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Research Paper

Reduction Formulae and Identities for Certain Generalized HFunction of two variables

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ABSTRACT:This article deals with certain identities and reduction formulae for generalized H- functions, which are of great interest and generalize many known and unknown interested results in literature especially the results given by Shewta and Srivastava[7] and cook [2].

I. NOTATIONS AND RESULT USED

${}_1\left(a_j;\alpha_j^{(k)}\right)_p$ Abbreviations the array of P Parameters $\left(a_1;\alpha_1^{(k)}\right),\ldots\ldots\ldots,\left(a_p;\alpha_p^{(k)}\right)$

${}_1\left(a_j;\alpha_j,A_j\right)_p$ Stands the array of P Parameters $\left(a_1;\alpha_1,A_1\right),\ldots\ldots\ldots,{}_1\left(a_p;\alpha_p,A_p\right)$

${}_1\left(a_j;\alpha_j\right)_p$ Abbreviations the array of P Parameters

$\left(a_1;\alpha_1\right),\ldots\ldots\ldots,\left(a_p;\alpha_p\right)$, $(a)_n$ Stands for the product of n factors

$a(a+1)(a+2)\ldots\ldots\ldots\ldots(a+n-1);(a)_0=1$

Erdelyi [3, P.210(12)]

$$\left(a_p-a_1\right)G_{p,q}^{m,n}\left[x\left|\begin{matrix}{}_1\left(a_j\right)_p\\{}_1\left(b_j\right)_q\end{matrix}\right.\right]=G_{p,q}^{m,n}\left[x\left|\begin{matrix}a_1-1,\ {}_2\left(a_j\right)_p\\{}_1\left(b_j\right)_q\end{matrix}\right.\right]+G_{p,q}^{m,n}\left[x\left|\begin{matrix}{}_1\left(a_j\right)_{p-1},a_p-1\\{}_1\left(b_j\right)_q\end{matrix}\right.\right]\quad\ldots(2.2.1)$$

for $1\leq n\leq p-1.$

Erdelyi[4, P.439]

$$G_{p,\ q}^{1,\ n}\left[x\left|\begin{matrix}{}_1\left(a_j\right)_p\\{}_1\left(b_j\right)_q\end{matrix}\right.\right]=\frac{\prod_{j=1}^{n}\Gamma\left(1+b_1-a_j\right)x^{b_1}}{\prod_{j=2}^{q}\Gamma\left(1+b_1-b_j\right)\prod_{j=n+1}^{p}\Gamma\left(a_j-b_1\right)}$$

$${}_2F_{q-1}\left[\begin{matrix}{}_1\left(1+b_1-a_j\right)_p\ ;\ (-1)^{p-n-1}x\\{}_2\left(1+b_1-a_j\right)_p\end{matrix}\right]\quad for\quad p\leq q.$$

Taking $m=1,b_1=0$ and using (2.2.2); (2.2.1) reduces to the relation

$$(a_1 - a_p)\; {}_pF_q\left[\begin{matrix} {}_1(a_j);\ x \\ {}_1(b_j); \end{matrix}\right] = a_1\; {}_pF_q\left[\begin{matrix} a_1+1,\ {}_2(a_j)_p;\ x \\ {}_1(b_j)_q; \end{matrix}\right] - a_p\; {}_pF_q\left[\begin{matrix} {}_1(a_j)_{p-1}\ a_p+1;\ x \\ {}_1(b_j)_q; \end{matrix}\right] \quad .....(2.2.2)$$

$for\ \ 2 \le p \le q+1.$

When p = 2 and q = 1; (1.1.3) agree with a relation given by Erdelyi [3, P. 103 (32)]. Jeta Ram [5,P.226 (3.11)].

$$\sum_{k=0}^{N} \binom{n}{k} \frac{\prod_{j=1}^{p} (a_j)_k}{\prod_{j=1}^{q} (b_j)_k}\; {}_{p+1}F_q\left[\begin{matrix} c+k,\ {}_1(a_j+k)_p;\ z \\ {}_1(b_j+k)_q; \end{matrix}\right] z^k = {}_{p+1}F_q\left[\begin{matrix} c+N,\ {}_1(a_j)_p;\ z \\ {}_1(b_j)_q; \end{matrix}\right] \quad ....(2.2.3)$$

Taking N = 1 and renaming c + 1 as c (2.2.4) gives rise to the relation

$${}_{p+1}F_q\left[\begin{matrix} c,\ {}_1(a_j)_p;\ z \\ {}_1(b_j)_q; \end{matrix}\right] = {}_{p+1}F_q\left[\begin{matrix} c-1,\ {}_1(a_j)_p;\ z \\ {}_1(b_j)_q; \end{matrix}\right] + Z\, \frac{\prod_{j=1}^{p} (a_j)}{\prod_{j=1}^{q} (b_j)}\; {}_{p+1}F_q\left[\begin{matrix} c,\ {}_1(a_j+1)_p;\ z \\ {}_1(b_j+1)_q; \end{matrix}\right] \quad .....(2.2.4)$$

Taking N = 1 and renaming c + 1 as c (2.2.4) gives rise to the relation

$${}_{p+1}F_q\left[\begin{matrix} c,\ {}_1(a_j)_p;\ z \\ {}_1(b_j)_q; \end{matrix}\right] = {}_{p+1}F_q\left[\begin{matrix} c-1,\ {}_1(a_j)_p;\ z \\ {}_1(b_j)_q; \end{matrix}\right] + Z\, \frac{\prod_{j=1}^{p} (a_j)}{\prod_{j=1}^{q} (b_j)}\; {}_{p+1}F_q\left[\begin{matrix} c,\ {}_1(a_j+1)_p;\ z \\ {}_1(b_j+1)_q; \end{matrix}\right] \quad .....(2.2.5)$$

The relation (2.2.5) can be restricted as

$${}_pF_q\left[\begin{matrix} {}_1(a_j)_p;\ z \\ {}_1(b_j)_q; \end{matrix}\right] = {}_pF_q\left[\begin{matrix} a_1-1,\ {}_1(a_j)_p;\ z \\ {}_1(b_j)_q; \end{matrix}\right] + Z\, \frac{\prod_{j=2}^{p} (a_j)}{\prod_{j=1}^{q} (b_j)}\; {}_pF_q\left[\begin{matrix} a_1,\ {}_2(a_j+1)_p;\ z \\ {}_1(b_j+1)_q; \end{matrix}\right] \quad .....(2.2.6)$$

2.3 Identities Involving Generalized H- Functions of Two Variables

Identity – I

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$$\sum_{r=0}^{\infty}\frac{t^r}{r!}\boldsymbol{H}^{M+m,\ \ N:\ M_1,\ N_1;\ M_2,\ N_2}_{P+m,\ Q+m:\ P_1,\ Q_1;\ P_2,\ Q_2}\left[\begin{matrix}x\\ y\end{matrix}\left|\begin{matrix}{}_1\left(a_j;\alpha_j,A_j\right)_p,{}_1\left(\delta_1+r;\eta_i',\eta_i''\right)_m:\\ {}_1\left(\upsilon_i+\mathrm{r};\mu_i',\mu_i''\right)_m,{}_1\left(\mathrm{b}_j;\beta_j,\beta_j\right)_Q:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}{}_1\left(c_j,C_j\right)_{p_2};{}_1\left(e_j,E_j\right)_{p_2}\\ {}_1\left(d_j,\mathrm{D}_j\right)_{Q_1};{}_1\left(f_j,\mathrm{F}_j\right)_{Q_2}\end{matrix}\right]\sum_{r=0}^{\infty}\frac{t^r}{r!}\boldsymbol{H}^{M+m+1,\ \ N:M_1,\ N_1;\ M_2,\ N_2}_{P+m+1,\ Q+m+1:\ P_1,\ Q_1;\ P_2,\ Q_2}$$

$$\left[\begin{matrix}x\\ y\end{matrix}\left|\begin{matrix}{}_1\left(a_j;\alpha_j,A_j\right)_p,\ {}_1\left(\upsilon_1-1;\mu_1',\mu_1''\right),\ {}_1\left(\delta_1+r;\eta_i',\eta_i''\right)_m:{}_1\left(c_j,C_j\right)_{p_2};{}_1\left(e_j,E_j\right)_{p_2}\\ {}_1\left(\upsilon_1;\mu_1',\mu_1''\right),\ {}_1\left(\upsilon_1+\mathrm{r}-1;\mu_1',\mu_1''\right),\ {}_2\left(\upsilon_1+\mathrm{r};\mu_1',\mu_1''\right)_m,{}_1\left(\mathrm{b}_j;\beta_j,\beta_j\right)_Q:{}_1\left(d_j,\mathrm{D}_j\right)_{Q_1};{}_1\left(f_j,\mathrm{F}_j\right)_{Q_2}\end{matrix}\right.\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^r}{r!}\boldsymbol{H}^{M+m,\ \ N\ :\ M_1,\ N_1\ ;\ M_2,\ N_2}_{P+m,\ Q+m\ :\ P_1\ ,\ Q_1\ ;\ P_2\ ,\ Q_2}$$

$$\left[\begin{matrix}x\\ y\end{matrix}\left|\begin{matrix}{}_1\left(a_j;\alpha_j,A_j\right)_p,\ {}_1\left(\delta_1+r+1;\eta_i',\eta_i''\right)_m:{}_1\left(c_j,C_j\right)_{p_2};{}_1\left(e_j,E_j\right)_{p_2}\\ \left(\upsilon_1+\mathrm{r};\mu_1',\mu_1''\right),\ {}_2\left(\upsilon_1+\mathrm{r}+1;\mu_1',\mu_1''\right)_m,{}_1\left(\mathrm{b}_j;\beta_j,\beta_j\right)_Q:{}_1\left(d_j,\mathrm{D}_j\right)_{Q_1};{}_1\left(f_j,\mathrm{F}_j\right)_{Q_2}\end{matrix}\right.\right]$$

$$where\ \ m>0 \qquad \text{..............}(2.3.1.)$$

Proof:
To prove (2.3.1), first express the H-function of two variables variables involved on the left- hand side in terms of Mellin- Barners contour integral (1.1.46), change the order of integration and summation and use the property of Gamma function to obtain.

$$L.H.S.=\frac{1}{(2\pi i)^2}\int_{L1}\int_{L_2}\varphi\left(s_1,s_2\right)\theta\left(s_1\right)\phi\left(s_2\right)x^{s_1}\,x^{s_2}$$

$$\frac{\prod_{i=1}^{m}\Gamma\left(\nu_1-\mu_i's_1-\mu_i''s_2\right)}{\prod_{i=1}^{m}\Gamma\left(\delta_1-\eta_i's_1-\eta_i''s_2\right)}\left[\sum_{r=0}^{\infty}\frac{t^r}{r!}\frac{\prod_{i=1}^{m}\Gamma\left(\nu_1-\mu_i's_1-\mu_i''s_2\right)_r}{\prod_{i=1}^{m}\Gamma\left(\delta_1-\eta_i's_1-\eta_i''s_2\right)_r}\right]ds_1\,ds_2\,.$$

$$\text{.....}(2.3.2)$$

By the definition of hyper geometric function (1.1.1), the expression (2.3.2) becomes

$$L.H.S. = \frac{1}{(2\pi i)^2} \int_{L_1} \int_{L_2} \psi(s_1, s_2)\ \theta(s_1)\ \phi(s_2)\, x^{s_1}\, x^{s_2}\ \frac{\prod_{i=1}^{m} \Gamma\left(\nu_1 - \mu_i^{'} s_1 - \mu_i^{''} s_2\right)}{\prod_{i=1}^{m} \Gamma\left(\delta_1 - \eta_i^{'} s_1 - \eta_i^{''} s_2\right)}$$

$$ {}_m\mathrm{F}_m \left[\begin{matrix} {}_1\left(\nu_1 - \mu_i^{'} s_1 - \mu_i^{''} s_2\right)_m ;\ t \\ {}_1\left(\delta_1 - \eta_i^{'} s_1 - \eta_i^{''} s_2\right)_m ; \end{matrix} \right] .....(2.3.3)$$

Now applying (2.2.6) for p = q = m on (2.3.3) expressing the resulting Hypergeometric functions as a series using (1.1.1). Taking the order of integration and summation and then interpreting the two double integrals with the help of (1.1.46) we arrive at the required result. The change of order of integration and summation is justified (1, P.500) Provided the series is uniformly convergent and the double integrals involved are absolutely convergent

Special Case:

When m = 2, the identity (2.3.1) reduces to the following form:

$$\sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}^{M+2,\ N:M_1,N_1;M_2,N_2}_{P+2,\ Q+2:\ P_1,\ Q_1\ ;P_2,\ Q_2} \left[\begin{matrix} x \\ y \end{matrix} \middle| \begin{matrix} {}_1\left(a_j;\ \alpha_j, A_j\right)_p, \left(\delta_1 + r; \eta_i^{'}, \eta_i^{''}\right)_m : \left(\delta + r;\ \xi_1 . \xi_2\right) : \left(c_j, C_j\right)_{p_2} ; {}_1\left(e_j, E_j\right)_{p_2} \\ {}_1\left(\upsilon + r; \mu_1, \mu_2\right), \left(\rho + r\ ;\ \gamma_1, \gamma_2\right), {}_1\left(b_j; \beta_j, \beta_j\right)_Q :\ {}_1\left(d_j, D_j\right)_{Q_1} ; {}_1\left(f_j, F_j\right)_{Q_2} \end{matrix} \right]$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}^{M+3,\ N:M_1,N_1;M_2,N_2}_{P+3,\ Q+3:\ P_1,\ Q_1\ ;P_2,\ Q_2} \left[\begin{matrix} x \\ y \end{matrix} \middle| \begin{matrix} {}_1\left(a_j; \alpha_j, A_j\right)_p, \left(\upsilon - 1; \mu_1, \mu_2\right), \left(\delta + r; \eta_1, \eta_2\right) \left(\sigma + r\ ;\ \xi_1,\ \xi_2\right) : {}_1\left(c_j, C_j\right)_{p_2} ; {}_1\left(e_j, E_j\right)_{p_2} \\ {}_1\left(\upsilon_1; \mu_1,\ \mu_2\right), \left(\upsilon + r - 1; \mu_1, \mu_2\right), \left(\rho + r; \gamma_1, \gamma_2\right)_1, \left(b_j; \beta_j, \beta_j\right)_Q : {}_1\left(d_j, D_j\right)_{Q_1} ; {}_1\left(f_j, F_j\right)_{Q_2} \end{matrix} \right]$$

$$+ \sum_{r=0}^{\infty} \frac{t^{r+1}}{r!} \boldsymbol{H}^{M+2,\ N:M_1,N_1;M_2,N_2}_{P+2,\ Q+2:\ P_1,\ Q_1\ ;P_2,\ Q_2} \left[\begin{matrix} x \\ y \end{matrix} \middle| \begin{matrix} {}_1\left(a_j; \alpha_j, A_j\right)_p, \left(\delta + r + 1; \eta_1, \eta_2\right), \left(\sigma + r + 1; \xi_1, \xi_2\right) : \\ {}_1\left(\upsilon + r; \mu_1, \mu_2\right), \left(\rho + r + 1; \gamma_1, \gamma_2\right), {}_1\left(b_j; \beta_j, \beta_j\right)_Q : \end{matrix} \right] ....(2.3.4)$$

Taking $M = N = P = Q = 0;\quad M_2 = 1;\quad N_2 = P_2;\quad E_j = F_j = 1;\quad f_1 = 0;\quad \eta_i^{''} = \mu_i^{''} = 0$ *with*

$y \to 0$ *and remaning the parameters* $(2.3.1)$ *gives rise to*:

$$\sum_{r=0}^{\infty} \frac{t^r}{r!} \quad \boldsymbol{H}^{m+2,\ n}_{P+k,\, q+k} \left[x \left| \begin{matrix} {}_1\left(a_j, \alpha_j\right)_p, \left(\delta_i + r,\ \eta_i\right)_n \\ {}_1\left(\upsilon + \mathrm{r}\ ;\ \mu_1\right), {}_1\left(\mathrm{b}_j, \beta_j\right)_q \ : \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}^{m+K+1,\, n}_{P+k+1,\, q+k+1} \left[x \left| \begin{matrix} {}_1\left(a_j, \alpha_j,\right)_p, \left(\upsilon - 1,\ \mu_1\right), \left(\delta + r\ ;\ \eta_i\right)_k \\ {}_1\left(\upsilon_1, \mu_1\right), \left(\upsilon_i + \mathrm{r} - 1,\ \mu_1,\right), {}_2\left(\upsilon_i + \mathrm{r}, \mu_i,\right)_k, {}_1\left(b_j, \beta_j\right)_q \end{matrix} \right. \right] + \sum_{r=0}^{\infty} \frac{t^{r+1}}{r!} \boldsymbol{H}^{m+k,\ n}_{P+k,\ q+k}$$

$$\left[x \left| \begin{matrix} {}_1\left(a_j, \alpha_j\right)_p, \quad {}_1\left(\delta + r + 1,\ \eta_1\right)_{kl} \\ \left(\upsilon_1 + \mathrm{r}, \mu_1\right), {}_2\left(\upsilon_i + \mathrm{r} - 1,\ \mu_i\right)_k, {}_1\left(\mathrm{b}_j, \beta_j\right)_q \end{matrix} \right. \right] \quad \ldots\ldots\ldots.(2.3.5)$$

(3.5) becomes

$$\sum_{r=0}^{\infty} \frac{t^r}{r!} \quad \boldsymbol{H}^{m+1,\ n}_{P+k,\ q+k} \left[x \left| \begin{matrix} {}_1\left(a_j, \alpha_j\right)_p, (\delta + r,\ \eta) \\ (\upsilon + \mathrm{r},\ \mu), {}_1\left(\mathrm{b}_j, \beta_j\right)_q \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}^{m+2,\ n}_{P+2,\ q+2} \left[x \left| \begin{matrix} {}_1\left(a_j, \alpha_j\right)_p, (\upsilon - 1,\ \mu), (\delta + r,\ \eta) \\ (\upsilon, \mu), (\upsilon + \mathrm{r} - 1, \mu), {}_1\left(b_j, \beta_j\right)_q \end{matrix} \right. \right]$$

$$+ \sum_{r=0}^{\infty} \frac{t^{r+1}}{r!} \boldsymbol{H}^{m+1,\ n}_{P+1,\ q+1} \left[x \left| \begin{matrix} {}_1\left(a_j, \alpha_j\right)_p, (\delta + r + 1,\ \eta) \\ (\upsilon + \mathrm{r}, \mu), {}_1\left(\mathrm{b}_j, \beta_j\right)_q \end{matrix} \right. \right] \quad \ldots.(2.3.6)$$

Identity – II

$$\sum_{r=0}^{\infty}\frac{t^r}{r!}\,\boldsymbol{H}^{M+m,\ \ N+m\ :\ M_1,N_1;\ M_2,\ N_2}_{P+2m,\ Q+m\ :\ P_1,\ Q_1\ ;\ P_2,\ Q_2}\left[\begin{matrix}x\\ \\ y\end{matrix}\middle|\begin{matrix}{}_1\left(\delta_1-r\ ;\eta_i',\ \eta_i''\right)_m,\ {}_1\left(a_j;\ \alpha_j,A_j\right)_p,\ \left(\varsigma_i+r\ ;\lambda_1',\lambda_i''\right):{}_1\left(c_j,C_j\right)_{p_1};{}_1\left(e_j,E_j\right)_{p_2}\\ {}_1\left(\upsilon_i+\mathrm{r};\mu_i',\ \mu_i''\right)_m,\ \left(\mathrm{b}_j;\beta_j,\beta_j\right)_Q:{}_1\left(d_j,\mathrm{D}_j\right)_{Q_1};\ {}_1\left(f_j,\mathrm{F}_j\right)_{Q_2}\end{matrix}\right]$$

$$=\sum_{r=0}^{\infty}\frac{t^r}{r!}\boldsymbol{H}^{M+m,\ \ \ N+m+1:\ M_1,\ N_1\ ;M_2,\ N_2}_{P+2m+1,\ Q+m+1:\ P_1,\ Q_1\ ;P_2,\ Q_2}\left[\begin{matrix}x\\ \\ y\end{matrix}\middle|\begin{matrix}\left(\delta_1;\eta_i',\eta_i''\right),\ {}_2\left(\delta_1-r+1;\eta_i',\eta_i''\right)_m,{}_2\left(\delta_1-r\ ;\eta_i',\eta_i''\right)_m,\ {}_1\left(a_j;\alpha_j,A_j\right)_p,{}_1\left(\varsigma_i+r+1;\lambda_1',\lambda_{i1}'''\right)_m:{}_1\left(c_j,C_j\right)_{p_2};\ {}_1\left(e_j,E_j\right)_{p_2}\\ {}_1\left(\upsilon+\mathrm{r}-1;\mu_i'',\mu_i'\right)_m,\ {}_1\left(\mathrm{b}_j;\beta_j,\beta_j\right)_Q,\ \left(\delta_1+1;\ \eta_i',\ \eta_i''\right):{}_1\left(d_j,\mathrm{D}_j\right)_{Q_1};{}_1\left(f_j,\mathrm{F}_j\right)_{Q_2}\end{matrix}\right]$$

$$+\sum_{r=0}^{\infty}\frac{t^{r+1}}{r!}\boldsymbol{H}^{M+m,\ N+m,:\ \mathrm{M}_1,\ N_1;M_2,N_2}_{P+2m,\ \ Q+m:\ P_1,\ Q_1\ ;P_2,\ Q_2}\left[\begin{matrix}x\\ \\ y\end{matrix}\middle|\begin{matrix}\left(\delta_1-r\ ;\eta_i',\eta_i''\right)_1,\ \left(\delta_1-r-1;\eta_i',\eta_i''\right)_m,\ {}_1\left(a_j;\alpha_j,A_j\right)_p,\ {}_1\left(\varsigma_i+r+1;\lambda_1',\lambda_{i1}'''\right)_m:{}_1\left(c_j,C_j\right)_{p_1};{}_1\left(e_j,E_j\right)_{p_2}\\ \left(\upsilon_i+\mathrm{r}+1;\mu_i'',\mu_i'\right)_m,\ {}_1\left(\mathrm{b}_j;\beta_j,\beta_j\right)_Q:{}_1\left(d_j,\mathrm{D}_j\right)_{Q_1};{}_1\left(f_j,\mathrm{F}_j\right)_{Q_2}\end{matrix}\right];m>0\text{.............}(.\,2.3.7)$$

Proof is obtained in similar lines to (2.3.1) , on using (2.2.6) for P = 2m and q = m.

SpecialCases:

Taking $M=N=P=Q=0;\ M_2=1;\ N_2=P_2;$ $E_j=F_j=1;\ f_1=0;\ \eta_l''=\mu_i''=\lambda_i''=0$ *with* $y\to 0$, *and remaning the parameters*, (3.7) *gives rise to*:

$$\sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}_{P+2k,\, q+k}^{m+k,\; n+k} \left[x \middle| \begin{matrix} {}_1(\delta_1 - r,\ \eta_i)_{m,\, k},\ {}_1(a_j,\ \alpha_j)_p,\ {}_1(\varsigma_i + r,\ \lambda_1) \\ {}_1(\upsilon_i + \mathrm{r},\ \mu_i)_k,\ (\mathrm{b}_j,\ \beta_j)_Q \end{matrix} \right]$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}_{P+2k+1,\; Q+k+1}^{\mathrm{m+k},\quad n+k+1} \left[x \middle| \begin{matrix} (\delta_1,\ \eta_1),\ (\delta_1 - r + 1,\ \eta_1),\ {}_2(\delta_i - r,\ \eta_i)_k,\ {}_1(a_j,\ \alpha_j)_p,\ {}_1(\varsigma_i + r,\ \lambda_i)_k \\ {}_1(\upsilon + \mathrm{r},\ \mu_i)_k,\ {}_1(\mathrm{b}_j,\ \beta_j)_q,\ (\delta_1 + 1,\ \eta_1) \end{matrix} \right]$$

$$+ \sum_{r=0}^{\infty} \frac{t^{r+1}}{r!} \boldsymbol{H}_{P+2k,\quad q+k}^{m+k,\quad n+k+1} \left[x \middle| \begin{matrix} (\delta_1 - r,\ \eta_1),\ {}_2(\delta_i - r - 1,\ \eta_i)_k,\ {}_1(a_j,\ \alpha_j)_p,\ {}_1(\varsigma_i + r + 1;\ \lambda_i)_k \\ {}_1(\upsilon_i + \mathrm{r} + 1;\ \mu_i)_k,\ {}_1(\mathrm{b}_j;\ \beta_j)_q \end{matrix} \right] \quad \ldots(2.3.8)$$

where $k > 0$

when k = 2, (2.3.8) becomes

$$\sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}_{P+4,\quad q+2}^{m+2,\quad n+2} \left[x \middle| \begin{matrix} (\delta - r,\ \eta),\ (\sigma - r,\ \xi),\ (a_j,\ \alpha_j)_p,\ (\varsigma + r,\ \lambda),\ (\tau + r,\ \zeta) \\ (\upsilon + \mathrm{r},\ \mu),\ (\rho + r,\ \gamma),\ (\mathrm{b}_j,\ \beta_j)_q \end{matrix} \right]$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \boldsymbol{H}_{P+5,\quad q+3}^{\mathrm{m+2},\ n+3} \left[x \middle| \begin{matrix} (\delta,\ \eta),(\delta - r + 1,\ \eta),\ (\sigma - r,\ \xi),\ {}_1(a_j,\ \alpha_j)_p,\ (\varsigma + r,\ \lambda),\ (\tau + r,\ \zeta) \\ (\upsilon + \mathrm{r},\ \mu),\ (\rho + r,\ \gamma),\ {}_1(\mathrm{b}_j,\ \beta_j)_q,\ (\delta + 1,\ \eta) \end{matrix} \right] + \sum_{r=0}^{\infty} \frac{t^{r+1}}{r!} \boldsymbol{H}_{P+4,\quad q+2}^{m+2,\quad n+2}$$

$$\left[x \middle| \begin{matrix} (\delta - r,\ \eta),\ (\sigma - r + 1,\ \xi),\ {}_1(a_j,\ \alpha_j)_p,(\varsigma + r + 1,\ \lambda),\ (\tau + r + 1,\ \zeta) \\ (\upsilon + \mathrm{r} + 1,\ \mu),\ (\rho + r + 1,\ \gamma),\ {}_1(\mathrm{b}_j,\ \beta_j)_q \end{matrix} \right]$$

$$\ldots\ldots\ldots(2.3.9)$$

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