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Research Paper

Vibration Reduction Performance of a Belleville Spring Mount under Resonance Conditions

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Abstract
To investigate the vibration reduction capability of the proposed Belleville spring mount, a physical model and a mathematical model were developed in this study. Operating condition with harmonic force, speed of punching machine is studied to evaluate effects of reducing vibration. The vibration isolation performance of the Belleville spring mount was compared with that of an equivalent coil spring system having the same stiffness characteristics. The results demonstrated that the dynamic responses of the Belleville spring mount, including displacement amplitude, velocity, and acceleration, showed better vibration isolation performance than the equivalent coil spring system.
KEYWORDS: *Belleville spring mount, vibration isolator, disc spring, vibration reduction*

I. INTRODUCTION

Belleville spring mount as shown in Figure-1 has ability of reducing vibration than a coil spring in the same space. This mount is suitable at work conditions with heavy loads and small deformations are required [4]. Some properties of Belleville washers include: high fatigue life, better use of space, low creep tendency, high load capacity with a small spring deflection and possibility for high hysteresis (damping) by stacking several belleville washers on top of each other in the same direction [27]. Cone springs are usually installed and arranged in layers. This type of mount has the ability to self-damp vibrations like leaf springs: Vibrations are quickly damped after compression. The conical disc of this type of spring has the cross-sectional dimensions and load as shown in Figure 1. The advantages of disc springs include small mounting space in the direction of the force, the ability to withstand lateral forces, and the force-deflection characteristics can be changed by adding or removing discs.

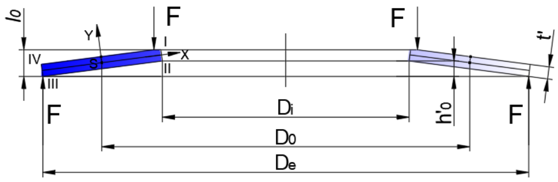

Figure 1. a single cone spring

The application range of individual cone springs can be extended to cover higher forces and/or greater deflection by stacking.

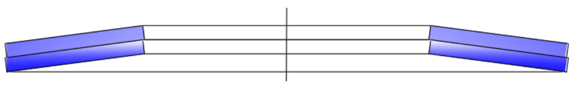

Figure 2. Belleville spring mount of parallel formation

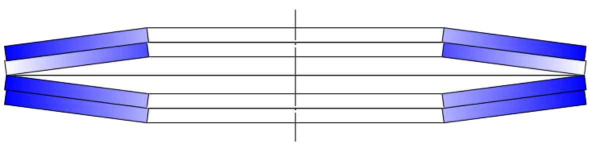

Figure 3. Belleville spring mount in parallel alternating formation.

Disc spring vibration isolators have attracted increasing attention in industrial applications because of their compact structure, high load capacity, and superior vibration damping characteristics. F. Jia and F. Y. Xu [26] developed a vibration damper composed of stacked disc springs sliding along a guide core. Compared with conventional torsion spring systems, this structure offers higher load capacity while significantly reducing installation space. Experimental results demonstrated that the proposed system could achieve vibration reduction efficiencies of up to 98%, making it suitable for industrial machinery such as punching and pressure-forming equipment.

To better understand the behavior of belleville springs, several researchers have investigated their mechanical and damping characteristics. L. J. Zheng [20] established a mathematical expression for accurately predicting the load–displacement relationship of disc springs. Through theoretical analysis, Saini [21] examined the influence of thickness variation on the load-carrying capacity and deformation behavior of disc springs. In addition, experimental investigations reported in [22] revealed that disc springs possess superior damping performance compared with conventional materials.

The influence of friction and nonlinear behavior on vibration dampers has also been widely studied. G. Curti [23] investigated the effect of friction on belleville spring performance using both finite element analysis and experimental methods. Similarly, X. S. Gong [24] proposed a dynamic modeling method for vibration dampers with nonlinear hysteresis characteristics. Furthermore, the mechanical properties of disc spring vibration dampers with viscous damping were analyzed in [25] through numerical simulation and experimental validation. The results confirmed that friction damping has a significant influence on the static stiffness characteristics of the vibration damper system. Unlike previous studies focusing mainly on general vibration isolation characteristics, the present study specifically investigates the dynamic behavior of Belleville spring mounts under resonance conditions.

II. DYNAMIC MODEL

To evaluate the vibration isolation performance of the Belleville spring mount, a simplified quarter-machine dynamic model was developed and analyzed, as illustrated in Fig. 4.

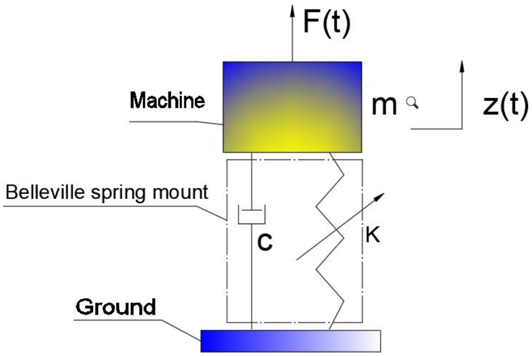

Figure 4. Quarter-machine dynamic model

According [3] the mathematical model of the system is a single-degree-of-freedom motion model (figure-1):

$$m\ddot{z} + c\dot{z} + k.z = F(t) \qquad (1)$$

Where c is the damping coefficient, k is the stiffness of the layers disc spring. F_t is the excitation force.
If viscous damping is ignored, the vertical motion of the system, z(t) can be represented as follows:

$$z(t) = \frac{F_0/k}{1-r^2} sin(\omega.t) \qquad (2)$$

where: $r = \dfrac{\omega}{\omega_n}$ with $\omega_n = \sqrt{\dfrac{k}{m}}$, when $r=1$ the resonance phenomenon occurs, causing the oscillation amplitude to increase sharply even though the excitation force is small.
Natural frequency f_n:

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}(Hz) \qquad (3)$$

Critical frequency:

$$f = 60 f_n = \frac{60}{2\pi}\sqrt{\frac{k}{m}} \qquad (4)$$

Force transmitted to the floor:

$$F_T = k.z \quad (5)$$

Transmission coefficient when considering the effect of damping:

$$\rho = \sqrt{\frac{1+(2\xi cr)^2}{(1-r^2)^2+(2\xi r)^2}} \quad (6)$$

In which ξ is the vibration reduction coefficient.

$$\xi = \frac{c}{2m\omega_n}\frac{c}{c_r}; c_r = 2\sqrt{k.m} \quad (7)$$

Assumptions:
The punching machine is simplified as a lumped-parameter system with an equivalent mass of 340 kg.
The supporting structure is assumed to be rigid, while the machine is connected to the foundation through a Belleville spring mount characterized by stiffness K and damping coefficient c.
The initial deformation of the spring corresponds to the static load generated by the machine weight.
The stiffness of the helical spring is equivalent to the stiffness of the belleville spring mount when it reaches deformation $s_0 = 0,25h_0 = 0,4$ *mm*, then the stiffness of the disc spring at deformation s_0 is equal to $K = 1607,78$ *N/mm* A quarter-vehicle dynamic model of punching machine is established to see the effect of the values of the damping coefficient of belleville spring mount, as shown in Fig-5.

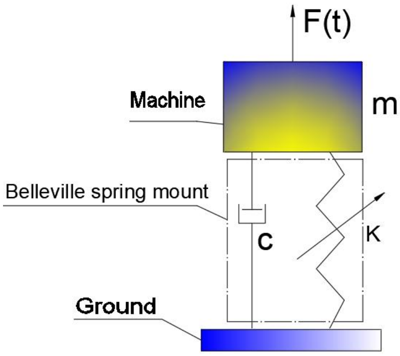

Figure 5. Quarter-machine dynamic model

In Fig-5, m is quarter-machine mass; k and c are the stiffness and damping coefficients of belleville spring mount; *z(t)* is the vertical displacements of machine; and $F(t)=F_0 sin(\omega t)$ is the force excitation of the vibrating machine; F_0 is the amplitude of force excitation; ω is the angular frequency of the machine.
The stiffness equation of a single disc spring is obtained [6]

$$k = \frac{dF}{dx} = 1163,52x^2 - 3397,8x + 2677 \quad (8)$$

According [4] The effect of vibration reduction is an important requirement in the design process isolator.
Usually, the vibration reduction efficiency needs to reach from 70% to 90%.
Transmission coefficient ρ determines the maximum transmission coefficient of the system based on the required vibration reduction efficiency
Minimum value of the excitation force frequency that ensures the vibration reduction efficiency of the bearing, according to [4] $f/f_n > \sqrt{2}$
In order to compare the vibration reduction characteristics of the belleville spring mount and the coil spring with equivalent stiffness and the same damping coefficient, calculate the damping coefficient based on the model as shown in Figure 6:

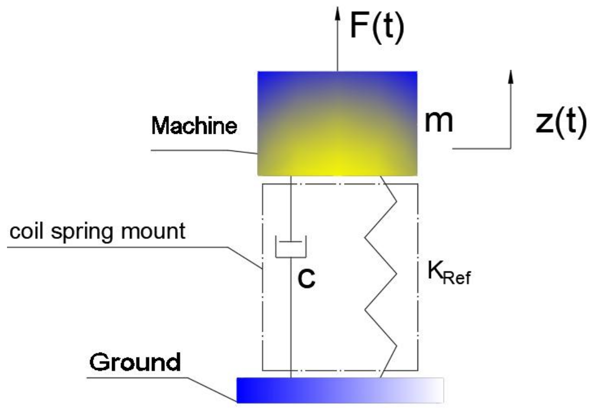

Figure 6. Vibration model using coil spring with stiffness K_{ref}

According [6]:

$$K_{ref} = \left.\frac{dF}{ds}\right|_{x^*=0,4} = 1163,52.(x^*)^2 - 3397,8.x^* + 2677$$

$$1163,52.(0,4)^2 - 3397,8.0,4 + 2677 = 1504.04\,N/mm \qquad (9)$$

$$= 1504,04\,N/mm$$

From (6) with the target of vibration damping efficiency being 90%, $\rho=0,1$ so we have:

$$r = \frac{\omega_{min}}{\omega_n} = 3.2 \qquad (10)$$

And have $c = 1357,94\,kg/s$ (11)

The natural frequency of the spring system with stiffness Kref:

$$\omega_n = \sqrt{\frac{K_{ref}}{m}} = \sqrt{\frac{1504,04.10\text{^}3}{85}} = 133,02\ (rad/s) \qquad (12)$$

$$r = \frac{\omega_{min}}{\omega_n} = 3.2 \Rightarrow \omega_{min} = \omega_n.r = 425.667(rad/s) \qquad (13)$$

Determine the amplitude F_0 according [27] the design working point of the disc spring is usually taken as $0,75h_0$. where h_0 is the height of the cone of the disc spring, the initial deformation is chosen as $s_0 = 0,25.h_0 = 0.4(mm)$ therefore the oscillation amplitude of the mass m in Figure – 4 $z_{max}=0,4mm$ so we have:

$$\frac{F_0}{K_{ref}} \le X_{max} \Rightarrow F_0 \le K_{ref}.X_{max} = 1504,04.10^3.0,0004 \qquad (14)$$

$$= 601,16N$$

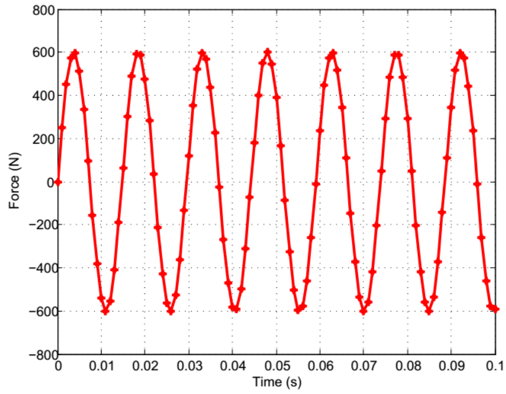

Figure 7. Harmonic Force

III.SIMULATION AND DISCUSSION

The governing dynamic equation described in Section 2 was numerically solved using MATLAB/Simulink software to investigate the vibration behavior of the proposed isolation system.

The simulation was carried out under resonance conditions, where the excitation frequency was selected close to the natural frequency of the vibration system. For the equivalent coil spring configuration, the natural angular frequency was determined to be 133.02 rad/s.

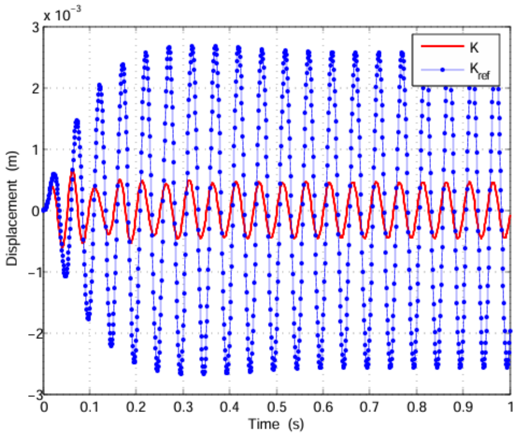

Figure 8. Vibration amplitude graph of mass m

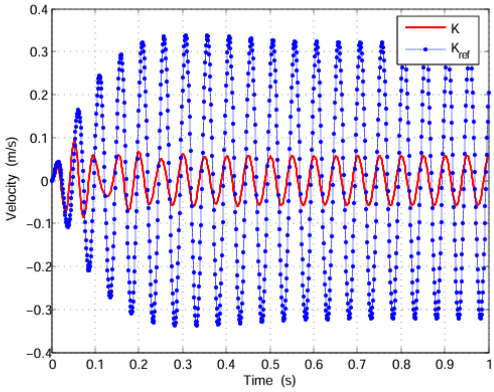

Figure 9. Vibration velocity of mass m

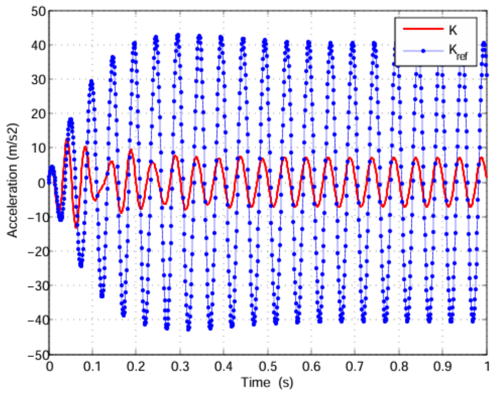

Figure 10. Vibration Acceleration of mass m

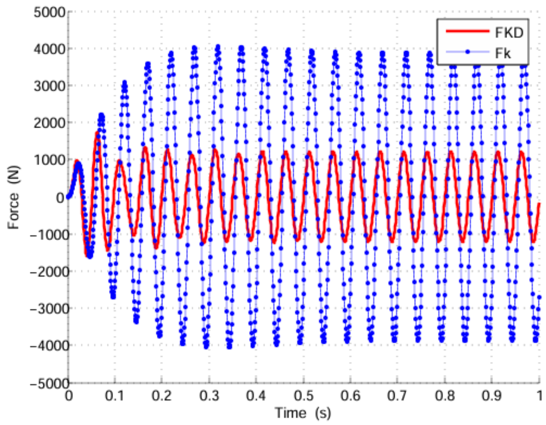

Figure 11. Force transmitted to the ground

IV. CONCLUSION

This study focused on evaluating the influence of the damping coefficient on the vibration reduction performance of the Belleville spring mount. A quarter-vehicle dynamic model of vibratory is established for analysis and evaluation. Parameters of small type punching machine are selected to apply on belleville spring mount. The results of matlab/Simulink software are shown.

The major conclusions can be drawn from the analysis and evaluation results as follows:

The obtained results demonstrated that, under resonance conditions, the Belleville spring mount provided more effective vibration isolation performance than the conventional coil spring system with equivalent stiffness.

Furthermore, the transmitted force from the excitation source to the foundation was significantly reduced when the Belleville spring mount was employed. The proposed isolation system is therefore suitable for industrial punching machines operating under severe vibration conditions. Future studies may focus on experimental validation and optimization of the nonlinear stiffness characteristics of the Belleville spring arrangement.

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