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Research Paper

Analysis of Yielding, Residual Stress and Creep in Rotating Composite Discs

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ABSTRACT

The present study investigates the creep deformation and stress analysis of rotating discs made of composite and functionally graded materials (FGMs) under thermo-mechanical loading conditions. Rotating discs are important structural components used in aerospace, automotive, gas turbines, and nuclear engineering applications where high rotational speed and elevated temperature produce severe stresses and time-dependent deformation. Conventional materials often fail to meet these demanding service requirements due to excessive creep, thermal stresses, and reduced structural efficiency. Therefore, advanced metal matrix composites and FGMs have been considered because of their superior mechanical and thermal properties. The research primarily focuses on Al–SiC metal matrix composites reinforced with silicon carbide particles or whiskers. Mathematical formulations based on elasticity, plasticity, and creep theories have been developed to evaluate radial stress, circumferential stress, strain distribution, and creep behavior in rotating discs. The influence of thermal gradients, residual stresses, anisotropy, reinforcement geometry, and composition gradients on the mechanical response of rotating composite discs has also been analyzed. Yielding behavior has been examined using von Mises and Hoffman yield criteria under different loading conditions. Functionally graded materials have been employed to achieve gradual variation of material properties across the disc radius, thereby reducing stress concentration and improving creep resistance. The analysis indicates that functionally graded composite discs possess better thermo-mechanical performance, lower creep deformation, and improved load-carrying capacity compared to homogeneous discs. The study demonstrates that proper control of reinforcement distribution and material gradation significantly enhances the structural reliability and service life of rotating components operating at elevated temperatures.

Keywords: *Functionally Graded Materials (FGMs), Metal Matrix Composites (MMCs), Creep Deformation, Rotating Discs, and Thermo-Mechanical Stress Analysis*

I. INTRODUCTION

Materials are the backbone of all natural and man-made structures. Technological advancement is associated with continuous improvement of existing material properties and the invention of new classes/types of structural material. The emergence of new materials is motivated by the necessity of improving efficiency and performance of existing structure. In addition, new materials offer opportunities to develop modified structures and technology, while the latter challenges material scientists with new problems. One of the best manifestations of this interrelated process in the development of materials, structures and technology is associated with the composite materials.

Composite materials have been the part of human life from the beginning of the civilization. Apart from natural composites, like the wood, people invented many multi-component materials even in ancient times. One of the most famous applications of the earlier invented composites is the Chinese wall, whose durability and stability was ensured by incorporation of contrastively different materials into a single structure. The age of modern composite materials began with the introduction of particles or fibres reinforced into thermoset phenolics in the early 1900's. Nowadays, composites play a very important role in engineering from aerospace and nuclear devices to microelectronics or structural engineering.

Composites are multifunctional material systems that provide characteristics not obtainable from any discrete material. They have cohesive structures made by physically combining two or more compatible materials, differing in composition and characteristics and sometimes in form.

The composites contain dispersed fibers, particles or whiskers embedded in a matrix (continuous phase). The properties of composites strongly depend on the properties of their constituents, their distribution and the interaction between them. The morphology of the discontinuous phase, size and distribution of the reinforcement and relative amounts of the phases present are important parameters that determine the extent of interaction between the reinforcement and the matrix phase. The composites have a special combination of room temperature strength and modulus, dimensional stability, excellent wear resistance, weldability and high temperature formability (Clyne and Withers, 1993).

Monolithic metals and their alloys cannot always meet the stringent needs as desired in advanced applications. In many cases, the use of composites is more efficient owing to their unique and tailored properties such as low density, exceptional strength and stiffness, fatigue and corrosion resistance, high thermal conductivity and low coefficient of thermal expansion (Srivatsan *et al*, 1995).

Metal Matrix Composites (MMCs) are extensively used in diverse areas like defence, aerospace, electronics, shipbuilding and automotive. MMCs possess metallic matrix with reinforcement in the forms of fibers, whiskers or particulates. MMCs can sustain high service temperature; possess high thermal and electrical conductivities and an excellent strength in shear and compression as compared to polymer matrix composites (PMCs). Due to superior stiffness, high strength at elevated temperatures and better creep characteristics, many of these MMCs are finding extensive applications in components exposed to elevated temperatures (Park *et al* 1990). MMCs are an attractive choice for variety of applications such as combustion engine, brake systems, stiff beams, load transfer elements in vehicles, aerospace devices, thermal management components in high power electronic devices, thermally cycled components and machine components requiring high wear resistance coupled with low weight.

In FGMs, the constituents or their contents are tailored with respect to some position coordinates, which enable them to provide unique performance (Gupta *et al*, 2005). The ceramic phase in an FGM offers thermal barrier effects and protects the metal from corrosion as well as oxidation and the FGM is toughened and strengthened by the presence of metallic phase. The development in processing techniques has made it possible to manufacture a mixture of ceramic and metal with a continuously varying volume fraction. This has made it possible to eliminate the interface problems, such as residual stress, in FGMs, that are caused due to difference in coefficient of thermal expansion (CTE) of the phases. FGMs have been developed for potential applications in aircrafts, space vehicles and other components exposed to elevated temperatures (Noda *et al*, 1998).

II. APPLICATIONS OF COMPOSITES

The mechanical properties and service-life of components evoked considerable interest, particularly for those made of composite materials. The tailor-made properties of composites could allow flexibility in design, since the required properties might be obtained by economic use of expensive constituents. As a result, new aspects of materials, such as the long-term behavior under mechanical, thermal and chemical loadings are being addressed. Composite materials have become more popular because of increased competition in the global market for lightweight components having greater strength and stiffness. Composite materials have the potential to replace widely used metals or their alloys such as steel, aluminum and their alloys. It is revealed that 60% to 80% weight saving can be achieved by replacing steel components with composite components, while 20% to 50% weight saving occurs if aluminum parts are replaced by composite parts (Mazumdar, 2002). The testing and evaluation of many available composite material systems are quite complex, costly and time consuming. Therefore, the prediction and analysis of creep properties, for assessing useful life of components made of composite materials, is of great practical importance for practicing engineers. The salient applications of composite materials along with required properties are summarized in Table 1.

Table 1: Typical applications of composites

Application Areas	Applications	Required Properties
Automobiles (Rohatagi *et al*, 1992; Yue *et al,* 1998; Peters, 1998; Fitzpatrick *et al*, 1998; Hunt, 2000; Surappa, 2003; Bayat *et al*, 2007; Singh, 2008; Bayat *et al*, 2008)	Combustion chambers (SiC-SiC), Engine cylinder liners (Al-SiC), CNG storage cylinders, Diesel Engine pistons (SiCw/Al-alloy), Brake rotors, Leaf springs (E-glass/epoxy), Drive shafts (Al-C), Flywheels, Racing car brakes (Al-SiC), Motorcycle drive sprocket, Pulleys, Torque converter reactor, Shock absorbers (SiCp/Al-alloy), Radiator end caps.	Increased stiffness, Improved wear resistance, Thermal fatigue resistance, Weight reduction, High thermal conductivity
Sub-Marine (Peters, 1998)	Propulsion shaft (Carbon and glass fibers), Cylindrical pressure hull (Graphite/Epoxy), Sonar domes (Glass/Epoxy), Composite piping system, Scuba diving cylinders (Al-SiC), Floats, Boat hulls.	Weight reduction, Durability
Commercial and Industrial (Fitzpatrick *et al*, 1998; Peters, 1998; Hunt, 2000)	Computer hard disk drive, Needle for carpet-weaving machine, Electronic Packaging/Thermal Management, Pressure vessels, Fuel tanks, Cutting tool inserts, Laptop cases, Wind turbine blades, Electric motors, Firefighting air	Weight reduction, Increased specific stiffness and strength, durability, High elastic modulus

	bottles, Artificial ligaments, MRI scanner cryogenic tubes, Wheelchairs, Hip joint implants, Eyeglass frames, camera tripods, Musical instruments, Drilling tubes, Drilling motor shaft, Drill casing, Crane components, High pressure hydraulic pipe, X-ray tables, Heart valves, Helmets, Crucibles, Beams.	
Aerospace equipment and structures, Space structure (Yue *et al,* 1998; Pitcher *et al*, 1998; Bache *et al*, 1998; Peters, 1998; Hunt, 2000)	Rocket nozzle (TiAl-SiC fibers), Heat exchanger panels, Engine parts (Be-Al), Wind tunnel blades, Spacecraft truss structure, Reflectors, Solar panels, Camera housing, Hublle space telescope metering truss assembly, Turbine rotor, Turbine wheels (operating above 40,000 *rpm*), Nose caps and leading edge of missiles and Space shuttle.	Light weight, Specific stiffness and specific strength at elevated temperature, Creep and fatigue resistance, Controlled CTE, Thermal conductivity, Dimensional stability
Aircraft, Missile structures (Pitcher *et al*, 1998; Shakesheff and Purdue, 1998; Peters, 1998; Hunt, 2000)	Wings, Rotary launchers, Engine casing, Rings (Al_2O_3/Al-alloy), Drive shaft, Propeller blades, Landing gear doors, Thrust reverser (Carbon/Bismaleimide), Helicopter components *viz.* Rotor drive shaft, Mast mount, Main rotor blades (Carbon/Epoxy).	Stiffness, Reduced weight, High temperature stability
Nuclear Reactor (Hunt, 2000)	Storage casks for spent fuel rods from nuclear reactors	Absorption of neutron radiation
Sports (Fitzpatrick *et al*, 1998; Hunt, 2000)	Tennis rackets, Golf shafts, Racing bicycle frame (SiCw/6061), Fishing rod, Pool cues.	Increased specific stiffness and strength, durability

III. YIELD CRITERIA

3.1 Mathematical Formulation

The yield criteria of materials limit the elastic domain during loading, whereas the failure criteria give the maximum stress that can be applied. Traditionally, we use the term yield criteria for metals or alloys and failure criteria for geomaterials such as soil and concrete. Safe and efficient use of materials is required for the successful design of any structural components. Therefore, for design purposes, the onset of plastic yielding under loading conditions is of great importance. The yield criterion gives the onset of plastic deformation. In other words, if a state of stress satisfies yield criterion, one can say that the plastification may start. It is assumed that the initial yielding is dependent only on the state of stress and not on how the stress is reached. It can be assumed that there exists a yield function $f(\sigma_{ij})$ such that:

$$\text{Material is elastic if :} \quad f(\sigma_{ij}) < 0$$

As the yield criterion does not depend on the path of loading, it does not tell anything about deformation. To develop a yield function, the components of the multiaxial stress state are combined into a single quantity known as the effective stress (σ_e). The effective stress is then compared with the yield stress in some appropriate form to check the inset of yielding.

The state of stress can be determined by specifying the principal stress and orientation of principal axes. The three principal stresses and three angles constitute the six-dimensional spaces. Thus, the yield function can be written as,

$$f = f(\sigma_1, \sigma_2, \sigma_3, \alpha_1, \alpha_2, \alpha_3)$$

where $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses and $\alpha_1, \alpha_2, \alpha_3$ are the orientations of principal axes.

The yield criterion must be consistent with a number of experimental observations, the chief of which is that pure hydrostatic pressure should not cause yielding in a continuous solid (Dieter, 1988).

3.2 Geometrical Representation

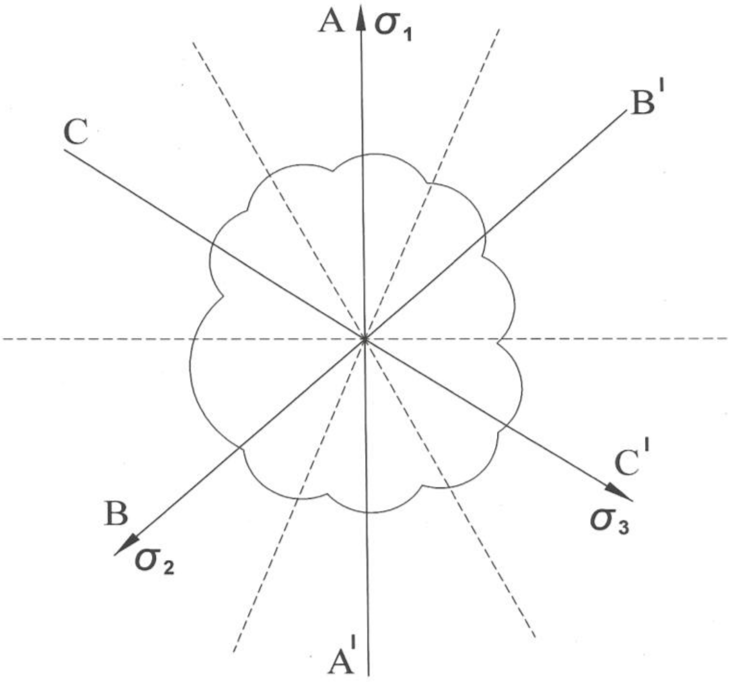

Fig. 1: Intersection of yield surface with π -plane

There exist three principal stresses and their orientation for a given state of stress at a point. Hence one can represent the state of stress as a point in three-dimensional vector space whose bases are the principal stresses for isotropic materials, where the orientation of the axes is not important. For isotropic materials $f(\sigma_1, \sigma_2, \sigma_3) = 0$ represents a surface in the principal stress space. If a material is acted upon by deviatoric stress, then the yield surface can be represented as,

$$f(\sigma_1, \sigma_2, \sigma_3) = 0 \tag{1}$$

Subject to the condition that,

$$\sigma_1 + \sigma_2 + \sigma_3 = 0 \tag{2}$$

Here, $\sigma_1 + \sigma_2 + \sigma_3 = 0$ is a plane passing through the origin and equally inclined to the principal axes. This plane is called the π - plane. Equations (1) and (2) represent a curve *C* on $\sigma_1 + \sigma_2 + \sigma_3 = 0$ plane (refer Fig. 1). If the yield is independent of hydrostatic stress, then the yield function is a right cylinder with generator perpendicular to the π - plane and whose cross-section from the hydrostatic line is the same as curve *C*.

3.3 Yield Criteria for Isotropic Material

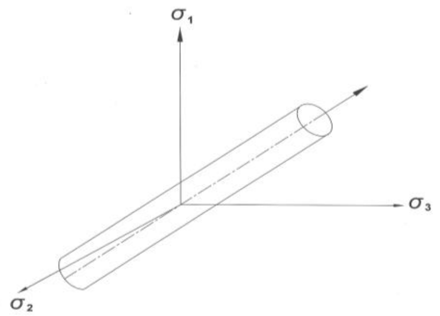

Fig. 2: Representation of von Mises yield criterion

For an isotropic material, the yield function must be independent of orientation of principal axes *i.e.* α_1, α_2 and α_3. Thus, in this case the yield function *f* is a function of σ_1, σ_2 and σ_3 alone *i.e*,

$$f = f(\sigma_1, \sigma_2, \sigma_3)$$

It was proposed by von Mises (1913) that the yielding would occur in an isotropic material when the second invariant of stress deviator J_2 exceeds some critical value.

$$J_2 = k^2 \tag{3}$$

where,

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

To estimate the value of constant *k* and to relate it with the yielding under tension test, it is realized that at the onset of yielding under uniaxial tension: $\sigma_1 = \sigma_e$, $\sigma_1 = \sigma_2 = 0$. Therefore, from Eqn. (3), we get,

$$\sigma_e^2 + \sigma_e^2 = 6k^2$$

or,

$$\sigma_e = \sqrt{3}\,k \tag{4}$$

Using Eqn. (4) into Eqn. (3), we obtain the following usual form of von Mises yield criterion,

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} \tag{5}$$

where σ_e is the equivalent stress.

Geometrically, von Mises yield criterion represents right circular cylinder whose generator is equally inclined to the principal stress axes, as shown in Fig. 2. The von-Mises criterion implies that the yielding under both uniaxial tension and compression would start at the same value of tensile and compressive stresses.

3.4 Yielding in Presence of Residual Stress

Arsenault and Taya (1987) pointed out that even in an isotropic metal matrix composite yielding does not begins at the same level of tensile and compressive stresses under uniaxial loading. Badini (1990) also noticed that the compressive yield strength of 15 *vol%* SiCw/6061Al composite is higher than its yield strength in tension. The processing of metal matrix composites often involves cooling from the higher temperature, which results in residual tensile thermal stress in the matrix due to restraint caused by ceramic reinforcements. To consider the effect of different yield stresses in compression and tension, some researchers (Schellekens and De Borst, 1990a, b; Bicanic *et al*, 1994; Moin, 1996) employed the following isotropic form of Hoffman yield criterion,

$$(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) + Q(\sigma_1 + \sigma_2 + \sigma_3) - 1 = 0 \tag{6}$$

where *Q* is a material parameter.

A number of other investigators (Bicanic *et al*, 1994; Moin, 1996) preferred the following alternate form of Hoffman's yield criterion that employs uniaxial compression and tensile yield stresses denoted respectively by f_c and f_t,

$$(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) + (f_c - f_t)(\sigma_1 + \sigma_2 + \sigma_3) - f_c f_t = 0 \tag{7}$$

The mechanical strength of whisker reinforced composites depends on the orientation of whiskers in the matrix (Ledrich and Shastry, 1982; Crowe *et al*, 1985; McDenels, 1985). Badini (1990) observed that the compressive yield strength of 15 *vol%* SiCw/6061Al composite in the longitudinal direction is higher than its yield strength in the transverse direction. Moreover, fiber-reinforced composites or components that are fabricated by processes such as forging, rolling or extrusion exhibit anisotropic properties. Therefore, the use of von-Mises yield criterion given by Eqn. (5) is not an appropriate choice for such cases.

The Eqn. (7) is used conveniently to describe the yielding of materials, which show different yield stresses under tension and compression. Several investigators (Arsenault and Taya, 1987; Shi *et al*, 1992) reported that thermal residual stresses generally develop during cooling of composites from high processing temperature. As a result of thermal residual stress, the yield strength of composites is different when their direction of loading is reversed during uniaxial loading. The magnitude of change in yield strength ($\Delta\sigma_y$) between compression and tension is dependent on the test temperature, as reported in Table 2 for 20 *vol%* SiCw/Al composite.

Table 2:Effect of processing temperature on yield strength of 20 *vol%* SiCw/Al (Shi *et al*, 1992)

Processing Temperature (^0C)	**Tensile Yield Stress (*MPa*)**	**Compressive Yield Stress (*MPa*)**
0	39.7	39.7
480	79.8	87.7

The yield stresses reported in the table above are deduced from the stress-strain curve (tension/compression) of SiC_w/Al composite with thermal history (*i.e.* without cooling and cooling from 480 ^0C).

Badini (1990) studied the correlation between the microstructure and the tensile and compressive properties of extruded bars made of 6061 Al alloy matrix composite reinforced with silicon carbide whiskers. The compressive strength in the longitudinal direction was observed to be considerably higher than the strength in the transverse direction. The compressive and tensile strengths of cylindrical samples, having 20 *mm* diameter, was found to be 218 *MPa* and 186 *MPa* respectively. Further, the mechanical strength of material was observed to depend on the loading direction, due to orientation of whiskers that contribute to reinforcing the composite to different degrees depending on their alignment in the direction of load applied (Lederich and Sastry, 1982; Crowe *et al*, 1985; McDanels, 1985).

IV. CONCLUSIONS

a) The present study investigates the creep deformation and stress analysis of rotating composite and functionally graded discs under thermo-mechanical loading conditions. The results show that metal matrix composites and functionally graded materials provide better mechanical and thermal performance than conventional homogeneous materials in high-temperature rotating applications.
b) The analysis indicates that reinforcement distribution, thermal gradients, anisotropy, and residual stresses significantly affect the stress and creep behavior of rotating discs. The incorporation of silicon carbide reinforcement improves creep resistance and reduces deformation. Functionally graded materials provide smoother stress distribution and lower stress concentration due to gradual variation in material properties.
c) The study also confirms that yield criteria such as von Mises and Hoffman criteria are effective in predicting the yielding behavior of composite materials. Overall, functionally graded composite discs exhibit improved load-carrying capacity, reduced creep deformation, and enhanced structural reliability, making them suitable for advanced engineering applications operating under severe thermo-mechanical conditions.

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