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Research Paper

Forecasting Rainfall in Saudi Arabia via Transfer Function Models

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ABSTRACT:
Nowadays, a lot of statistical techniques for studying the dynamic relationship between the variables have been used; this paper aims at modeling and predicting the rainfall amounts at Al-Baha rejoin in Saudi Arabia with transfer function of time series analysis technique. Prediction procedures constructed through several stages, namely; identification, pre whitening and identifying the value (b,r,s) via the cross-correlation function. Monthly data representing rainfall mean temperature and air humidity collected from King Khalid airport. The empirical finding revealed that based on transfer function model of multi input with single output , the rainfall at Al-Baha rejoin in Saudi Arabia on a certain month is directly affected by mean temperature and air humidity.
KEY WORDS: *Transfer Function, Rainfall, Temperature, Air Humidity*

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I. INTRODUCTION:

Al-Baha rejoins has a moderate desert climate affected by its varying geographic features. The rainfall amount falls over Al-Baha area is very important factor to evacuate the amount of water available to meet the demands of industry, agriculture, and other human activities.

II. MATERIALS AND METHODS:

In modeling and forecasting time series, there are some cases, multivariate time series are used to study the dynamic relationship between the variables; this case is described by multiple time series. For example, if we have two time series, one is called the input series and the other is called the output series, if we denotes the input series by X_t and the output series by Y_t, then the effect of the input series X_t to the output series Y_t can be studied through a relationship or function which known as the transfer function.

Transfer function is a multivariate time series analysis technique, it used to study the dynamic relationship among input and output series, it is also use to analyze the impact of input series X_t on the output series Y_t.[2-3]

Consider the two time series variables y_t and x_t both variables stationary, the transfer function model (TFM) can be expressed as follows:

$$y_t = \mu_j + \sum_{j=1}^{k} \frac{w(B)}{\delta(B)} x_{jt-b} + \frac{\theta(B)}{\phi(B)} \varepsilon_t \qquad (1)$$

where:
$y_t \equiv$ the dependent variable (output series).
$x_{jt} \equiv$ the independent variable (input series).
$\mu_j \equiv$ constant term,
$\varepsilon_t \equiv$ the noise series of the system that is independent of the input series (the stochastic disturbance).
$w(B) = w_0 + w_1 B + w_2 B^2 + \cdots + w_h B^h \,;\, \delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r$ (2)
The parameters r, b, h represent the order of the numerator polynomial , the delay parameter which called dead-time or delay time, and order of the denominator polynomial respectively.
It's assume that the roots of all the polynomials $w(B), \delta(B), \theta(B), \phi(B)$ lies outside the unit circle root as well as differencing may be required to produce Stationarity of both input and output series.

$$\frac{\theta(B)}{\phi(B)} \varepsilon_t = noise\ ARMA \qquad (4)$$

The construction of transfer function models (TFM) is likewise ARIMA modelling, it can be done through the stages of identification, estimation and diagnostic checking.[4-6]
In identification state the researcher must be finding out the orders (*b*, *r*, *h*) of a rational form transfer function, and then nonlinear least square method can be used to estimate parameters, after estimation of the model, researcher must have to check for randomness of disturbance term series of identified and estimated model. Figure (1) bellow illustrates the diagram of transfer function model.

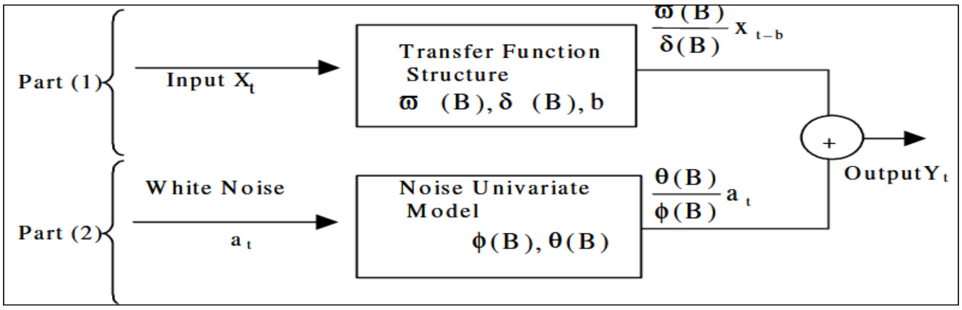

Figure (1) The Transfer Function Model Diagram**.** The cross-correlation function between two time series Y_t and X_t is expressed as follows:

$$r_{x,y}(k) = \frac{C_{x,y}(k)}{S_x S_y} \qquad (5)$$

where:

$$C_{x,y}(k) = \begin{cases} \frac{1}{n}\sum_{t=1}^{n-k} (x_t - \underline{x})(y_{t+k} - \underline{y}) & k \geq 0 \\ \frac{1}{n}\sum_{t=1}^{n+k} (x_{t-k} - \underline{x})(y_t - \underline{y}) & k \leq 0 \end{cases} \qquad (6)$$

$$S_x = \sqrt{\frac{1}{n}\sum_{t=1}^{n} (x_t - \underline{x})^2} \qquad (7)$$

$$S_y = \sqrt{\frac{1}{n}\sum_{t=1}^{n} (y_t - \underline{y})^2} \qquad (8)$$

The cross correlation function is not symmetric at $k = 0$.[6]
The objective of the identification stage is to obtain some idea of the order ranks of the model and to derive initial guesses for the parameter ll, wand the delay parameter b. In the same way that the auto-correlation function is used to identify p, d, q parameters of the univariate model, the r, s and b parameters for the transfer function models are identified by the cross correlation between input and output series. Following Box-Jenkins the whole process of identification, estimation and diagnostic and forecasting can be outlined as follows:
The basic steps of transfer function modeling are as follows (Wei, 2006; Box et al, 1994):
1- Prewritten the input series. Transfer function models require both input and output series to be stationary. This step includes the traditional way of modeling time series: inspect the autocorrelation functions (ACF) and partial autocorrelation functions (PACF) for possible differencing and determining the order of the ARIMA model, then investigating the behavior of the noise series as diagnostic checking. The pre whitened input series is expressed as follows:

$$\alpha_t = \frac{\phi_x(B)}{\theta_x(B)} x_t \qquad (9)$$

Where:
$\alpha_t \equiv$ a white noise process with zero mean and constant variance σ_α^2.
2- Prewritten the output series. The output series will then be filtered using the pre whitened input series defined in step 1. The transformed output series is written by the following equation:

$$\beta_t = \frac{\phi_x(B)}{\theta_x(B)} y_t \qquad (10)$$

3- Calculate the sample cross correlation function (CCF) between α_t and β_t and estimate the transfer function.. It is important to note that both input and output series must be pre whitened in order to have meaningful interpretations of the CCF.

4- Estimate the noise series and combine it with the function in step 4 to have the estimated transfer function model. In transfer function modeling, the noise process is not limited to a white noise process.

III. LITERATURE REVIEW:

Modeling and forecasting rainfall at Al-Baha rejoin in Saudi Arabia requires finding a model that reasonably represents it. In the literature many academics and practitioners suggest a number of approaches for building a time series models are discussed however, the suitability of any of these methods to a given time – series data has to be judged on the basis of its fit to that data. In this study transfer function model discussed by Box and Tiao will applied to data representing rainfall, mean temperatures and air humidity in Saudi Arabia – Al-Baha region for the period 1/1990 – 12/2019. With the objective of modeling and forecasting rainfall at Al-Baha rejoin in Saudi Arabia. Priska Arindya et at (2017) obtained the appropriate transfer function to model rainfall data in Batu City based on air temperature, humidity, wind speed and cloud. Their results of rainfall forecasting in Batu City using multi input with single output transfer function model was accurate based on the result of model validation using t test and MAPE statistic that less than 20%. Rashmi .N and Sudhir .N (2017) carried out a simulation study of rainfall and runoff data of Kulfo River of Southern Ethiopia, twenty years rainfall and runoff time series of Kulfo River are used to develop Transfer Function Model used to forecast the river runoff, their empirical results indicate that transfer function model performs excellently. Asep rusyana ea at (2019) Predicted rainfall based on its components using transfer function, they used monthly data included rainfall, air temperature, and wind speed data from January 2009 to December 2016 Aceh Statistics Center Board (BPS Aceh), they conclude that based on the model, rainfall in a particular month was affected by the rainfall of the previous one month up to the previous four months also the temperature of the previous four months up to the previous seven months.

IV. RESULTS AND DISCUSSION:

This section provides the application of transfer function technique in order to model and forecast rainfall at Al-Baha rejoin in Saudi Arabia, the data used in the analysis of this paper are rainfall quantity, average of temperature and air humidity rate. Monthly time data covered the period from January 1990 to December 2019 are used in the analysis of this paper, the data were supplied by King Khalid International Airport. Figure. 2 bellow shows the sequence chart of rainfall, mean temperatures and air humidity in Saudi Arabia – Al-Baha region for the period 1/1990 – 12/2019.

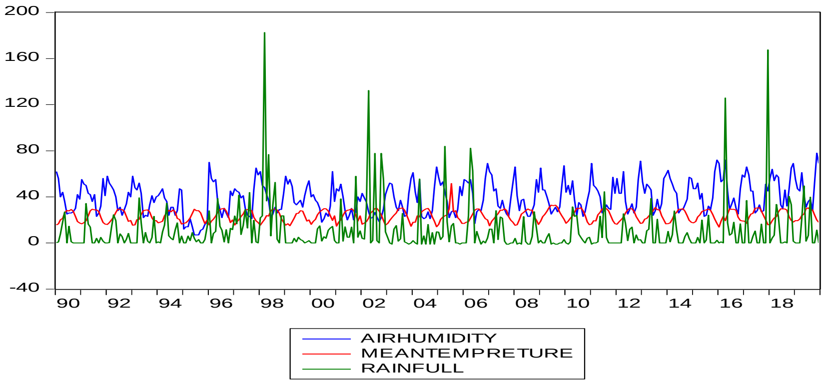

Figure (2) Rainfall, Mean Temperatures and Air Humidity multi graph Saudi Arabia – Al-Baha from the period 1/1990 – 12/2019.

Table 2 bellow reports the application of ADF test on data representing rainfall, mean temperatures and air humidity at Al-Baha rejoin in Saudi Arabia for the period January 1990 to December 2019.

Table (1) Augmented Dickey Fuller test results:

Variable	Augmented Dickey Fuller test	
	Level	First difference
Rainfall	$-16.81*$ Pob (0.000)	−
Mean Temperature	-3.34 Pob (0.142)	$-16.56*$ Pob (0.000)
Air Humidity	-2.30 Pob (0.171)	$-13.75*$ Pob (0.000)

∗ Indicate the test is significance at 0.05 significance level

The ADF test results in table. 1 above confirmed that rainfall series was stationary at series level whereas mean temperatures as well as mean temperatures and air humidity were stationary at first difference.

Based on Box-Tiao (1975) building a transfer function model for describing the dynamic relationship among rainfall and mean temperatures and air humidity at Al-Baha regions in Saudi Arabia requires identifying the ranks ***(b, r, h)*** of ARIMAX- TF model, estimation and finally diagnostic checking. Following the construction of prewritten both output rainfall and input mean temperatures and air humidity series for building a transfer function model it can be seen that:

Figure. 3 show the coreelogram of rainfall data in Saudi Arabia – Al-Baha from the period 1/1990 – 12/2019, both ACF and PACF shows not clear pattern to identify a suitable ARIMA models, hence numerous ARIMA models has been suggested in order to select an appropriate model for forecasting rainfall at Al-Baha rejoin using certain criteria such as AIC, BIC, of model selection criteria.

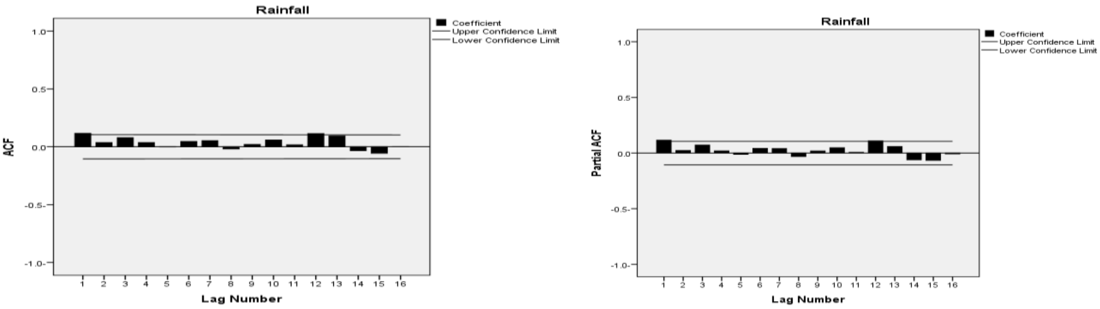

Figure (3) Correlogram of rainfall data in Saudi Arabia for the period (1990 – 2019)

Table. 1 bellow report the suggested ARIMA models to predict rainfall in at Al-Baha rejoin.

Table (2) Suggested ARIMA models for rainfall data:

$ARIMA(p,d,q)$	AIC	BIC	HQC
ARIMA(0,0,0)	8.93	8.94	8.93
ARIMA(1,0,1)	8.93	8.97	8.94
ARIMA(2,0,1)	8.93	8.98	8.95
ARIMA(1,0,2)	8.93	8.98	8.95
ARIMA(2,0,2)	8.93	9.00	8.96
ARIMA(1,1,1)	9.28	9.32	9.29

According to models selection criteria in table 2 above, the stochastic model **ARIMA(**0,0,0**)** was chosen as a an appropriate model for rainfall data. The output rainfall series estimated equation is written as follows:

$$Rainfall = 10.57 + \varepsilon_{1t}$$

Figure. 4 show the coreelogram of mean temperatures data in Saudi Arabia – Al-Baha from the period 1/1990 – 12/2019. It can be seen that the ACF decay to zero in a sign wave, while the PACF shows positive and negative peaks decays slowly to zero at increasing lags. Seasonal variation was also preset in the dada. ADF test confirmed that the mean temperatures series was stationary at first difference, therefore the multiplicative model **ARIMA(**1,1,0)$(1,1,0)^{12}$ was identified for prewritten the mean temperatures series.

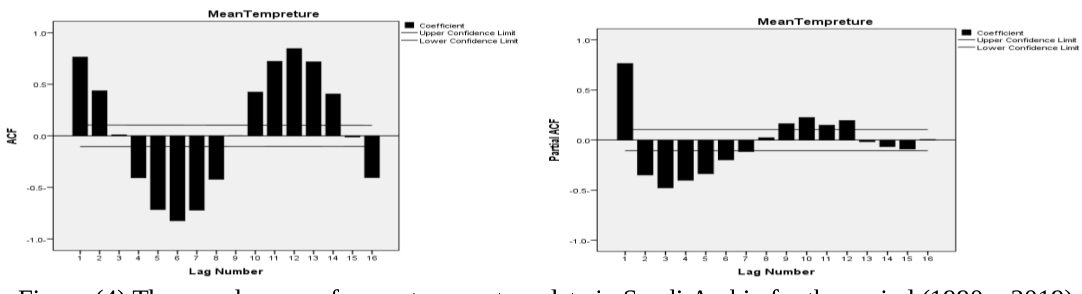

Figure (4) The correlogram of mean temperature data in Saudi Arabia for the period (1990 – 2019)

The input mean temperatures series estimated equation is written as follows:

$$(1 + 0.048B)(1 + 0.46B^{12})Mean\ Temperatures = \varepsilon_{2t}$$

In this identified and estimated model both nonseasonal as well as seasonal parameters are statistically significant

Figure. 5 show the coreelogram of air humidity data in Saudi Arabia -Al-Baha from the period 1/1990 – 12/2019. It can be seen that the ACF decay to zero in a sign wave, while the PACF shows positive and negative peaks decays slowly to zero at increasing lags. Seasonal variation was also preset in the dada. ADF test confirmed that air humidity series was stationary at first difference, therefore the multiplicative model **ARIMA(**$(1,1,0)(1,1,0)^{12}$ was identified for prewritten the air humidity series

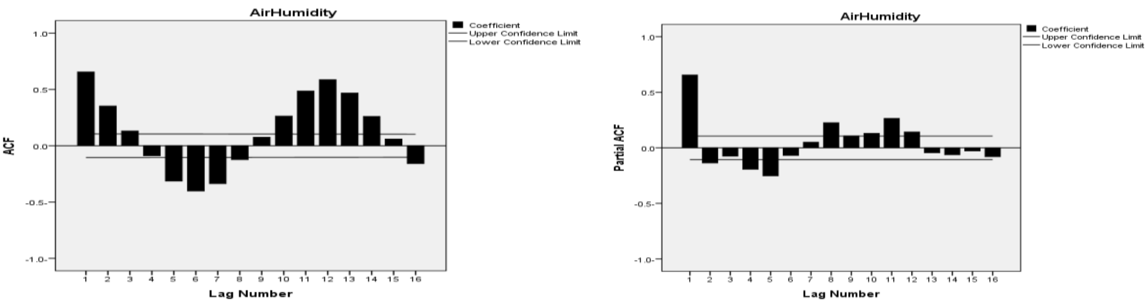

Figure (5) The correlogram of air humidity data in Saudi Arabia for the period (1990 – 2019)

The input air humidity series estimated equation is written as follows:

$$(1+0.33B)(1+0.53B^{12})air\ humidity\ =0.04+\varepsilon_{3t}$$

In this identified and estimated model both no seasonal as well as seasonal parameters are statistically significant.

The cross correlation between prewritten rainfall and mean temperature series are shown in figure. 6 bellow, the value of b, s, r for the mean temperature input series and the rainfall output series are (b = 0, s = 0, r = 0).

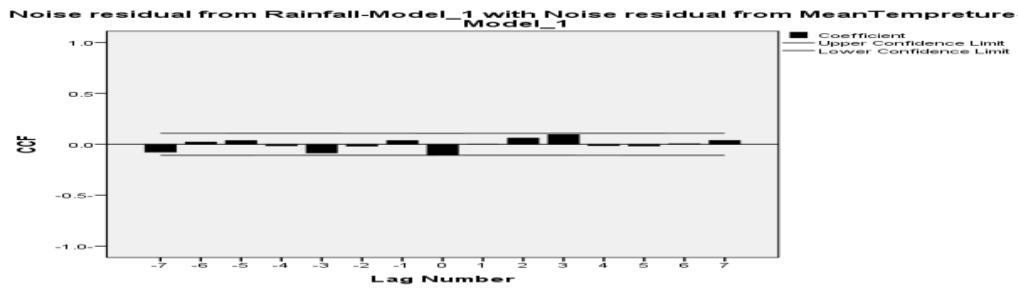

Figure (6) The cross correlation between prewritten rainfall and mean temperature series

The estimated equation of the transfer function model TFM*(0, 0, 0)* model is expressed as follows:

$$Rainfall_t = 11.32 - 0.03\ Mean\ Tempreture\ _t + \varepsilon_{1t}$$

The cross correlation between prewritten rainfall and air humidity series are shown in figure. 7 bellow, the value of b, s, r for the humidity input series and the rainfall output series are ($b\ =\ 0, s\ =\ 0, r\ =\ 0$)

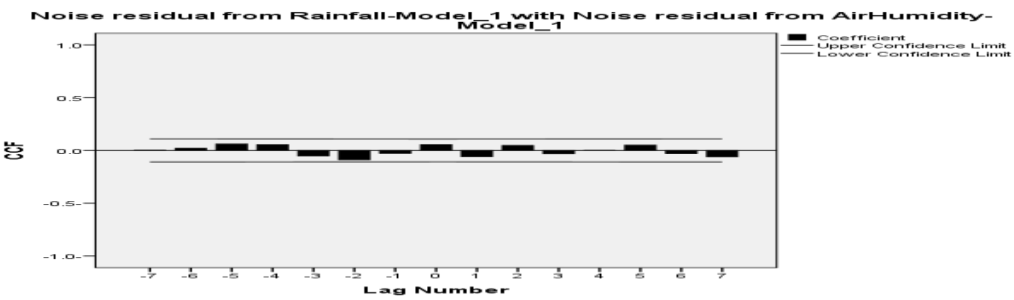

Figure (6) The cross correlation between prewritten rainfall and mean air humidity series

The estimated equation of the transfer function model TFM*(0, 0, 0)* model is expressed as follows:

$$Rainfall_t = 6.4 + 0.11 Air\ Humidity\ _t + \varepsilon_{2t}$$

The multi input transfer function with single output for rainfall prediction at Albaha rejoins in Saudi Arabia is expressed as follows:

$$Rainfall_t = -3.7 + 0.03\ Mean\ Tempreture_t + 0.11\ Air\ Humidity\ _t + \varepsilon_{3t}$$

V. CONCLUSION:

In this paper Transfer function technique of time series models discussed by Box and Tiao was applied to data representing rainfall, mean temperatures and air humidity at Al-Baha rejoin in Saudi Arabia for the period January 1990 to December 2019. Augmented Dickey-Fuller (ADF) tests confirmed that rainfall series was stationary at series level whereas mean temperatures as well as mean temperatures and air humidity were stationary at first difference. Both ACF and PACF confirmed that **ARIMA(**$1,1,0)(1,1,0)^{12}$) was appropriate model for modeling both mean temperatures and air humidity, however the stochastic model **ARIMA(**$0,0,0$**)** was chosen as a an appropriate model for rainfall data. Based on Transfer function identification technique, TF Model $(0, 0, 0)$ model was chosen as an appropriate for modeling both mean temperatures and air humidity data. Multi input transfer function with single output for rainfall prediction at Albaha rejoins in Saudi Arabia was performed and tested, the findings conclude that for this particular type of data, TF Models are highly recommended.

REFERENCES:

[1]. G. Box, G. Jenkins dan J. Reisel, Time Series Analysis Forecasting and Control, 4th penyunt, New Jersey: Wiley, 2015.
[2]. Asep rusyana ea at (2019), Forecasting Rainfall Using Transfer Function in Aceh Province, Indonesia vol. 13, No. 7, pp: 299 - 307
[3]. Khalid A. Alkhuzai (2019), Hydrological Analysis of Rainfall Data for Drought Investigation in Albaha City, International Journal of Scientific & Engineering Research Vol 10, Issue 1, pp: 88-107
[4]. Liu, L. (1991). Use of Linear Transfer Function Analysis in Econometric Time Series Modeling. Statistisca Simica1:503-525
[5]. Marek, L.(2000).Transfer function models. Acta O economica Pragnesia vol 8:pp: 83-94
[6]. Priska Arindya et at (2017), Modelling Multi Input Transfer Function for Rainfall Forecasting in Batu City, Jurnal Matematika Murni dan Aplikasi, Volume 5, pp: 29-35
[7]. R. H. Shumway and D. S. Stoffer, Time Series Analysis and Its Applications With R Examples Third Edition. New York, Dordrecht, Heidelberg, London, Springer, 2006.
[8]. S. Makridakis, S. C. Wheelwright, and V. E. Mc GEE (1983), Forcasting: Methods and Applications, Second Edition. New York: Wiley,.
[9]. Rashmi .N and Sudhir .N (2017), Multivariate Rainfall - Runoff Modeling of Kulfo River, Journal of Current Environmental Engineering Continued as Current Environmental Management, vol 4, issue 3, pp: 177 – 188.
[10]. Tsay R. (2005). Analysis of Financial Time Series, 2nd Edition. A Wiley-Inter science publication. John Wiley & Sons. Inc. New York.
[11]. Tasy, R. (2009). Transfer function Model. School of Business, University of Chicago, USA**.**
[12]. Rublikova, E. and Marek, L. (2001). Linear transfer function model for out flow rates. Ekonomické rozhľady.vol 30 pp: 457- 466.