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Research Paper

Quantification of slip along deformation using Finite Element Method

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ABSTRACT:- *Dislocation models can be simulated in active deformation caused by slip along a fault. The high-precision Global Positioning System (GPS) is used to geodetically constrain the motion of stations in the seismological areas and examine the deformation using IGS (International GPS Service) fiducial stations. Both forward and inverse modelling is used to understand the information about the deformation area. Finite Element Method (FEM) can be used also in 2-D and 3-D system to understand slip along deformation. In this paper slip has been calculated using 3-D system and techniques has been presented to calculate the slip and the model the causative fault(s) that could have produced the observed deformation.*

I. INTRODUCTION

The dislocation theory was first introduced in the field of seismology [1, 2]. Numerous theoretical formulations have been described for the deformation of an isotropic homogeneous semi-infinite medium and have been developed with increasing completeness and simplification of source type and geometry [1]. Anderson [3] recognized that since the principal stress directions are directions of zero shear stress, the fault can be placed in the context of principal stress. The faults have a common meaning, to shorten the crust one direction and extend the crust in other direction. In Cartesian Co-ordinate system (x,y,z) the half space occupied region z<0 if fault is located at (0,0,-d) the point force distribution can be given in following form [4]:

$$
\begin{aligned}
F_1 &= \frac{\mu}{\lambda+\mu}\left\{\frac{x^2-y^2}{r^4}(R-d)-\frac{x^2}{r^2R}\right\}\\
F_2 &= \frac{\mu}{\lambda+\mu}\left\{\frac{2xy}{r^4}(R-d)-\frac{xy}{r^2R}\right\}\\
F_3 &= \frac{\mu}{\lambda+\mu}\left\{\frac{x}{r^2}\left(1-\frac{d}{R}\right)\right\}\\
\text{where}\quad R &= \left(r^2+d^2\right)^{1/2}=\left(x^2+y^2+d^2\right)^{1/2}
\end{aligned}
\qquad (1)
$$

Thrust faults: F_1 and F_2 will be horizontal and F_3 will be vertical.
Normal faults: F_2 and F_3 will be horizontal and F_1 will be vertical
Strike-slip faults: F_1 and F_3 will be horizontal and F_2 will be vertical [3]
Martin, H. Sadd [5] explain Navier,s equation of elasticity can be expressed in vector form

$$\mu\nabla^2\vec{u}+(\lambda+\mu)\nabla\left(\nabla.\vec{u}\right)+F=0$$

or written out in terms of the three scalar equations

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+F_1=0$$

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+F_2=0 \qquad (2)$$

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+F_3=0$$

Where F_1,F_2, and F_3 represent the body forces in x,y and z direction and u, v ,w are corresponding displacements. The above equation can be solve by Galerkin [6] formulation for stress analysis which gives the solution

$$\{f\}_s+\{f\}_X=[k]\{u\} \qquad (3)$$

Where $\{f\}_s$=surface force
$\{f\}_x$= body force , [k]=global stiffness matrix , {u}= displacement matrix
The Forces equation (1) will work as a body force in equation (3).

II. FINITE ELEMENT MODEL

The fundamental equation of Finite Element method (FEM), the displacements U on node of a body (Fig: 1) having area cross section A is given by:

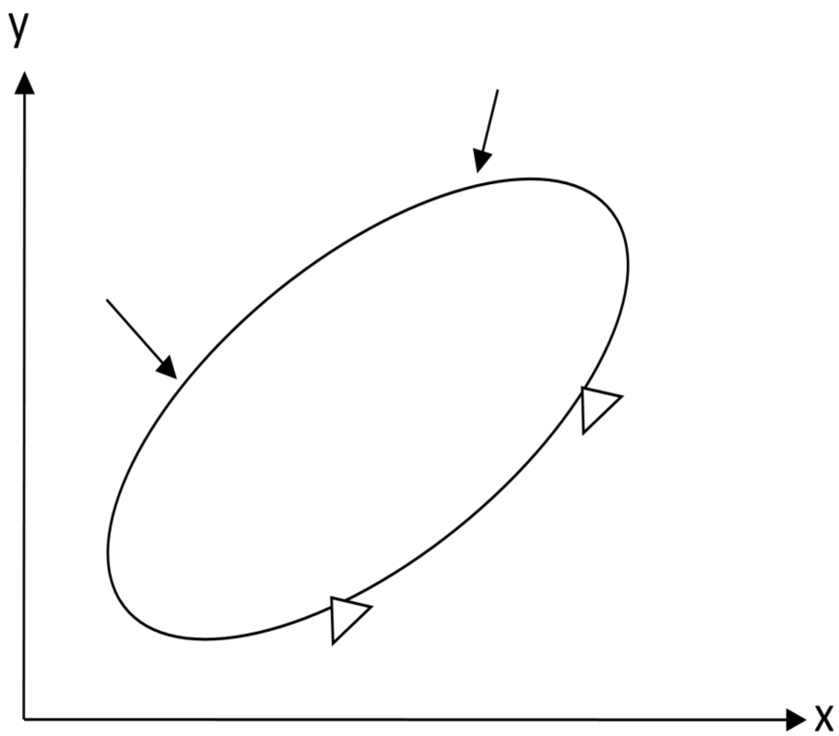

Fig 1: Body showing surface area in which force are acting in x and y direction

$$U=\frac{Fl}{AE}$$

$$F=\frac{AE}{l}U$$

$$F=KU \qquad (4)$$

Where F is acting force, E is elasticity constant and l is the length of the body and K is called Stiffness constant. The body and surface forces will act on body nodes surface displacement will take place in x and y direction (Fig: 2)

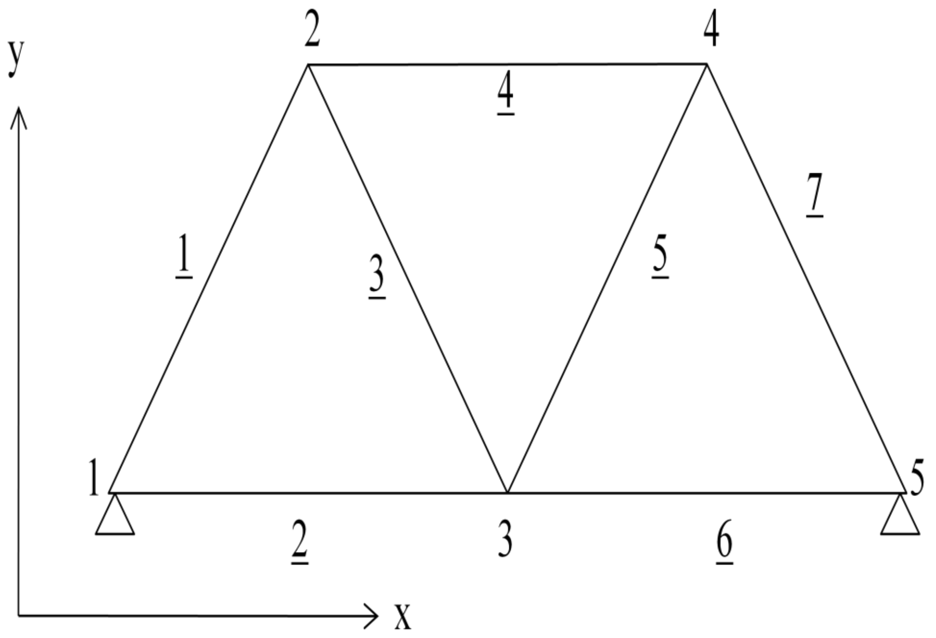

Fig 2: Displacements of node in x, y direction. The numbers are showing no. of nodes and Bar numbers are showing no. of elements

That is

$$\begin{bmatrix} k_{11}^1 & k_{12}^1 & k_{13}^1 & k_{14}^1 \\ k_{21}^1 & k_{22}^1 & k_{23}^1 & k_{24}^1 \\ k_{31}^1 & k_{32}^1 & k_{33}^1 & k_{34}^1 \\ k_{41}^1 & k_{42}^1 & k_{43}^1 & k_{44}^1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} F_1^x \\ F_1^y \\ F_2^x \\ F_2^y \end{bmatrix} \qquad (5)$$

Melosh and Raefsky present a solution in 2-D system that an active deformation causative fault(s) fracture node will remove force [7]. Similarly for 3-D system due to the active deformation causative fault(s) fracture (Fig: 3) the above equation no. (5) will become like this

$$\begin{bmatrix} k_{11}^1 & k_{12}^1 & k_{13}^1 & k_{14}^1 \\ k_{21}^1 & k_{22}^1 & k_{23}^1 & k_{24}^1 \\ k_{31}^1 & k_{32}^1 & k_{33}^1 & k_{34}^1 \\ k_{41}^1 & k_{42}^1 & k_{43}^1 & k_{44}^1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} F_1^x - k_{13}^1 \Delta u_2^1 - k_{14}^1 \Delta v_2^1 \\ F_1^y - k_{23}^1 \Delta u_2^1 - k_{24}^1 \Delta v_2^1 \\ F_2^x - k_{33}^1 \Delta u_2^1 - k_{34}^1 \Delta v_2^1 \\ F_2^y - k_{43}^1 \Delta u_2^1 - k_{44}^1 \Delta v_2^1 \end{bmatrix} \qquad (6)$$

Similarly for other element the above equation can be written like this

$$\begin{bmatrix} k_{11}^2 & k_{12}^2 & k_{13}^2 & k_{14}^2 \\ k_{21}^2 & k_{22}^2 & k_{23}^2 & k_{24}^2 \\ k_{31}^2 & k_{32}^2 & k_{33}^2 & k_{34}^2 \\ k_{41}^2 & k_{42}^2 & k_{43}^2 & k_{44}^2 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} F_2^x - k_{11}^2 \Delta u_2^1 - k_{12}^2 \Delta v_1^2 \\ F_2^y - k_{21}^2 \Delta u_1^2 - k_{22}^2 \Delta v_1^2 \\ F_3^x - k_{31}^2 \Delta u_1^2 - k_{32}^2 \Delta v_1^2 \\ F_3^y - k_{41}^2 \Delta u_1^2 - k_{42}^2 \Delta v_1^2 \end{bmatrix} \qquad (7)$$

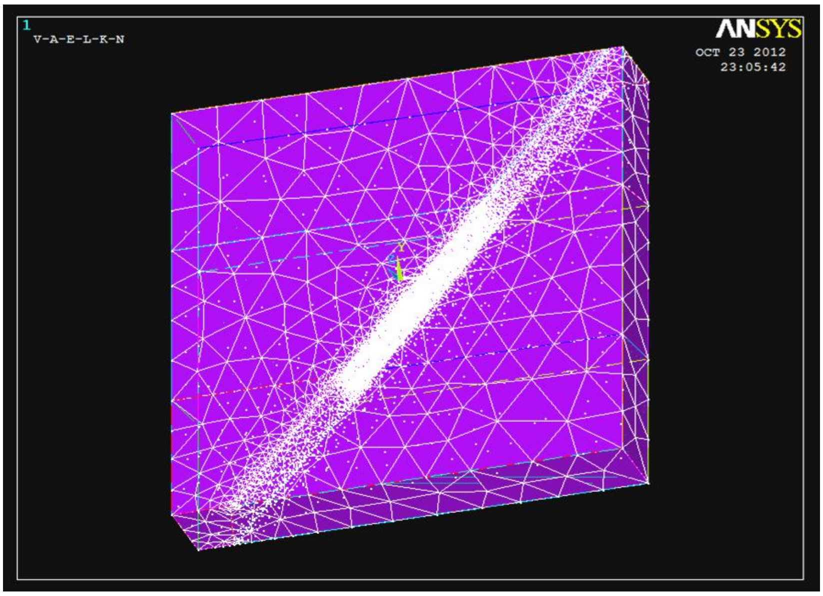

Fig: 3, ANSYS (Brick 8 node 185) element [8], White concentrated area is showing finite rectangular fault.

On adding both the elements equations (6 & 7) the final equation will be like this

$$\begin{bmatrix} k_{11}^1 & k_{12}^1 & k_{13}^1 & k_{14}^1 & 0 & 0 \\ k_{21}^1 & k_{22}^1 & k_{23}^1 & k_{24}^1 & 0 & 0 \\ k_{31}^1 & k_{32}^1 & k_{33}^1+k_{11}^2 & k_{34}^1+k_{12}^2 & k_{13}^2 & k_{14}^2 \\ k_{41}^1 & k_{42}^1 & k_{43}^1+k_{21}^2 & k_{44}^1+k_{22}^2 & k_{23}^2 & k_{24}^2 \\ 0 & 0 & k_{31}^2 & k_{32}^2 & k_{33}^2 & k_{34}^2 \\ 0 & 0 & k_{41}^2 & k_{42}^2 & k_{43}^2 & k_{44}^2 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} P_1^x - k_{13}^1\Delta u_2^1 - k_{14}^1\Delta v_2^1 \\ P_1^y - k_{23}^1\Delta u_2^1 - k_{24}^1\Delta v_2^1 \\ P_2^x - k_{33}^1\Delta u_2^1 - k_{34}^1\Delta v_2^1 - k_{11}^2\Delta u_1^2 - k_{12}^2\Delta v_1^2 \\ P_2^y - k_{43}^1\Delta u_2^1 - k_{44}^1\Delta v_2^1 - k_{21}^2\Delta u_1^2 - k_{22}^2\Delta v_1^2 \\ P_3^x - k_{31}^2\Delta u_1^2 - k_{32}^2\Delta v_1^2 \\ P_3^y - k_{41}^2\Delta u_1^2 - k_{42}^2\Delta v_1^2 \end{bmatrix}$$

Then modeled slip will be $\left\{\sqrt{\left(\Delta u_2^1\right)^2+\left(\Delta v_2^1\right)^2}+\sqrt{\left(\Delta u_1^2\right)^2+\left(\Delta v_1^2\right)^2}\right\}$ (8)

The modelled slip can be calculated with the help of equation (8) easily.

III. DISCUSSION

I present a simple method to calculate the slip of finite rectangular fault. The above technique can be used in forward modelling method. Post processing high-precision campaign mode GPS station data gives observed displacements in x and y directions. Body force and surface force can be calculated with the help of equations (1, 2 & 3) on GPS stations. Then stiffness matrix will work as a model parameter of finite rectangular fault. This suggests that my uniform geologic structure of FEM model may be sufficient for simulating large continental earthquakes.

REFERENCES

[1]. Steketee, J. A., On Volterra's dislocation in a semi-infinite elastic medium, Can. J. Phys. 36, 1958,192-205.
[2]. Rongved, L. and Frasier, J. T., Displacement discontinuity in the elastic half-space, J. Appl: Mech., 25, 1958, 125-128.
[3]. http://www.geology.cwu.edu/facstaff/charlier/courses/g360/anderson.html
[4]. Sato, R. and M. Matsu'ura, Strains and tilts on the surface of a semi-infinite medium, J.Phys.Earth 22, 1974, 213-221.
[5]. Martin,H. Sadd, Elasticity Theory Applications and Numerics, page 103
[6]. http://en.wikipedia.org/wiki/Galerkin_method
[7]. Melosh H.J. and Raefsky ,A., A simple and efficient method for introducing faults into finite element computations, Bulletin of the Seismological Society of America, 71(5), 1981, 1391-14000
[8]. http://www.ansys.com/