



# Basic Principles of Least Squares Adjustment Computation Comparison in a Baseline Calibration Surveying

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## ABSTRACT:

This research study presents the approaches of basic principles' techniques of least square adjustment computation, which are first principle, observation equations and condition equations techniques in an EDM baseline Calibration Surveying. EDM instruments are modern surveying distance measuring equipment that needs baseline for regular and proper calibrations for their performance in terms of instrument constant, standard error and its accuracy. An EDM Calibration baseline was proposed at the Surveying and Geoinformatics Department Jambutu Campus for such purposes. The 39.715m baseline consisted of four (4) concrete pillars forming a straight line on relatively the same slope. The Total Station instrument with reflectors was used in making the EDM distance measurement. After the calibration baseline measurement carried out; least square adjustment computations applied yielded absolute values (MPV) for all the six (6) possible combination of the baseline distances. The known baseline distance could be used to determine standards and zero errors of EDM surveying equipment. Possible pillar movements of the baseline points needed regular checks and monitoring system to have full benefit of the calibration. Also, the baseline could be improved on by establishing a 3D calibration scheme whereby GPS instrumentation performance could be tested and adjusted.

**KEYWORDS:** Least squares Adjustment, Electronic Distance Measurement EDM, Baseline, Calibration

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## I. INTRODUCTION:

Basically, Electronic Distance Measurement (EDM) calibration instruments are primarily made up of the determination or verification of instrument constants and the assurance that the measured distances meet accuracy specifications. To assure that the measured accuracy as well as operating precision capabilities of an instrument has not significantly deteriorated, a known distance of high accuracy or preferably, a sequence of distances forming a calibration range or base line is required (Majid and Halim, 2015). With the development of the modern EDM equipment in surveying operations; distance measurement performance in terms of accuracy, precision and speed has greatly improved. Nonetheless, this improved capability in distance measurement instruments definitely arrived with problems in accuracy and precision. (Ashkenazi, and Dodson 2005).

Generally, the accuracy of the EDM is given by the following relationship;

$$\text{Standard deviation} = t(A \text{ mm} + B \cdot 10^{-6} \cdot D) \dots\dots\dots (1)$$

Where A is the variability of the EDM instrument separate from the distance, B is that portion of the variability of the EDM measurements dependent on the distance like the scale errors, while D is the measured distance and t is the period; t is the phase time taken for the electromagnetic rays to travel from the instrument to a reflector (sensor) and back to the instrument Diacupet al (2002), pointed out that; due to the fact that systematic errors which most times occur during observations because of frequent normal usage as a result of reduction in the efficiency of the electronic and mechanical components of the instrument. It becomes of necessity to calibrate all EDM instruments so as to determine the performance of the instruments and instrumental constants. Basically a sequence of lengths forming a calibration baseline was needed for a proper and systematic calibration of the EDM Instrument. Therefore, the necessity for a calibration baseline cannot be overemphasizing. This research paper therefore, discusses the work done in proposing an EDM calibration baseline at the School of Environmental sciences (SES) Department of Surveying and Geoinformatics Jambutu Campus Adamawa State Polytechnic, Yola (ASPY).

Mathematical adjustments are required in most if not all surveying works that require high accuracy attainment. This is due to the fact that data gathering or collection in terms of measurements and observations in surveying exercises are usually compromised by errors and this research on baseline calibration surveying is not an exception. Since a typical survey measurement may involve several elementary operations, such as centering, pointing, setting, reading and booking. In performing these operations and due to human limitations, imperfection in instrument, and environmental changes and carelessness on the part of the observer, certain amount of error is bound to creep into the measurement or observation (Helsharn, 2010).

After removing blunders and systematic errors, the errors which remain in the measurements are residual, random or accidental errors. These errors literally are unavoidable as no true value of a measurement really exists because a true value is believed to be free from all errors and it is not possible to eliminate all errors completely from a measured quantity hence in the real sense the true value cannot be determined. But we can only have what surveyors refer to as the most probable value of a measured quantity or value; which is the value of a quantity which has more chances of being true than any other value; and the least square adjustment method which has been proven to be one of the best method of adjustment of surveying observations and measurements was adopted for this research.

**Least Squares Adjustment and Computations (First Principle)**

The method of least squares may be defined as a method which makes use of redundant observations in the mathematical modeling of a given problem with a view to minimizing the sum of squares of discrepancies between the observations and their most probable (adjusted) values subject to the prevailing mathematical model. The discrepancies between the observation and their most probable values are known as residuals (Ayeni 2001).When redundant observations are made, the number of discrepancies may occur between repeated observations of the same quality, since each measurement has a certain precision attached to it. Such discrepancies (residuals) have to be adjusted so as to obtain the most probable (adjusted) values of the measured quantities.

The role of adjustment is to derive values for residuals such that:

$$L^a = L^b_1 + V_1 = L^b_2 + V_2 = L^b_3 + V_3 \dots\dots\dots(2.0)$$

Is satisfied.

Where,  $V_i = L^a - L^b_i$

$L^b_i$  = Measured quantity

$L^a$  = Adjusted values, and is expected to be the closet to the true value.

$V_1, V_2, V_3, \dots$  are residuals associated with the measured values. Each estimate observation can be looked as a corrected observation, obtained from the measured value  $L$ , by adding correction  $V$ , to it as in equation (2.0).

The least square method is a systematic procedure needed for application to situation and it is based on the following criterion. The sum of the squares of the observational residuals must be equal to minimum.

$$\Phi = \sum V_i^2 = (V_1^2 + V_2^2 + V_3^2 + \dots\dots\dots V_n^2) = \text{Minimum} \dots\dots(2.1) \text{ (with equal reliability)}$$

$$\Phi = \sum P_i V_i^2 = (P_1 V_1^2 + P_2 V_2^2 + P_3 V_3^2 + \dots\dots P_n V_n^2) = \text{Minimum} \dots\dots(2.2) \text{ (with unequal precision)}$$

In equation (2.2), it is assumed that all observations are uncorrelated and of equal reliability. While in equation (2.1) weight matrix ( $P$ ) is a unit matrix with the diagonal entries equal to unity (1) and non-diagonal element equal to zero (0), that is when the observations are non-correlated but of equal precisions, the weight matrix is assumed to be one (1).  $\Phi$  is the quadratic form of the sum of weighted squares of residuals.

Okwuashi (2014),made it abundantly clear that apart from this first principles in Least Square Adjustment; Observation Equation and Condition Equation Methods of Least Square Adjustment could and were also used to further solidify the accuracy and precision of the adjustment process.

**The Observation Equation Method (OEM) Derivation:**

Ayeni (2001) explained that; the method is those set up among a series of unknown which are independent of each other in the sense that they are subject to no restriction, except those imposed by observations themselves. The functional relationship between each observation to be adjusted and the parameters to be adjusted can be expressed in the mathematical model of the form;

$$L^a = F(X^a) \dots\dots\dots 3.0$$

$X^0$  = appropriate values of unknown parameters.

$x$  = the correction to  $X^0$

So, equation (3.0) becomes

$$L^b + V = F(X^0 + x) \dots\dots\dots 3.1$$

$$L^b + V = F(X^0) + \left. \frac{\partial f(x^b)}{\partial x} \right| \dots\dots\dots 3.2$$

$$= F(X^0) + AX \dots\dots\dots 3.3$$

When  $A = \frac{\partial F(x^0)}{\partial x^0}$  and let  $L = F(x)$  equation.....3.4 yields

$$L^b + V = Ax + L^0 \dots\dots\dots 3.5$$

$$V = Ax + (L^0 - L^b) \dots\dots\dots 3.6$$

The above 3.6 is the general observational model

By using 3.1 and solving in matrix form with weight P, i.e  $\Phi = [PV^2] = P^T PV = \min. \dots 3.7$

Applying 3.6 in 3.7 we then have

$$\begin{aligned} \Phi &= V^T PV = (Ax + L)^T P(Ax + L) \\ &= X^T A^T P A X + X^T A^T P L + L^T P A X + L^T P L \dots\dots\dots 3.8 \end{aligned}$$

$$X = -(A^T P A)^{-1} A^T P L \dots\dots\dots 3.9 \text{ Full details see (Ayeni 2001)}$$

**Condition Equation Method (CEM)**

The method of condition equations otherwise known as the method of correlates establishes a set of equations which must be satisfied by the true values of observations, given certain geometric conditions or physical laws of nature imposed by the configuration of the problem. Since the true values of observations exist only in the super – sensible world it is only practicable to set up condition equations which relate together some adjusted (most probable value of) observations. Idowu (2014) derived this method as follows;

$$F(L^a) = 0 \dots\dots\dots 4.0$$

Where  $L^a$  = adjusted observations.

The general case for a non – linear will be treated since the linear model can easily be derived from it;

$$\text{If we define } L^a \text{ as: } L^a = L^b + V \dots\dots\dots 4.1$$

Then we have;

$$F(L^b + V) = 0 \dots\dots\dots 4.2$$

Where  $L^b, V$  are defined in equations 1.1 as

Expanding equations 2.1 by Taylor's series, and neglecting the 2<sup>nd</sup> order terms, we then have;

$$F(L^b + V) = f(L^b) + \left. \frac{\partial f(L^b)}{\partial L^a} \right| V \dots\dots\dots 4.3$$

$$\text{i.e } BV + W = 0 \dots\dots\dots 4.4$$

$$\text{Where } B = \left. \frac{\partial f(L^a)}{\partial L^a} \right|_{L^a = L^b}$$

$V = L^a - L^0, W = f(L^b) = \text{Vector of misclosure}$

r = number of condition equations

n = number of observations

r < n according to rule 1

Note that equation 4.4 is true for both linear and non – linear problems. The least squares require the minimization of  $V^T PV$ . This will however be done subject to the linearized condition in equation 4.4 by the use of correlates (Langrange multipliers) hence the name adjustment correlates. The function to be minimized is given by;

$$F = V^T PV - 2K^T (BV + W) \dots\dots\dots 4.5$$

$$V^T PV = (P^{-1} B^T K)^T P (P^{-1} B^T K) \dots\dots\dots 4.6 \text{ see details in (Idowu 2014)}$$

## II. METHODOLOGY

### Baseline Calibration Requirement(Site Selected)

The proposed calibration baseline site for selection was subjected to a number of considerations that includes the following:

- i. **Access:** The portion selected for such purposes was be stress-free to reach without any limitations.
- ii. **Terrain:** The qualities of the terrain required was that the site is geologically stable and not susceptible to movement.
- iii. **Manufactured and natural obstacles:** These baseline calibration points were not established at any area close to any obstacles so as to ensure that microwave equipment can also be properly calibrated;
- iv. **Location:** This site for establishing the baseline calibration measurement was not close to infrastructural development sites.
- v. **Total Length:** For an accurate and more precise determination of scale factor error of short range EDM, a significantly long distance were needed in establishing the baseline calibration as required.

Taking in cognizance the above factors afterwards together with several other physical and economic constraints, a suitable site, was located within the campus. With easy access, the site was oriented approximately in the west -east direction. Sandy soil is largely the geology of the area that the soil is composed of, and a relatively flat topography exist.

**Design:** Radiation, baseline and network techniques are basically the three methods involved in establishing a baseline calibration network(Rueger, 1977). However, in this study, the baseline design which is made up of distance measurements in all combinations was adopted. The merits of the baseline design include: many observations with few stations, Only little space is needed (but linear), high precision for zero error, even if known distances are not available or out of date and easy computation, with or without known distances.

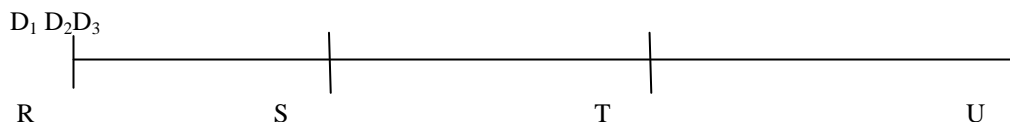
**Equipment used:** Total Station Instrument (Ruide Series RTS 876), Tripod legs, Total station Reflector and Reflector Pole with its other accessories, Linen Tapes and Four (4) Pillars.

**Field procedures and Results:**All the six combinations of the baseline distances were measured and reverse; then their averages taken. Ultimately, the baseline consists of four (4) pillar station, thus dividing the observed or measured distances into three (3) unknown inter pillar distances approximately from 10 meters to 15 meters apart. For this baseline, the number of distinct distances is six (6); and these values are shown in table 1 and figure 1 below;

Stations		Distance Measured
From	To	
R	S	12.153
S	T	14.501
T	U	13.061
R	T	26.649
S	U	27.563
R	U	39.718

**Table 1:** below shows the Observed measurement

Source: Field Work



**Figure 1:** Baseline Calibration Measurement (not to scale)

Source: Field Work

### Data Processing and Analysis

In order to adjust measurements of the baseline in figure 1 above RS, ST, TU; additional measurements of RT, SU and RU were designated as  $D_1$ ,  $D_2$  and  $D_3$  respectively.

### Solution Using the First Principles Method (FPM)

First  $P_i V_i^2$  was stated for the six(6) observations:

$$P_1V_1^2 = 1(D_1^a - 12.153)^2, P_2V_2^2 = 1(D_2^a - 14.501)^2, P_3V_3^2 = 1(D_3^a - 13.061)^2, P_4V_4 = 1(D_1^a + D_2^a - 26.649)^2$$

$$P_5V_5 = 1(D_2^a + D_3^a - 27.563)^2, P_6V_6 = 1(D_1^a + D_2^a + D_3^a - 39.718)^2$$

$$\Phi = (D_1^a - 12.153)^2 + (D_2^a - 14.501)^2 + (D_3^a - 13.061)^2 + (D_1^a + D_2^a - 26.649)^2 + (D_2^a + D_3^a - 27.563)^2 +$$

$(D_1^a + D_2^a + D_3^a - 39.718)^2$  is minimum.

$$\frac{\partial \Phi}{\partial D_1^a} = 2(D_1^a - 12.153) + 2(D_1^a + D_2^a - 26.649) + 2(D_1^a + D_2^a + D_3^a - 39.718)$$

$$= 2D_1^a - 24.306 + 2D_1^a + 2D_2^a - 53.298 + 2D_1^a + 2D_2^a + 2D_3^a - 79.436$$

$$= 6D_1^a + 4D_2^a + 2D_3^a = 157.040 \dots\dots\dots (i)$$

$$\frac{\partial \Phi}{\partial D_2^a} = 2(D_2^a - 14.501) + 2(D_1^a + D_2^a - 26.649) + 2(D_2^a + D_3^a - 27.563) + 2(D_1^a + D_2^a + D_3^a - 39.718)$$

$$= 2D_2^a - 29.002 + 2D_1^a + 2D_2^a - 53.298 + 2D_2^a + 2D_3^a - 55.126 + 2D_1^a + 2D_2^a + 2D_3^a - 79.436$$

$$= 4D_1^a + 8D_2^a + 4D_3^a = 216.862 \dots\dots\dots (ii)$$

$$\frac{\partial \Phi}{\partial D_3^a} = 2(D_3^a - 13.061) + 2(D_2^a + D_3^a - 27.563) + 2(D_1^a + D_2^a + D_3^a - 39.718)$$

$$= 2D_3^a - 26.122 + 2D_2^a + 2D_3^a - 55.126 + 2D_1^a + 2D_2^a + 2D_3^a - 79.436$$

$$= 2D_1^a + 4D_2^a + 6D_3^a = 160.684 \dots\dots\dots (iii)$$

Solving equations (i), (ii) and (iii) simultaneously to obtain

$$D_1^a = 12.152m, D_2^a = 14.500m \text{ and } D_3^a = 13.063m$$

**Solution Using the Observation Equation Method (OEM)**

This method is largely based on the fact that the number of observation equations must be equal to the number of field observations formed carried out. Hence six(6) equations is formed because of the fact that six field observations was made.

These observations equations formed are as follows:

$$L_1^a = D_1^a, L_2^a = D_2^a, L_3^a = D_3^a, L_4^a = D_1^a + D_2^a, L_5^a = D_2^a + D_3^a, L_6^a = D_1^a + D_2^a + D_3^a$$

Recall that  $\bar{X} = \bar{D} = (A^T P A)^{-1} A^T P L^b$

$$\bar{X} = \bar{D} = \begin{matrix} D_1^a \\ D_2^a \\ D_3^a \end{matrix}$$

$$\text{Where; } A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \quad P = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \quad L^b = \begin{matrix} 12.153 \\ 14.501 \\ 13.061 \\ 26.649 \\ 27.563 \\ 39.718 \end{matrix}$$

$$A^T P A = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{vmatrix} \quad * \quad \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \quad * \quad \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\text{Therefore } A^T P A = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \end{vmatrix}$$

1    2    3

While  $(A^T P A)^{-1}$  = Determining determinant, Cofactors and Adjoin, then the Inverse.

Determinant det. = 16

$$\text{Cofactors } C = \begin{vmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{vmatrix} \quad \text{Adjoin } C^T = \begin{vmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{vmatrix}$$

$$\text{Hence } (A^T P A)^{-1} = \frac{1}{16} \begin{vmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{vmatrix} = \begin{vmatrix} 0.5 & -0.25 & 0 \\ -0.25 & 0.5 & -0.25 \\ 0 & -0.25 & 0.5 \end{vmatrix}$$

$$A^T P L^b = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 108.431 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 80.342 \end{vmatrix} * \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} 12.153 \\ 14.501 \\ 13.061 \\ 26.649 \\ 27.563 \\ 39.718 \end{vmatrix} = \begin{vmatrix} 78. \\ \end{vmatrix}$$

Now combining the relation

$$\text{Hence } (A^T P A)^{-1} A^T P L^b = \begin{vmatrix} 0.5 & -0.25 & 0 \\ -0.25 & 0.5 & -0.25 \\ 0 & -0.25 & 0.5 \end{vmatrix} * \begin{vmatrix} 78.520 \\ 108.431 \\ 80.342 \end{vmatrix} = \begin{vmatrix} 14.500 \\ 12.152 \\ 13.063 \end{vmatrix}$$

This then implies that the observation equation methods result obtained are;

$$\begin{vmatrix} D_1^a \\ D_2^a \\ D_3^a \end{vmatrix} = \begin{vmatrix} 12.152 \\ 14.500 \\ 13.063 \end{vmatrix}$$

**Solution Using the Condition Equation Method (CEM)**

For condition equation method the number of field observation minus the number of unknown parameters is equal to the number of condition equations to be adopted.

Therefore for the cause of our baseline field observations three (3) condition equations will be formed.

$$L_1^a + L_2^a - L_4^a = 0 \dots (1), L_3^a + L_2^a - L_5^a = 0 \dots (2)$$

$$L_1^a + L_2^a - L_3^a = 0 \dots (3)$$

We recall that:  $V = X = -P^{-1} B^T (B P^{-1} B^T)^{-1} W$  and that  $L^a = L^b + V$

$$\begin{array}{l}
 B = \begin{matrix} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 & -1 \end{matrix} \qquad \begin{matrix} 12.153 + 14.501 - 26.649 \\ W = 14.501 + 13.061 - 27.563 \\ 12.153 + 14.501 + 13.061 - 39.718 \end{matrix}
 \end{array}$$

Hence  $W = \begin{vmatrix} 0.005 \\ -0.001 \\ -0.003 \end{vmatrix}$

Solving  $(BP^{-1}B^T)^{-1}W$  First

$$P = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$BP^{-1}B^T = \begin{vmatrix} 1 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 & * & 0 & 1 & 0 & 0 & 0 & * & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & -1 & & 0 & 0 & 1 & 0 & 0 & & -1 & 0 & 0 \\ & & & & & & & 0 & 0 & 0 & 0 & 1 & 0 & & 0 & -1 & 0 \\ & & & & & & & 0 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 & -1 \end{vmatrix}$$

Hence  $BP^{-1}B^T = \begin{vmatrix} 1 & 2 \\ 1 & 3 & 2 \\ 2 & 2 & 4 \end{vmatrix}$

Next we compute  $(BP^{-1}B^T)^{-1} = \text{Determinant, Cofactor, Adjoin and then Inverse}$

Determinant = 16

Cofactor  $C = \text{Adjoin } C^T = \begin{vmatrix} 8 & 0 & -4 \\ 0 & 8 & -4 \\ -4 & -4 & 8 \end{vmatrix}$

Hence Inverse  $(BP^{-1}B^T)^{-1} = \frac{1}{20} \begin{vmatrix} -1/4 \\ 0 & 1/2 & -1/4 \\ -1/4 & -1/4 & 1/2 \end{vmatrix}$

While  $(BP^{-1}B^T)^{-1}W = \frac{1}{20} \begin{vmatrix} -1/4 & 0.005 \\ 0 & 1/2 & -1/4 \\ -1/4 & -1/4 & 1/2 \end{vmatrix} \begin{vmatrix} 0.00325 \\ -0.001 \\ -0.003 \end{vmatrix} = \begin{vmatrix} 0.00025 \\ -0.00250 \end{vmatrix}$

Next solving  $-P^{-1}B^T = \begin{vmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & -1 \end{vmatrix}$

$$\begin{array}{cccccc|ccc}
 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & = & -1 & -1 & -1 \\
 0 & 0 & -1 & 0 & 0 & 0 & * & 0 & 1 & 1 & = & 0 & -1 & -1 \\
 0 & 0 & 0 & -1 & 0 & 0 & & -1 & 0 & 0 & & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & -1 & 0 & & 0 & -1 & 0 & & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & -1 & & 0 & 0 & -1 & & 0 & 0 & 1
 \end{array}$$

$$\text{Computing } V = -P^{-1}B^T (BP^{-1}B^T)^{-1}W = \begin{array}{ccc|ccc|ccc}
 -1 & 0 & -1 & & & 0.00325 & & & & & 0.00075 & & & \\
 & & -1 & -1 & -1 & & & & & & & & -0.0010 & \\
 & & 0 & -1 & -1 & * & 0.00025 & & = & & & & 0.00225 & \\
 & & 1 & 0 & 0 & & & & & & & & 0.00325 & \\
 & & 0 & 1 & 0 & & -0.00250 & & & & & & 0.00025 & \\
 & & 0 & 0 & 1 & & & & & & & & -0.0025 & 
 \end{array}$$

Finally, Computing;  $L^a = L^b + V$  Therefore, the final adjusted distances are;

$$\begin{array}{ccc|ccc}
 D_1^a & & & 12.152 & & \\
 D_2^a & & = & 14.500 & & \\
 D_3^a & & & 13.063 & & 
 \end{array}$$

### III. FINDINGS AND DISCUSSION:

S/N	Stations	FPM	OEM	CEM	Difference Between FPM, OEM & CEM
1.	D <sub>1</sub> <sup>a</sup>	12.152m	12.152m	12.152m	<b>0.000m</b>
2.	D <sub>2</sub> <sup>a</sup>	14.500m	14.500m	14.500m	<b>0.000m</b>
3.	D <sub>3</sub> <sup>a</sup>	13.063m	13.063m	13.063m	<b>0.000m</b>

**Table 2:** Computed and Adjusted MPV of D<sub>1</sub>, D<sub>2</sub> & D<sub>3</sub>

Source: Field & Office Work

The final adjusted calibration baseline distances  $D_1^a = 12.152m$ ,  $D_2^a = 14.500m$  and  $D_3^a = 13.063m$  reveals a perfectly the same solutions from the three different computational process i.e first principles (FP), observation equation (OB) and condition equation (CE) techniques of least square (LS) adjustment computation at 3 decimal places of a meter. That is as the MPV's from the three basic principles of LS adjustment shows no difference (Table.2) above; hence this shows the reliability of the LS adjustment in survey measurements. Also, it clearly demonstrates the LS adjustment high precision and reliability when determining the most probable value of field measurement in surveying. In summary, recalling, from the adjustment computations, some of the primary conditions for Least Square adjustment among others are that: (i) the number of field observations must exceed the number of parameters to be determined (ii) the number of observation equations formed must be equal to the number of field observations (iii) the number of condition equations formed must equal the difference between the number of observations and the number of unknown parameters to be determined were all indeed adhered to.

### IV. CONCLUSION AND RECOMMENDATIONS:

The baseline calibration system determined and proposed in SES will indeed be of massive benefit to the public, Polytechnic and particularly the Environmental Science School because of the fact that most of the department in this School use lots of instruments and equipment that require frequent calibration. This research could serve also as a spring board for further studies for those who are interested in this field. The baseline could be improved on by the following recommendations:



- i. Establishing a 3D calibration scheme whereby GPS instrumentation performance could be tested and adjusted; as it is expected that monitoring of the baseline could be achieved by utilizing techniques of GPS survey.
- ii. Moreover, a full EDM calibration could be achieved by incorporating laboratory calibration, which will enable the evaluation of modulation frequency error, cyclic error and pointing error.
- iii. The baseline itself needs to be monitored at regular interval to investigate on possible of pillars movement.

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