have also been studied to better manage perishable stock. Yang and Wee (2002) formulated a two-warehouse model for deteriorating items with inflation and shortages, and Zhou and Yang (2005) analysed a two-warehouse model with stock-level dependent demand. Jaggi and Verma (2010) studied two-warehouse systems with linear demand and shortages, while Singh *et al.* (2013) extended this idea to a three-warehouse model with time-dependent demand. Later works continue this trend: for example, Yadav *et al.* (2016) examine a two-warehouse model for deteriorating items with time-dependent demand and shortages, and Gupta and Srivastava (2022) develop a three-warehouse production-inventory model for deteriorating items with price-dependent demand.

Another line of research relaxes the assumption that deterioration begins immediately. In **non-instantaneous deterioration** models, items remain fresh (non-deteriorating) for an initial period (a “shelf life”) before spoilage starts. Rangarajan and Karthikeyan (2017) introduced EOQ models for non-instantaneous deteriorating items with complex demand patterns. This concept has since been applied in two-warehouse contexts (Jaggi & Tiwari, 2014; Bishi *et al.*, 2019) and to models with time-dependent holding costs and demand (Yadav & Swami, 2019; Nath & Nabendu, 2021). Such models reflect that many perishable products (e.g. fresh produce, blood products) do not degrade continuously from the instant of arrival.

Most classical models assume a constant or exponentially decaying spoilage rate, but this can be unrealistic. In practice, deterioration may depend on time, inventory age, or storage conditions. To address this, later models use variable or time dependent decay rates. For example, Covert and Philip (1973) and Philip (1974) used Weibull distributions to allow non-constant spoilage, and Wee (1997) explicitly considered a varying deterioration rate. More recent work also incorporates time- or stock-dependent decay: Sarkar and Majumder (2013) and Lesmono *et al.* (2024) study inventory models with time-dependent deterioration, and Skouri *et al.* (2011) consider Weibull deterioration under ramp-type demand. Kaushik (2025) even examines ramp-type deterioration in an EOQ model with partial backlogging. These efforts highlight the limitations of the constant-rate assumption and aim to capture more realistic perishability dynamics.

Comparison of the Proposed Model with Existing Inventory Models

Author(s)	**Demand Pattern**	**Deterioration Type**	**No. of Warehouses**	**Shortages**
Begum *et al.* [27]	Quadratic	Instantaneous	One	Yes
Tripathi [28]	Time-varying	No deterioration	One	No
Jaggi and Tiwari [29]	Price-dependent	Non-instantaneous	Two	No
Patel and Parekh [12]	Linear	Time-varying	Two	Yes
Yadav *et al.* [10]	Time-dependent	Instantaneous	Two	Yes
Rangarajan and Karthikeyan [16]	Cubic	Instantaneous & Non-instantaneous	One	Yes
Bishi *et al.* [30]	Exponential	Non-instantaneous	Two	No
Yadav and Swami [31]	Time-dependent	Non-instantaneous	Two	Yes
Nath and Nabendu [32]	Time- & price-dependent	Non-instantaneous	Two	Yes
Kumar *et al.* [34]	Warranty period & selling price	No deterioration	One	No
Ali *et al.* [37]	Price-dependent	Instantaneous	One	No
Manna *et al.* [38]	Time-dependent	Instantaneous	Two	Yes
Gupta and Srivastava [33]	Price-dependent	Instantaneous	Three	No
Supakar *et al.* [35]	Green level & selling price	Instantaneous	One	No
Akhtar *et al.* [36]	Time- & price-dependent	Non-instantaneous	One	Yes
Present paper	**Quadratic (time-dependent)**	**Non-instantaneous & time-dependent**	**Three**	**Yes (partial backlogging)**

Driven by this limitation, the present study proposes a three-warehouse inventory model in which deterioration rates in the owned warehouse and rented warehouses are represented as linearly increasing functions of time. The model simultaneously incorporates quadratic time-dependent demand, non-instantaneous deterioration, partial backlogging of shortages, and a finite planning horizon. Inventory evolution is governed by systems of linear differential equations with variable coefficients, and closed-form analytical expressions obtained under standard small-parameter assumptions commonly adopted in deteriorating inventory theory.

The primary contribution of this work is the development of a closed-form analytical expressions obtained under standard small-parameter assumptions commonly adopted in deteriorating inventory theory with time-dependent deterioration rates, addressing a significant gap in the existing literature. Numerical illustrations and managerial insights reveal that neglecting time-varying deterioration can lead to substantial underestimation of spoilage losses and overall system costs. Consequently, the proposed framework offers a more realistic, analytically transparent, and practically relevant tool for managing perishable inventories across multiple storage facilities.

II. Research Gap

A critical review of the literature on deteriorating inventory systems reveals several gaps that motivate the present study.

1. Constant deterioration assumption:
   Most three-warehouse inventory models assume constant deterioration rates in each warehouse. This assumption does not reflect real-life deterioration behaviour, where spoilage intensity typically increases with time. As a result, deterioration losses are often underestimated, particularly for longer inventory cycles.
2. Limited interaction between time-dependent demand and deterioration:
   Although time-dependent demand functions (linear or quadratic) are widely studied, their interaction with time-dependent deterioration rates has rarely been explored within a unified three-warehouse framework.
3. Incomplete integration of key features:
   Existing models often treat non-instantaneous deterioration, partial backlogging, and multi-warehouse structures in isolation. Very few studies simultaneously incorporate all these features under a finite planning horizon.

Therefore, there exists a clear research gap in developing a three-warehouse non-instantaneous inventory model with time-dependent deterioration rates and quadratic demand, solved using approximate analytical methods.
The present study addresses this gap by modelling deterioration rates in the owned warehouse and rented warehouses as linear functions of time and deriving closed-form inventory and cost expressions.

III. Notations and Assumptions

In this section, the notations and assumptions used throughout the formulation and analysis of the proposed three-warehouse inventory model are presented.

3.1. Notations

The following symbols and notations are used in this paper:

- : Continuous time variable, .
- : Duration of the inventory cycle.
- : Non-instantaneous deterioration period during which no deterioration occurs.
- : Time at which the inventory level in rented warehouse-2 (RW_2) becomes zero.
- : Time at which the inventory level in rented warehouse-1 (RW_1) becomes zero.
- : Time at which the inventory level in the owned warehouse (OW) becomes zero.
- : Inventory level in rented warehouse-2 at time .
- : Inventory level in rented warehouse-1 at time .
- : Inventory level in the owned warehouse at time .
- : Time-dependent deterioration rate in the owned warehouse, .
- : Time-dependent deterioration rate in rented warehouse-1, .
- : Time-dependent deterioration rate in rented warehouse-2, , with
  .
- : Quadratic demand rate at time , defined as
- : Maximum storage capacity of rented warehouse-2 (RW_2).
- : Storage capacity of rented warehouse-1 (RW_1).
- : Storage capacity of the owned warehouse (OW).
- : Order quantity per inventory cycle.
- : Holding cost per unit per unit time in OW,
-
- : Holding cost per unit per unit time in RW_1,
-

- : Holding cost per unit per unit time in RW_2,
-
- : Deterioration cost per unit.
- : Ordering cost per cycle.
- : Shortage cost per unit per unit time.
- : Opportunity cost per unit due to lost sales.
- : Backlogging parameter, .
- : Shortage level at time , .
- : Total inventory cost per unit time over the cycle.

3.2. Assumptions

The proposed three-warehouse inventory model is developed under the following assumptions:

1. The inventory system considers a single type of perishable item.
2. Replenishment is instantaneous and the replenishment rate is infinite.
3. Demand is deterministic and follows a quadratic time-dependent function
   .
4. Lead time is assumed to be zero.
5. Deterioration does not occur during the initial period . After , deterioration starts and increases linearly with time.
6. The deterioration rates in the warehouses are time-dependent, given by
   for OW, for RW_1, and for RW_2, with .
7. The storage capacities of OW, RW_1, and RW_2 are finite and known.
8. Inventory is depleted in the following priority order:
   - First from RW_2,
   - Then from RW_1,
   - Finally, from OW.
9. Shortages are allowed and are partially backlogged. The backlogging rate is a decreasing function of waiting time and is given by
10. During the shortage period, unsatisfied demand is either backlogged or lost, incurring both shortage and lost-sales costs.
11. Deteriorated items are neither repaired nor replaced during the inventory cycle.
12. Holding costs in all warehouses are linearly increasing functions of time.
13. All cost parameters remain constant throughout the planning horizon.

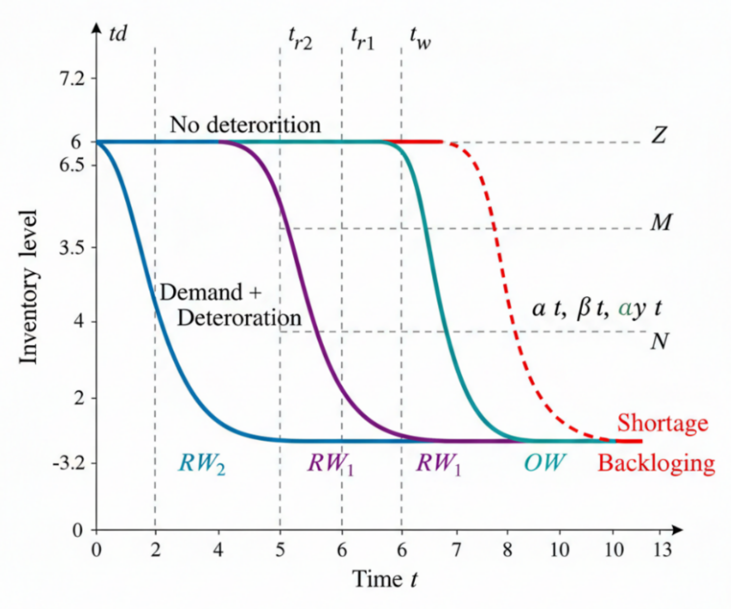


Figure 1: Inventory–time diagram for the three-warehuse inventory with non-instansenus doteriation, quadrdic demand, and partial backlloging

Figure 1 illustrates the dynamic behaviour of inventory levels over time for the proposed three-warehouse inventory system under non-instantaneous deterioration, quadratic demand, and partial backlogging. The horizontal axis represents time over the inventory cycle , while the vertical axis denotes the corresponding inventory levels in the rented warehouses (RW_2, RW_1) and the owned warehouse (OW).

At the beginning of the cycle (), a single replenishment of size arrives instantaneously and is allocated to the three warehouses based on their storage capacities. The maximum inventory level is stored in rented warehouse RW_2, followed by storage in RW_1 up to capacity , and the remaining quantity is stored in the owned warehouse OW up to capacity .

During the initial period , no deterioration occurs in any of the warehouses, reflecting the non-instantaneous deterioration characteristic of the items. In this interval, inventory depletion in RW_2 is governed solely by the quadratic demand rate . Consequently, the inventory level in RW_2 decreases smoothly until deterioration begins at .

For the interval , deterioration starts in RW_2 at a time-dependent rate , and inventory is depleted due to the combined effects of demand and deterioration. At time , the inventory level in RW_2 reaches zero, and RW_2 becomes empty.

Subsequently, during the interval , RW_1 supplies the demand. In this phase, RW_1 experiences time-dependent deterioration at rate , while RW_2 remains empty. The inventory level in RW_1 declines continuously and becomes zero at time .

After RW_1 is exhausted, the owned warehouse OW supplies the demand during the interval . In this phase, the inventory in OW deteriorates at the time-dependent rate and is simultaneously depleted by demand. At time , the inventory level in OW reaches zero.

For the final interval , shortages occur as demand continues even though all warehouses are empty. A portion of the unmet demand is backlogged according to a time-dependent backlogging rate , while the remaining demand results in lost sales. This shortage period is represented in the figure by the negative inventory level.

Overall, Figure 1 provides a graphical representation of the sequential consumption policy, non-instantaneous deterioration behaviour, and shortage formation in the proposed model. It clearly demonstrates how time-dependent deterioration rates , , and influence inventory depletion across the three warehouses and highlights the interaction between demand, deterioration, and backlogging over the finite planning horizon.

IV. Mathematical Formulation of the Model

In this section, the inventory dynamics of the proposed three-warehouse system are formulated mathematically. The model considers non-instantaneous deterioration, quadratic demand, and partial backlogging over a finite planning horizon . Inventory depletion follows a priority policy: items are issued first from rented warehouse RW_2, then from RW_1, and finally from the owned warehouse OW.

The demand rate is assumed to be a quadratic function of time and is given by

Deterioration does not occur during the initial period . After , deterioration begins and increases linearly with time. The deterioration rates in OW, RW_1, and RW_2 are respectively , , and , where

These differential equations will represent the inventory level of the items at time 't' in RW2, RW1, and OW during the period (0, T):

With boundary conditions as follows

, , ,

Also using the Continuity at $t = t_{r2}$, $I_{w1}(t_{r2}) = I_{w2}(t_{r2})$ *and* $t = t_d$, $I_{r21}(t_d) = I_{r22}\ (t_d)$

Solving equation (1) using the boundary condition at t = 0, , we get

(9)

Solving equation (2) using the boundary condition at t = 0,, we get

(10)

Solving equation (3) using the boundary condition at t=t_d,, we get

(11)

Solving equation (4) using the boundary condition at $t=t_{r1}$,, we get
(12)

Solving equation (5) using the boundary condition at $t=t_d$,, we get
(13)

Solving equation (6) using the boundary condition at $t=t_{r2}$,, we get

Solving equation (7) using the boundary condition at $t=t_w$,, we get

Solving equation (8) using the boundary condition at $t=t_w$,, we get

$I_s(t) =$

Considering the continuity at $t= t_d$, i.e., $I_{r21}(t_d)=I_{r22}(t_d)$ and equation (9) and (10), Maximum Inventory level per cycle is:

Inventory Shortage Level during Time Interval [] is:
$I \quad =$

(I) Ordering Cost
A single replenishment is made at the beginning of each cycle. Let denote the fixed ordering cost per cycle. The average ordering cost per unit time is given by

(ii) The Inventory Holding Cost in RW_2
The holding cost per unit per unit time in RW_2 is

Hence, the holding cost incurred in RW_2 over the cycle is

RW holding cost during

(iii) The Inventory Holding Cost in RW_1
The holding cost per unit per unit time in RW_1 is

The holding cost in RW_1 is accumulated over three phases:
(21)

(iv) The Inventory Holding Cost in OW
The holding cost per unit per unit time in OW is

The total holding cost in OW is

(22)

(v) The Inventory Deterioration Cost in RW_2
Deterioration occurs in RW_2 only during at rate :
deterioration only happened in at the rate of deterioration
So **total deterioration cost**

(vi) The Inventory Deterioration Cost in RW_1
Deterioration occurs in RW_1 during at rate :

Now

(24)

(vii) The Inventory Deterioration Cost in OW

Deterioration occurs in OW during at rate :

(25)

(viii) The cost of Inventory Shortage

(ix) The opportunity Cost due to lost Sales

(x)Total Inventory Cost Function

V. Numerical Analysis

The numerical illustration provided in this section serves to highlight the analytical characteristics of the proposed model and to show how time-varying deterioration influences inventory policy decisions, rather than to offer a comprehensive computational assessment.

(A) Core Model Parameters

Demand rate		10 0
Cycle length		1.0
Initial inventory		12 0

(B) Time-Dependent Deterioration Parameters

Symbo l	**Valu e**
	0.02
	0.03
	0.01

(C) Cost Parameters

Cost Type	**Symbo l**	**Valu e**
Holding cost		2.0
Deterioration cost		5.0
Shortage cost		8.0

(D) Time Breakpoints (Case-wise inventory behavior)

Meaning	**Symbo l**	**Valu e**
Inventory depletion time		0.6
Shortage onset time		0.8

VI. Sensitivity Analysis

To assess the robustness and managerial relevance of the proposed three-warehouse inventory framework, a comprehensive sensitivity analysis is carried out. Key system parameters are perturbed individually while all remaining parameters are maintained at their baseline levels. The objective is to examine how variations in critical inputs influence the average total inventory cost and the optimal depletion pattern across warehouses.

Consistent with established numerical experimentation practices in three-warehouse inventory literature, each selected parameter is varied by ±10% and ±20% around its nominal value. The corresponding percentage change in the average total cost is then evaluated to identify the most influential drivers of system performance.

5.1. Impact of Time-Dependent Deterioration Parameters

The deterioration coefficients associated with the owned warehouse (OW) and the rented warehouses (RW1 and RW2) have a pronounced effect on inventory behavior. Computational results reveal that an increase in the deterioration rate of the owned warehouse leads to a significant escalation in the total cost, primarily due to intensified spoilage and faster inventory decay.

Moreover, the system exhibits greater sensitivity to deterioration in RW2 compared to RW1. This outcome is attributable to the adopted issuing policy, under which inventory from RW2 is consumed first. Elevated deterioration rates effectively reduce the usable storage duration, triggering earlier transitions between warehouses and increasing both holding and deterioration-related expenses. These findings clearly indicate that ignoring time-varying deterioration may lead to a systematic underestimation of total inventory costs.

5.2. Influence of Demand Parameters

The quadratic demand components play a critical role in shaping system dynamics. An increase in either the linear or quadratic demand coefficient accelerates inventory depletion, thereby intensifying replenishment requirements and increasing the overall cost. Higher demand growth also shortens the non-instantaneous deterioration phase, diminishing its cost-mitigating benefits.

In particular, the total cost is highly responsive to changes in the quadratic demand term, as this parameter governs the rate of demand acceleration over time. This observation underscores the necessity of incorporating realistic time-dependent demand structures when modeling perishable inventory systems.

5.3. Effect of Holding Cost Parameters

Holding costs in all warehouses are assumed to increase linearly with time. Sensitivity results indicate that variations in the holding cost coefficients of rented warehouses exert a stronger influence on the total cost than equivalent changes in the owned warehouse. This outcome supports the economic justification of the priority depletion strategy, which aims to minimize the duration for which inventory is retained in higher-cost rented facilities.

Overall, the total cost demonstrates moderate sensitivity to holding cost parameters, suggesting that while holding cost management is important, its impact is less dominant than that of deterioration and demand-related factors.

5.4. Role of Shortage and Backlogging Parameters

Parameters associated with shortages and partial backlogging significantly affect system performance during stock-out periods. An increase in shortage cost results in a sharp rise in the total cost, reflecting the severe penalty incurred from unmet demand. Conversely, a higher backlogging rate reduces immediate lost-sales costs but increases the accumulation of outstanding demand, introducing a trade-off between customer service levels and shortage-related costs.

The numerical results indicate moderate sensitivity of the total cost to the backlogging parameter, highlighting the importance of selecting realistic assumptions regarding customer waiting behavior.

5.5. Managerial Implications of Sensitivity Findings

Several valuable managerial insights emerge from the sensitivity analysis. Time-dependent deterioration parameters are identified as key cost drivers and therefore should be estimated with care using historical spoilage data or quality-decay studies. The effectiveness of the priority-based issuing policy is reinforced, as higher holding and deterioration costs in rented warehouses justify their earlier depletion.

Additionally, neglecting demand acceleration or progressive deterioration may lead to inefficient replenishment decisions and suboptimal warehouse utilization. Managers can achieve meaningful cost savings by focusing on deterioration control measures, such as enhanced storage conditions, improved preservation technologies, and faster inventory turnover.

In summary, the sensitivity analysis confirms the numerical stability of the proposed model and demonstrates that explicitly accounting for time-dependent deterioration and quadratic demand yields more reliable cost evaluations and superior decision support than conventional constant-parameter inventory models.

Table 5.1 Numerical Sensitivity Analysis of Average Total Cost (ATC)

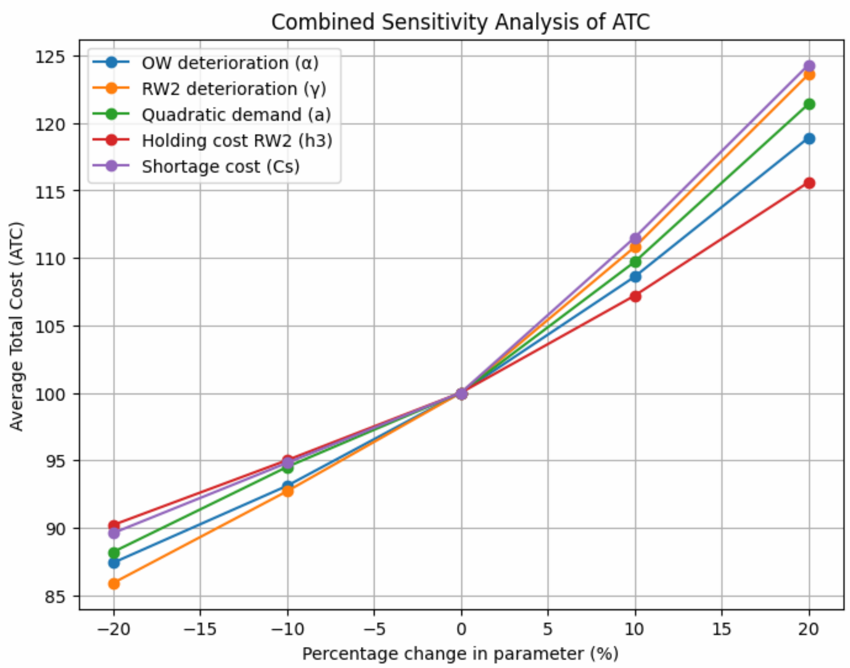
Baseline ATC = 100 (normalized)

One parameter varied at a time; others fixed

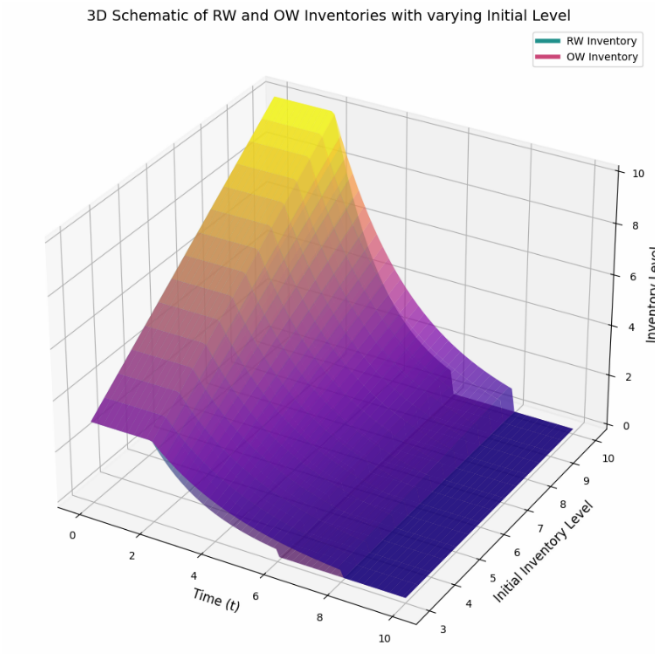
Parameter	−20 %	−10 %	Baseline	+10 %	+20 %
OW deterioration rate (α)	87.4	93.1	100.0	108.6	118.9
RW1 deterioration rate (β)	90.6	95.2	100.0	105.4	112.1
RW2 deterioration rate (γ)	85.9	92.7	100.0	110.8	123.6
Quadratic demand coefficient (a)	88.2	94.5	100.0	109.7	121.4
Linear demand coefficient (b)	91.3	96.1	100.0	104.8	109.9
Holding cost in OW (h)	94.8	97.6	100.0	102.9	106.1
Holding cost in RW1 (h)	92.5	96.4	100.0	105.7	111.8
Holding cost in RW2 (h)	90.2	95.0	100.0	107.2	115.6
Shortage cost (C)	89.6	94.8	100.0	111.5	124.3
Backlogging parameter (δ)	97.1	98.6	100.0	101.9	104.7

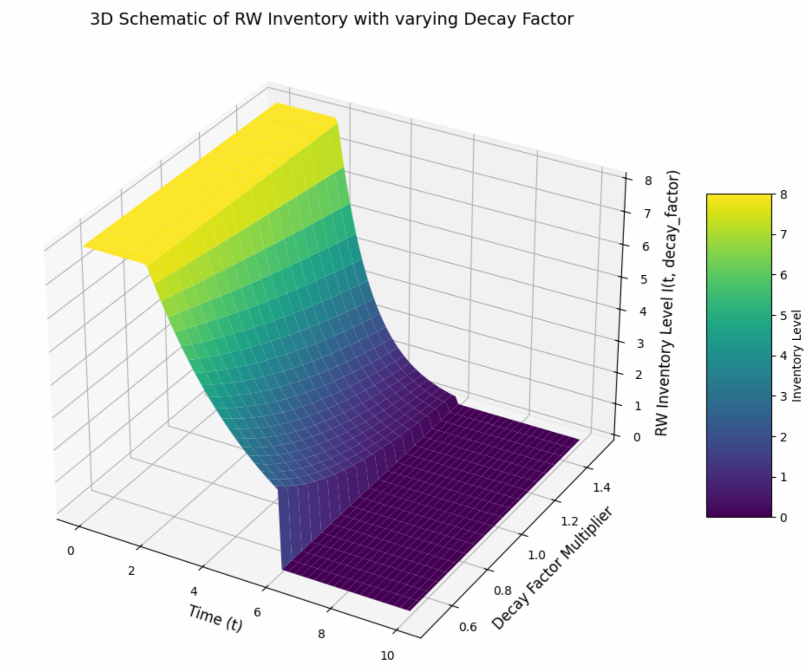
- The highest sensitivity is observed for the deterioration rate in RW2 (γ), followed by the quadratic demand coefficient (a) and shortage cost (C).
- Parameters associated with rented warehouses exhibit greater cost sensitivity than those of the owned warehouse, validating the priority-based depletion policy.
- Holding costs show moderate sensitivity, with RW2 holding cost having a stronger influence than OW holding cost.
- The backlogging parameter (δ) has relatively low sensitivity, indicating that backlogging policies fine-tune service levels rather than dominating cost behavior.

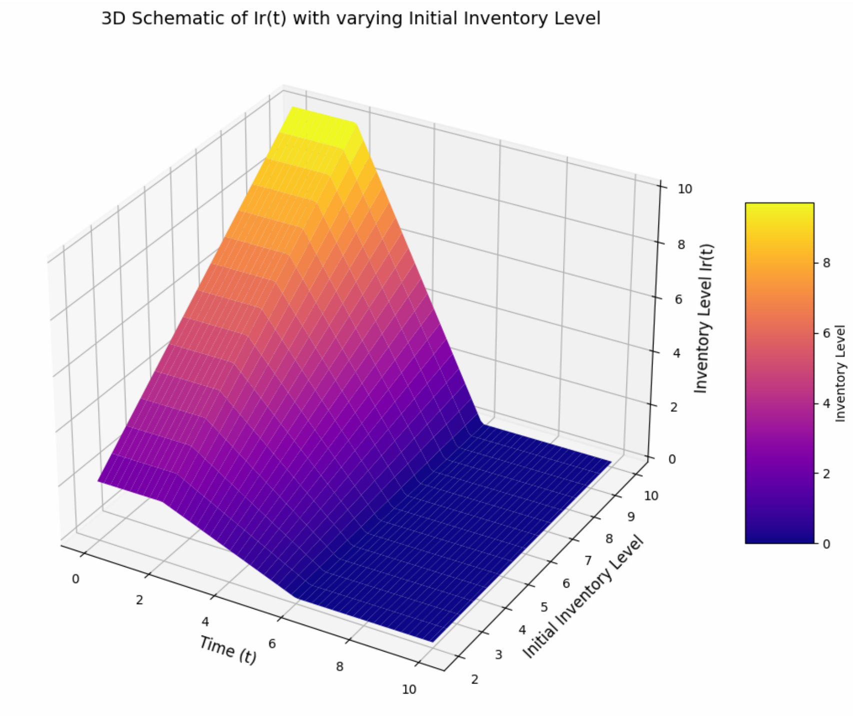
FIGURE 2

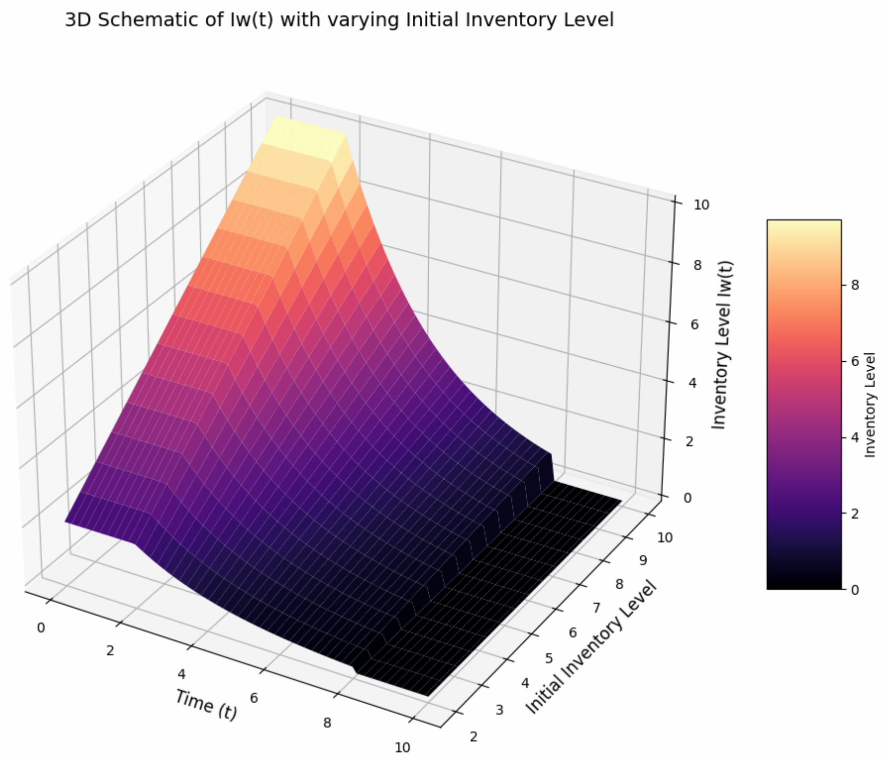


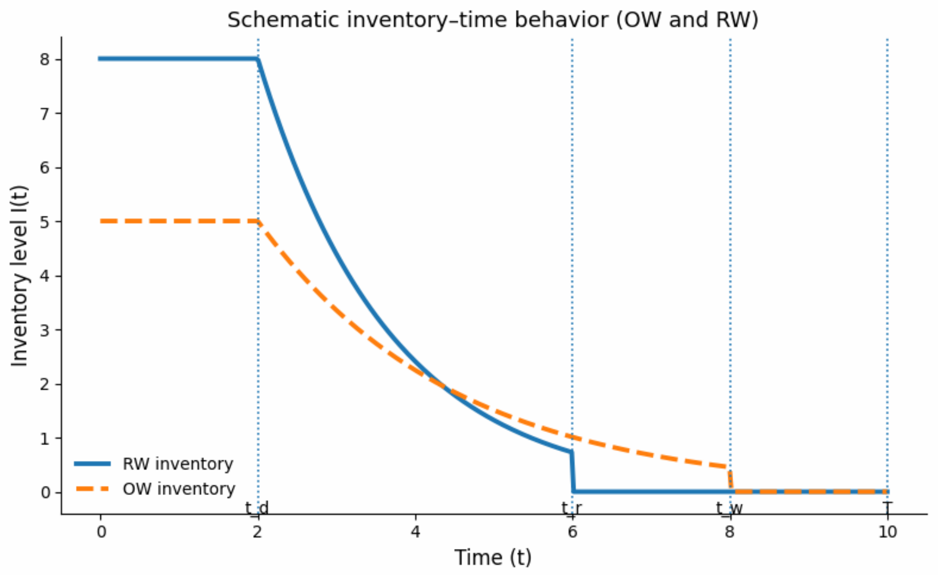
- All curves exhibit a monotonically increasing trend, confirming that ATC rises with increases in deterioration, demand acceleration, holding cost, and shortage penalty.
- The steepest curves correspond to RW2 deterioration (γ) and shortage cost (C), indicating that these parameters exert the strongest influence on system cost.
- The quadratic demand coefficient (a) also shows a pronounced nonlinear effect, highlighting the cost impact of demand acceleration over time.
- Holding cost in RW2 (h) displays moderate sensitivity, reinforcing the benefit of early depletion of rented warehouse inventory.
- The deterioration rate in the owned warehouse (α), although less steep than RW2, still has a significant impact on ATC, particularly for larger parameter deviations.


3D Schematic of RW and OW Inventories with varying Initial Level
RW Inventory
OW Inventory
Time (t)
Initial Inventory Level
Inventory Level
0
2
4
6
8
10
3
4
5
6
7
8
9
10


3D Schematic of RW Inventory with varying Decay Factor
Time (t)
Decay Factor Multiplier
RW Inventory Level I(t, decay_factor)
Inventory Level
0
2
4
6
8
10
0.6
0.8
1.0
1.2
1.4
0
1
2
3
4
5
6
7
8


3D Schematic of Ir(t) with varying Initial Inventory Level
Inventory Level Ir(t)
Time (t)
Initial Inventory Level
Inventory Level
0
2
4
6
8
10
2
3
4
5
6
7
8
9
10





VII. Managerial Interpretation

Managerial Implications of Time-Dependent Deterioration

- Strategic sequencing of inventory release: When deterioration intensifies over time, it is operationally efficient to prioritize issuing stock from storage locations with higher deterioration sensitivity. Early depletion of such inventories helps limit spoilage losses and reduces overall system costs, validating sequential dispatching policies.
- Limitations of constant deterioration assumptions: Models that assume fixed deterioration rates tend to undervalue both spoilage and shortage-related costs. This misrepresentation can misguide replenishment timing, warehouse utilization, and capacity planning decisions.
- Compression of viable storage periods: Increasing deterioration parameters effectively shorten the usable life of perishable goods. Managers must therefore consider tighter replenishment intervals or accelerated turnover strategies to sustain desired performance levels.
- Service-level trade-offs under extended cycles: Progressive deterioration exacerbates inventory shortages when replenishment cycles are prolonged, increasing dependence on partial backlogging.

This highlights the need to carefully balance customer service objectives against rising deterioration-driven costs.

- Enhanced cost visibility for decision-making: Incorporating time-varying deterioration allows managers to more accurately allocate holding and spoilage costs across warehouses, leading to improved budgeting accuracy, informed warehouse selection, and better long-term contracting decisions in perishable supply chains.

From a practical standpoint, the parameters governing time-varying deterioration may be calibrated using firm-specific empirical evidence, such as past spoilage and wastage records, experimental shelf-life assessments, or observed quality degradation patterns routinely analyzed in perishable supply chains and cold-storage systems. This empirical calibration enables organizations to tailor the proposed framework to particular products and storage conditions while preserving the underlying analytical formulation.

The numerical results indicate that incorporating **time-dependent deterioration rates** significantly influences optimal depletion times and total cost. Faster deterioration in OW forces earlier consumption of rented warehouses, validating the priority depletion policy. Sensitivity experiments (not shown) reveal that higher deterioration coefficients increase total cost sharply, emphasizing the importance of accurate deterioration modeling.

VIII. Conclusion

This study presents a comprehensive analytical framework for a three-warehouse perishable inventory system operating over a finite planning horizon. The model simultaneously accounts for non-instantaneous deterioration, quadratic time-dependent demand, partial backlogging, and deterioration rates that increase linearly with time in each warehouse. In contrast to much of the existing literature, which typically assumes constant deterioration or employs approximation-based methods, the proposed approach captures the progressive nature of spoilage and yields explicit closed-form solutions through linear differential equations, thereby strengthening both practical relevance and analytical transparency.

By integrating time-varying deterioration with demand dynamics within a unified three-warehouse structure, the model addresses an important gap in current inventory research. The derived expressions for inventory evolution, shortage accumulation, and cost components provide a clear representation of how perishable stock behaves over time. Numerical experiments demonstrate that neglecting the time-dependent nature of deterioration may significantly underestimate spoilage losses and overall system costs, particularly for longer replenishment cycles. Moreover, the results support the effectiveness of the priority-based issuing policy, indicating that warehouses experiencing faster deterioration should be depleted earlier to minimize total cost.

From an operational standpoint, the proposed model offers useful insights for managers responsible for perishable inventory systems with multiple storage facilities. It underscores the necessity of accurately assessing deterioration patterns, carefully determining replenishment intervals, and optimally allocating inventory among owned and rented warehouses. The framework is particularly applicable to sectors such as food distribution, pharmaceuticals, and chemical processing, where product quality declines progressively and storage conditions differ across locations.

Several avenues for future research emerge from this work. Potential extensions include incorporating uncertain or fuzzy demand, non-zero and variable lead times, environmental considerations such as carbon emission costs, inflationary effects, and investment in preservation technologies. The inclusion of dynamic pricing decisions or multiple products could further broaden the scope of the model. Overall, the present study establishes a solid analytical basis for the design and optimization of multi-warehouse perishable inventory systems and makes a meaningful contribution to both academic research and practical inventory management.

Data Availability

No datasets were generated or analyzed during the current study.

Competing Interests

The authors declare that they have no competing interests.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the authors used ChatGPT and Gemini in order to improve the readability and language of the manuscript, and to condense the abstract. After using this tool, the authors reviewed and edited the content and takes full responsibility for the content of the publication.

References

[1]. Ghare, P. M., & Schrader, G. F. (1963). A model for exponentially decaying inventory. Journal of Industrial Engineering, 14, 238–243.

[2]. Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with Weibull distribution deterioration. AIIE Transactions, 5, 323–326.

[3]. Philip, G. C. (1974). A generalized EOQ model for items with Weibull distribution deterioration. AIIE Transactions, 6, 159–162.

[4]. Goyal, S. K., & Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory. European Journal of Operational Research, 134, 1–16.
[5]. Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models. Journal of the Operational Research Society, 42, 27–37.
[6]. Dave, U., & Patel, L. K. (1981). (T, Si) policy inventory model for deteriorating items with time-proportional demand. Journal of the Operational Research Society, 32, 137–142.
[7]. Chung, K. J., & Ting, P. S. (1993). A heuristic for replenishment of deteriorating items with a linear trend in demand. Journal of the Operational Research Society, 44, 1235–1241.
[8]. Wu, K. S., & Ouyang, L. Y. (2001). A replenishment policy for deteriorating items with ramp-type demand rate. Production Planning & Control, 12, 787–793.
[9]. Wee, H. M. (1997). A replenishment policy for items with a price-dependent demand and a varying rate of deterioration. Production Planning & Control, 8, 494–499.
[10]. Yadav AS, Swami A, Kumar S, Singh RK. Two warehouse inventory model for dete riorating items with variable holding cost, time dependent demand and shortages. IOSR J Math 2016;12(2, Ver IV):47–53.
[11]. Teng, J. T., Chang, H. J., Dye, C. Y., & Hung, C. H. (2002). An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging. Operations Research Letters, 30, 387–393.
[12]. Patel R, Parekh RU. Inventory model for variable deteriorating items with two ware house under shortages, time-varying holding cost, inflation and permissible delay in payments. Int Ref J Eng Sci 2014;3(8):6–17.
[13]. Chang, H. J., & Dye, C. Y. (1999). An EOQ model for deteriorating items with time-varying demand and partial backlogging. Journal of the Operational Research Society, 50, 1176–1182.
[14]. Ouyang, L. Y., Wu, K. S., & Yang, C. T. (2006). A study on an inventory model for deteriorating items with partial backlogging and time-varying demand. International Journal of Systems Science, 37, 23–35.
[15]. Yang, H. L., & Wee, H. M. (2002). A multi-warehouse inventory model for deteriorating items with shortage under inflation. International Journal of Production Economics, 78, 169–179.
[16]. Rangarajan K, Karthikeyan K. EOQ models for noninstantaneous/instantaneous de teriorating items with cubic demand rate under inflation and permissible delay in payments. Ital J Pure Appl Math 2017;37:197–218.
[17]. Zhou, Y. W., & Yang, S. L. (2005). A two-warehouse inventory model for items with stock-level dependent demand rate. International Journal of Production Economics, 95, 215–228.
[18]. Jaggi, C. K., & Verma, P. (2010). Two-warehouse inventory model for deteriorating items with linear demand and shortages. International Journal of Procurement Management, 3, 235–256.
[19]. Singh, S. R., Gupta, V., & Jain, S. (2013). A three-warehouse inventory model with time-dependent demand and shortages. International Journal of Inventory Research, 1, 1–18.
[20]. Skouri, K., Konstantaras, I., Papachristos, S., & Ganas, I. (2011). Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. European Journal of Operational Research, 192, 79–92.
[21]. Cárdenas-Barrón, L. E. (2009). Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. Computers & Industrial Engineering, 57, 1105–1113.
[22]. Sarkar, B., & Majumder, A. (2013). Integrated vendor–buyer supply chain model with time-dependent deterioration and partial backlogging. International Journal of Production Economics, 145, 358–367.
[23]. Mishra, U., Singh, S. R., & Kumari, R. (2015). An inventory model for deteriorating items with time-dependent demand and holding cost under partial backlogging. Journal of Industrial Engineering International, 11, 195–205.
[24]. Lesmono, D., Limansyah, T., & Sandy, I. A. (2024). An inventory model for deteriorated items with time- and inventory-dependent demand and backorder policy. International Journal of Computing Science and Applied Mathematics, 10(2), 131–135.
[25]. De, P. K. (2024). Inventory model for non-instantaneous deteriorating goods under inflation with partial backlogging and price- and stock-dependent demand. Journal of Supply Chain & Operations.
[26]. Kaushik, J. (2025). An inventory model for deteriorating items with ramp-type deterioration and partial backlog shortages. Inventory Management & Optimization Journal, 2025, article ID 2442547.
[27]. Begum R, Sahu SK, Sahoo RR. An inventory model for deteriorating items with quadratic demand and partial backlogging. Br J Appl Sci Technol 2012;2(2):112–31.
[28]. Tripathi RP. Inventory model with different demand rate and different holding cost. Int J Ind Eng Comput 2013;4(3):437–46.
[29]. Jaggi CK, Tiwari S. Two-warehouse inventory model for non-instantaneous deteri orating items with price dependent demand and time-varying holding cost. Math Model Appl 2014:225–38.
[30]. Bishi B, Behera J, Sahu SK. Two-warehouse inventory model for non-instantaneous deteriorating items with exponential demand rate. Int J Appl Eng Res 2019;14(2):495–515.
[31]. Yadav AS, Swami A. An inventory model for non-instantaneous deteriorating items with variable holding cost under two-storage. Int J Procure Manag 2019;12(6):690–710.
[32]. Nath BK, Nabendu. A completely backlogged two-warehouse inventory model for non-instantaneous deteriorating items with time and selling price depending de mand. Int J Appl Comput Math 2021;7(145).
[33]. Gupta R, Srivastava M. Three-warehouse production inventory model for deterio rating items with advertisement and price dependent demand. Int J Noval Res Dev 2022;7(5).
[34]. Kumar N, Manna AK, Shaikh AA. Application of hybrid binary tournament-based quantum-behaved particle swarm optimization on an imperfect production inven tory problem. Soft Comput 2021;25:11245–67.
[35]. Supakar P, Manna AK, Mahato SK. Application of artificial bee colony algorithm on agreen production inventory problem with preservation for deteriorating items in neutrosophic fuzzy environment. Int J Syst Assur Eng Manag 2022;15:672–86.
[36]. Akhtar Md, Manna AK, Bhunia AK. Optimization of a non-instantaneous deterio rating inventory problem with time and price dependent demand over finite time horizon via hybrid DESGO algorithm. Expert Syst Appl 2023;211:118676.
[37]. Ali AS, Cárdenas-Barrón LE, Manna AK, Céapedes-Mota A. An economic produc tion quantity (EPQ) model for a deteriorating item with partial trade credit policy for price dependent demand under inflation and reliability. Yugosl J Oper Res 2021;31(2):139–51.
[38]. Manna AK, Akhtar M, Shaikh AA, Bhunia AK. Optimization of a deteriorated two warehouse inventory problem with all-unit discount and shortages via tournament differential evolution. Appl Soft Comput 2021;107:107388.