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Research Paper

An Overview of Recent Trends in Constraint-Based Fracture Mechanics

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Abstract
*Constraint-based fracture mechanics is a specialized field within the broader realm of fracture mechanics that focuses on understanding and predicting the behavior of cracks and defects in materials under various loading conditions. Unlike traditional fracture mechanics, which often relies on a single stress intensity factor to assess the criticality of a crack, constraint-based fracture mechanics takes into account the influence of constraint, or the stress field around the crack tip. This approach recognizes that the severity of a crack is not solely determined by its size and the applied load but also by the local stress state. Constraint-based fracture mechanics employs parameters like T-stress, Q-stress, and the stress triaxiality to provide a more comprehensive assessment of crack behavior. It considers factors such as crack tip plasticity, residual stresses, and material properties, making it a powerful tool for predicting crack growth and failure in real-world engineering applications. Researchers and engineers use constraint-based fracture mechanics to improve the safety and reliability of structures and components, particularly in industries like aerospace, automotive, and civil engineering. By considering the effect of constraint, they can make more accurate predictions about when and how cracks will propagate, enabling the development of better materials and design practices.***

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I. Introduction

Fracture mechanics is a pivotal field of study within materials science and engineering, essential for understanding the behavior of materials when subjected to external forces and stresses. It plays a critical role in various industries, from aerospace and civil engineering to the automotive and energy sectors. In the quest for safer and more reliable structures and components, researchers and engineers continually strive to enhance their grasp of fracture mechanics, seeking innovative ways to predict, prevent, and control fractures.

One compelling and evolving facet of fracture mechanics is "constraint-based fracture mechanics." This approach represents a paradigm shift in our understanding of how materials respond to stress concentrations, addressing not only the magnitude of applied loads but also the effects of geometric and material constraints. In essence, it recognizes that the mere assessment of stress intensity factors is insufficient to predict crack growth accurately.

This exploration delves into the core concepts, methodologies, and applications of constraint-based fracture mechanics. This research will embark on a journey to uncover how this approach augments our comprehension of fracture behavior in diverse materials and structures. By examining the principles of constraint, its theoretical underpinnings, and its practical implications, we aim to shed light on the innovative solutions it offers for safer and more resilient designs across various industries. Constraint-based fracture mechanics, with its ability to provide a more comprehensive view of crack growth and failure, stands as a promising frontier in the pursuit of engineering excellence and materials innovation

II. Theory of Constraint-based Fracture Mechanics

(Rice, 1968) proposed the J integral theory, which marks the beginning of the elastic-plastic fracture mechanics (EPFM) theory and method. Now, the J integral is regarded as a major parameter to characterize the fracture behavior of nonlinear materials. The J-integral theory and testing techniques have been widely applied to derive crack tip stresses of ductile material under plastic deformation,is crucial in structural integrity assessment of engineering structures, flaw evaluation, evaluation of the mechanical performance of materials, and the nuclear pressure vessel and piping, oil, and gas pipeline assessment. In fracture mechanics, the conservative approach is via a two-parameter conservative plain strain solution reported by (Hutchinson, 1968 and Rice & Rosengren, 1968), commonly referred to as the HRR Singularity. The HRR solutionwasdeveloped for materials with power-law stress-strain relation, given in Equation (1)

$$\frac{\varepsilon}{\varepsilon_Y} = \frac{\sigma}{\sigma_Y} + \alpha\left(\frac{\sigma}{\sigma_Y}\right)^n \text{……………………………Equation 1}$$

Where (σ_Y) is usually the yield stress, $\varepsilon_Y = \frac{\sigma_Y}{E}$ represents the yield strain, and E is Young'smodulus, α and nrepresentmaterial constants; hardening coefficient and hardening exponent, respectively. (n>1). HRR stress field around the crack tip is given by Equation (2).

$$\sigma_{ij} = \sigma_0\left(\frac{J}{\alpha\sigma_0\varepsilon_0 I_n r}\right)^{\frac{1}{(n+1)}} \widetilde{\sigma_{ij}}\,(n,\theta) \quad \text{………………………Equation 2}$$

$\widetilde{\sigma_{ij}}$ is the universal function that changes with the angle of strain hardening exponent tabulated by(Shih, 1983)and I_n is a dimensionless constant that depends on strain hardening exponent *n*, while Japart from being the strain energy release rate, it can also be viewed as a stress intensity parameter. (Rice, 1968)exploited a supposition based nonlinear material on proposing the J contour integral parameter that defines the vicinity of a crack. However, before publishing his findings, Rice found out that(Eshelby, 1968)(Cherepanov, 1967)had independentlypreviously published a sequenceof conservative integrals, some identical to the J integral of Rice. Nevertheless,it was Rice who established the relevance of his findings to solving crack related problems.

The J integral is found by integrating equation (3), an expression along an arbitrary path around the crack tip.

$$J = \int_\Gamma \left(W dy - T\frac{\delta u}{\delta x} ds\right) \quad \text{…………………………Equation 3}$$

Where Γ is the path of integration, *T*and *u* are the traction and displacement vector, *ds* is an increment along Γand *W* is the strain energy density or work of deformation per unit volume.*W* is mathematically expressed as shown in Equation (4)

$$W = \int_0^{\varepsilon_{ij}} \sigma_{ij}\, d\varepsilon_{ij} \quad \text{……………………………………Equation 4}$$

Where σ_{ij} *and* ε_{ij} are the stress and strain tensors, respectively.

The J integral as a nonlinear elastic release rate can also be defined accordingly as in the equation (5) below.

$$J = -\frac{1}{B}\left(\frac{\delta U}{\delta a}\right) \quad \text{…………………………………………Equation 5}$$

Where U is the strain energy per thickness or referred to as potential energy in a plate of thickness B.

In beginning of 1970s, (Begley & Landes, 1972) and (Landes & Begley, 1972) were the pioneers that successfully measured the J integral and it critical value J_{Ic}experimentally for standard laboratory test specimens. They used a sequence of test specimens of same material, size, and geometry but of cracks of different length and $a+\delta a$ introduced by fatigue pre-cracking method, using the energy release rate definition of J as given in equa. (5). This testing method has the weakness of complicated experimental procedures and multiple specimen test are needed to achieve a single experimental result of J, that is also expensive. Nevertheless, their early contribution encouraged anextensivedevelopment in the J integral fracture mechanics. The first main achievement of this work is the development of a simple analysis for computing J as a function of crack length at a point on the load–displacement curve from a single specimen test. A further advancement of practical experimental test technique for the J integral was the contribution by(Rice et al., 1973)that proved the possibility of a single specimen method, to estimate J resistance curve (J-R curve) directly from single load P against load line displacement (LLD) record for a compact tension specimen. Since strain energy U can be obtained from the area under of the P against LLD curve, further expressions for J integral in term of displacement control conditions and load control conditions can be obtained from equations(6) and (7)as follows:

$$J = -\frac{1}{B}\int_0^\Delta \left(\frac{\delta P}{\delta a}\right)_\Delta d\Delta \quad \text{………………………………Equation 6}$$

$$J = -\frac{1}{B}\int_0^\Delta \left(\frac{\delta \Delta}{\delta a}\right)_P dP \quad \text{………………………………Equation 7}$$

For compact tension specimen, (Merkle & Corten, 1974)suggested additional modification to consider tensile component of the applied load on the test specimen for accurate valuation of J. Since the total displacement Δ can be expressed separately as elastic and plastic component, as in $\Delta = \Delta_{el} + \Delta_{pl}$, the total J-integral was also correspondingly written as two separate parts:

$J = J_{el} + J_{pl}$ ……………………………………Equation 8

The elastic component J_{el} can be regarded as elastic strain energy rate, G and is most simply calculated from the stress intensity factor K_I

$J_{el} = \frac{K_I{}^2}{E'}$ …………………………………Equation 9

Where $E' = E/1 - v^2$ for plain strain while K is obtained from the load relation specified in (E399-09e2, 2011).Stress intensity factor solutions for various fracture specimens were documented in the work of (Tada et al., 2000)

Meanwhile, the plastic component,J_{pl} ,was stated similarly to equations (6) and (7)as follows:

$J_{pl} = -\frac{1}{B}\int_0^{\Delta_{pl}}\left(\frac{\delta P}{\delta a}\right)_{\Delta_{pl}} d\Delta_{pl} = \frac{1}{B}\int_0^{P}\left(\frac{\delta \Delta_{pl}}{\delta a}\right)_P dP$ ……………………..Equation 10

(Merkle & Corten, 1974) applied plastic analysis to provide a more reliable J estimate for C(T) specimens:

$J_{pl} = -\frac{2(1+\alpha)}{b(1+\alpha^2)}\int_0^{\Delta_{pl}}\left(\frac{P}{B}\right) d\Delta_{pl} + \frac{2}{b}\frac{\alpha(1-2\alpha+\alpha^2)}{(1+\alpha^2)^2}\int_0^{\frac{P}{B}}\Delta_{pl} d\left(\frac{P}{B}\right)$ ……………………Equation 11

Where the parameter αin equation 11 is defined as:

$\alpha = 2\sqrt{\left(\frac{a}{b}\right)^2 + \frac{a}{b} + \frac{1}{2}} - 2\left(\frac{a}{b} + \frac{1}{2}\right)$ ……………………………………Equation 12

It has been shown that equation (11) can accurately compute J-integral, based on the total displacement for a linear or nonlinear load-deflection curve provided $\frac{a}{W} \geq 0.5$. Otherwise, the elastic and plastic component of J should be calculated separately using equation(9), (11), (8).

It is a well-established phenomenon that the stress field around the crack tip solidly depends on the so-called constraint effect, (Chao & Lam, 1996)which is interrelated to geometries, loading mode, and the properties of material, and the magnitude of constraint is closely related to the material's fracture toughness. It is essential to note that the study of the constraint effects at fracture is linked to the basis and the restriction of the HRR-solution (Hutchinson, 1968)(Rice & Rosengren, 1968)Over the past few years, various two parameter approaches explaining elastic-plastic fracture mechanics were presented by researchers to highlight some of the limitations associated tothe single-parameter approach based on the J-integral. Some of the notable approaches been developed to describe crack tip fields and characterize the in-plane constraint effects which are mainly influenced by specimen geometry, crack size etc. are as follows: (Betegón & Hancock, 1991)(Al-Ani & Hancock, 1991)extended the K-T approach earlier suggested by (Williams, 1957)that described elastic materials to elastic-plastic by proposing J-T approach,J-Q approach proposed by(O'dowd & Shih, 1991)(O'dowd & Shih, 1992), later J-A_2 approach was proposed by (S. Yang et al., 1993b)(Chao et al., 1994) in which J integral and the second fracture parameter A_2 are both included in a three-term expansion of crack tip field. The A_2 constraint parameter is defined as the dimensionless amplitude of the second order term of that expansion. Based on the this, a similar two parameter approach J-A was proposed by (Nikishkov, 1995)(Nikishkov et al., 1995) where the magnitude of the second term in the expansion is represented by A, and it is a different normalizing form of the parameter A_2. The initial parameters K or J measures the level of crack tip deformation whereas the second parameter T/Q/A(A_2) measures triaxiality or magnitude ofconstraint at the crack tip. Some renown structural integrity standards such as the (R6, 2001) and the (BS7910, 2005) are still based the conservative two dimensional plain strain techniques looking at the capacity of materials to demonstrate improve toughness with reduced thickness which is caused by the out of plane effect. Similarly, the out of plane constraint is largely due to the influence of specimen thickness, the parameter $T_z$was proposed by(Guo, 1993a)(Guo, 1993b)(Guo, 1995) in a series of publications to characterize the out of plane constraint effect accurately, which is important foundation for three parameter dominant stress field, and permits the tendency to characterize the stress state in a three dimensional cracked structures. Citing the J-T_Z approach, (Graba, 2017) suggested that an elastic-plastic crack tip constraint in 3D can be computed through:

$Q^* = \frac{\sigma_{22}^{3D(FE)} - \sigma_{22}^{(J-Tz)}}{\sigma_0}\ ; r = \frac{2J}{\sigma_0}\ at\ \theta = 0^o$ …………………………………..Equation 13

Where Q^* represent the stress difference between the opening stress σ_{22} (3DFE) result and J-Tz deduced opening stress. (Zhao et al., 2008) observed the inherent drawback of the HRR field to measure the in plane constraint effect which was apparent in J-Tz approach and suggested a correction through a solution of the form:- $\mathrm{J} - \mathrm{Tz} - Q^{2D}$. Nevertheless, the insensitivity of this solution $\mathrm{J} - \mathrm{Tz} - Q^{2D}$ in characterizing highly compressive T or Q^{2D} geometries was reported in the work of (Hebel et al., 2007)(Shlyannikov et al., 2014)(Yusof & Leong, 2019). A solution that combines the advantages of both J-Tz and J-A_2 approaches was developed, a high order J-Tz-A_T solution was suggested by (Cui & Guo, 2019)Some of the other similar out-of–plane constraint parameter were suggested such as the crack tip stress normalized area of a zone defined by $\frac{\sigma_1}{\sigma_y}$(Anderson & Dodds, 1991)(Dodds et al., 1991)but was later disputed by (Theiss & Bryson, 1993) as limited in it application, while the h parameter which was reported to be an equivalent of the Q stress parameter was proposed by(Henry & Luxmoore, 1997). In another angle of contribution to measure crack tip constraint effect fracture resistance, a partially empirical approach was offered to characterized the transitional toughness of crack test specimen that moves from a relatively thick size specimen to thin size specimen, (Mostafavi et al., 2010) proposed J-ϕbased on the crack tip plastic deformation zone earlier suggested (Anderson & Dodds, 1991). Then recently, J-$A_P$an equivalent plastic strain parameter was introduced by (Yang et al., 2013)based on some modification to the parameter J-ϕ approach. It has been reported that this parameter has the ability to measure both in-plane and out-of–plane constraints in many test specimens of different crack depths and thickness.Forsimplicity of computation and engineering applications, another unified constraint parameter A_d was proposed by (Xu et al., 2018) which was based on crack tip opening displacement (CTOD) with modifications to A_P. The parameters $A_d$and A_P has reported to have same capability to concurrently characterize in plane and out of plane constraints. (Y. H. Wang et al., 2019) applied the crack driving force principle for envisaging ductile fracture based on two parameter $J–A_p$ and $J–A_d$ with exhaustive investigation and development of this parameters. In a similar development (Yusof, 2019)developed J-Δσ where it was demonstrated that three dimensional constraint field differs hydrostatically however, similar in terms of maximum stress deviator which remain independent of length, r and position along the crack front, z,

2.1 K -T Approach

(Rice, 1967)(Rice, 1968) proposed Boundary Layer Formulation (BLF) to study crack tip plasticity in controlled yielding without having to model geometry of the entire cracked part. When crack tip plasticity is minimal in relation to the dimension of the finite model, small scale yielding occurs, allowing for asymptotic elastic behavior. Traction equivalent to the K field are applied on the outer boundary of the crack tip region. (Rice & Tracey, 1973) maintainedthat the BLF remain valid as per as the plasticity is limited to less than one tenth of the mesh radius.

The BLF estimation has been observed to have limited range of validity, and there were inconsistencies found between BLF and a number of specimen geometries for crack tip plastic zones even when they were within the ASTM standard limit for fracture toughness measurement.(Larsson & Carlsson, 1973)discovered that the second term in William's expansion influences the shape and size of the plastic zone that forms at the crack tip. As denoted by (Rice, 1974), the T-stress is the second term in the Williams expansion, was claimed to be reason for the inconsistency mentioned above. The BLF solution and full field solutions from different geometries are compared to calculates for T-stress based on the discrepancies.The addition of the calculated non-singular T-stress to the remote K field in the boundary condition is referred to as the Modified Boundary Layer Formulation (MBLF) approach. The load in MBLF can be added using the superposition principle since the T-stress is directly proportional to the load applied in an isotropic linear elastic body. The MBLF approach or corresponding K-T characterization was also applied by (Bilby et al., 1986) where effect of non-singular T-stress on large geometry change deformation in void growth mechanism was examined. The hydrostatic stress ahead of the crack tip was described to be lowered by the compressive T- stress.

(Bilby et al., 1986) applied displacement field as boundary condition for modified boundary layer formulation instead of the applying the K-T field in a mode of stress on the outermost traction as suggested by Larsson & Carlsson, (1973)

The plane strain displacement u_i^{mblf} under the load due to parameter K and T is given by:

$u_i^{mblf} = u_i^{K_1} + u_i^T$ ……………………………..*Equation 14*

Where $u_i^{K_1} and\ u_i^T$ are displacement due to K and T-stress respectively.

The MBLF approach can be run by the finite element method under plain strain deformation, $\varepsilon_{zz} = 0$

$\sigma_{11} = T\ ,\ \sigma_{22} = 0,\ \sigma_{zz} = VT$ ………………….*Equation 15*

Thus, the remote displacements field for a mode I plain strain MBLF in terms of cylindrical coordinate (r, θ)are represented as:

$u_1^{mblf}(r,\theta) = K_I \frac{1-v}{E}\sqrt{\frac{r}{2\pi}} cos\frac{\theta}{2}(3-4v-cos\theta) + T\frac{1-v^2}{E} rcos\theta$ ……….*Equation 16*

$u_2^{mblf}(r,\theta) = K_I \frac{1-v}{E}\sqrt{\frac{r}{2\pi}} sin\frac{\theta}{2}(3-4v-cos\theta) - T\frac{v(1-v^2)}{E} rsin\theta$ ………*Equation 17*

Where u_1^{mblf} and u_2^{mblf} are the remote displacement field with respect to the x_1 and $x_2$axes, v is the poisson's ratio and E is the Young's modulus.

The remote displacement field for plane stress MBLF can also be obtained for a state of remote plane stress. Under plane stress, $\sigma_{zz} = 0$:

$\sigma_{11} = T\ ,\ \sigma_{22} = 0,$

The mode I remote displacement field for plane stress in term of cylindrical coordinates (r, θ) is given as:

$u_1^{mblf}(r,\theta) = K_I \frac{1-v}{E}\sqrt{\frac{r}{2\pi}} cos\frac{\theta}{2}(3-4v-cos\theta) + \frac{T}{E} rcos\theta$ …………..*Equation 18*

$$u_2^{mblf}(r,\theta) = K_I \frac{1-v}{E}\sqrt{\frac{r}{2\pi}} sin\frac{\theta}{2}(3-4v-cos\theta) - \frac{vT}{E} rsin\theta \text{ ………… Equation 19}$$

2.2 J-T Approach

(Rice, 1974) had admittedthat T-stress has no much influence on the J-integral. This means that the T-stress can be safely used in conjunction with other parameters to measure the crack tip field where the single parameter application fails.

(Bilby et al., 1986) in their contribution proved the consequence of the second-order term on the large geometry change (LGC) within the ratio $\frac{2J}{\sigma_0}$of the crack tip vicinity. Most importantly, negative T-stresses were found to substantially minimize triaxial stress value ahead of the crack. Conversely, the influence of T on the crack tip field was detailed through the systematic contributions of Hancock and his team in (Al-Ani & Hancock, (1991)(Betegón & Hancock, 1991)Du & Hancock, (1991)) they observed the importance of T-stress in the application as a constraint parameter in measuring the deviation between stress field and HRR fields, and thus developed the K/J-T crack tip quantification approach.

(Betegón & Hancock, 1991) analyze plane-strain elastic-plastic crack tip field by applying modified boundary layer formulation and a strain hardening material response. Their findings specifythat the configuration characterized by zero and positive T stress makes the stress field to approach HRR field, while configurations with Negative T stress make the opening stress ahead of the crack tip to reduce significantly. The influence of T stresses lower direct stresses ahead of the tip of the crack by an amount that is proportional to T and independent of the distance $\frac{r\sigma_0}{J}$. Bategon and Hancock with reference to this result suggested a stress field familydiffering only by a distance independent higher order term which depends on T:

$$\left(\frac{\sigma_{\theta\theta}}{\sigma_0}\right)_{(r,T)} = \left(\frac{\sigma_{00}}{\sigma}\right)_{(r,T=0)} + A_n\left(\frac{T}{\sigma_0}\right) + B_n\left(\frac{T}{\sigma_0}\right)^2 \text{ ………………………………Equation 20}$$

$A_n$and $B_n$are constants that depends on the strain hardening exponent, n of the material. The first term of equation (20)$\left(\frac{\sigma_{00}}{\sigma}\right)_{(r,T=0)}$ denotes the opening stress estimated by using the J-based plain strain HRR solution where T = 0. While the second and the third term in the equation (20) estimates the deviation from the HRR field due to negative T-stress. The sign of T-stress related to the cracked body governs the J-dominance, therefore the requirement of specimen size and loading criteria is no longer applicable to establish J-dominance in a typical J-T approach.

Al-Ani & Hancock, (1991) reported that J-dominance in shallow crack geometries could be lost or retained according to the sign of T-stress. Submitting that sign of T-stress changes from positive to negative as the cracked bend bar transition from a deep crack to a shallow crack. An unconstrained flow field with plasticity covering the area of cracked and un-cracked ligament was revealed in a shallow crack with negative T-stress. While a completely constrained flow field was linked with a deeply crack bars that shows positive T-stress. This observation served as the basis for stress field to be measured by two parameters, J and T respectively. The fracture mechanics approach uses the latter parameter to quantify constraint while the former is used to scaled applied load. (Parks, 1992)

The effect of T on structure of the crack tip field in plane strain conditions has been reported in (Du & Hancock, 1991) using modified boundary layer formulations. Crack tip deformations was depicted by slip line fields within small-strain theory. It was proved that tensile T-stresses increases the magnitude of crack tip stress triaxiliaty towards the Prandtl field though the plastic zone reduces and rotates towards the crack flank. Compressive T-stresses decrease the level of stress traixiality and makes plastic zone to swing forward. They observed that tensile positive T-stress results plasticity to envelope the crack tip and displays a Prandtl field, equaling to the limiting HRR solution for non-hardening material. This leads to a loss of J-dominance, and stress distributions represented by an incomplete Prandtl field

The progress demonstrated in the work of Hancock and co-workers (Du & Hancock, 1991)(Betegón & Hancock, 1991)on the parameterization of J-T indicated that J-T approach can characterize stresses around the crack tip field irrespective of the geometry and loading. Even though T-stress was an elastic parameter, J-T approach based characterization can be applied to beyond the limit of contained yielding as reported by (Karstensen et al., 1997). But, other researchers in the field (O'dowd & Shih, 1991, 1992)challenged the argument intensely by claiming that T-stress was achieved from an asymptotic expansion series of elastic crack, therefore it application should be limited to only small scale yielding conditions.

2.3 J-Q Approach

The K/J-T approach has shortcomings, limited to small scale yielding conditions, essentially due to the T-stress been an elastic parameter, and tends to be less meaningful in characterizing crack tip fields as the plastic zone enlarges around the crack tip. To address this shortcomings, a different second parameter Q was examined for an elastic-plastic crack under small and large scale yielding conditions. (O'dowd & Shih, 1991)proposed a new two parameter approach termed J-Q approach. The second parameter Q wasconsidered as a dimensionless amplitude to designate the non-singular second order term in the crack tip expansion in (Williams, 1957). Based on the work of (Y. Li & Wang, 1986), (Sharma & Aravas, 1991) where two term asymptotic expansions of crack tip deformations was detailed. (O'dowd & Shih, 1991, 1992) assuming that FEA result are exact and calculated the difference between numerical and HRR. They suggested that the crack tip stress field can be shown in the form of the two term expansion by following equation (21)

$$\sigma_{ij} = \sigma_0 \left(\frac{J}{\alpha\sigma_0\varepsilon_0 I_n r}\right)^{\frac{1}{(n+1)}} \widetilde{\sigma_{ij}}(n,\theta) + Q \left(\frac{r}{J/\sigma_0}\right)^{q} \widetilde{\sigma\sigma_{ij}}(n,\theta) \quad \text{……………….. Equation 21}$$

where $\widetilde{\sigma_{ij}}(n,\theta)$ are functions computed numerically, q denotes power exponent ranging from (0 to 0.071) and Q as defined earlier is the parameter an amplitude of second term asymptotic solution. (O'dowd & Shih, 1991, 1992)experimented the Q parameter within the range of $J/\sigma_0 < r < 5J/\sigma_0$ near the crack tip. They reported that the parameter Qdimly defends on the distance of crack tip within the range of angle$|\theta| \leq \pi/2$.O'Dowd & Shih suggested only two terms to define the crack tip deformations in the vicinity of the crack.

$$\sigma_{ij} = (\sigma_{ij})_{HRR} + Q\sigma_0\widetilde{\sigma_{ij}}(\theta) \quad \text{………………………………………………….Equation 22}$$

Where $(\sigma_{ij})_{HRR}$ is the HRR field, $\sigma_0$is the yield stress and Q is the stress triaxiality parameter to represent the hydrostatic stress level at the crack tip. It was observed that from FEA result of crack tip opening stress $(\sigma_{\theta\theta})_{FEA}$ the Q-stress may be evaluated from the following formulae:

$$Q = \frac{(\sigma_{\theta\theta})FEA - (\sigma_{\theta\theta})HRR}{\sigma_0} \; at \; \frac{r\sigma_0}{J} = 2 \;\; and \; \theta = 0 \quad \text{…………………………….Equation 23}$$

Where $(\sigma_{\theta\theta})_{FEA}$ is the stress value evaluated from FEA and$(\sigma_{\theta\theta})_{HRR}$ is the stress value obtained from HRR solution.

The distance $r = \frac{2J}{\sigma_0}$from crack tip lies outside the finite strain blunting zone, this makes the values of Q obtained from small or finite strain analysis nearly the same(McMeeking, 1977). Furthermore, (O'dowd & Shih, 1991) and (O'dowd, 1995) proposed anotherreference stress field to substitute the HRR solution in equation (22) using the small scale yielding stress field $(\sigma_{ij})_{SSY}$ with T=0 and where $(\sigma_{ij})_{SSY}$ is to be define by FEA applying the modified boundary layer formulation. Consequently, two reference stress field emanate in two forms of Q definition. The difference in Q due to these two reference stress field were analyzed by (O'dowd, 1995)Generally, the theoretical HRR field is recommended to be applied as the reference field as used in equation (22) and (23) to decrease the FE computations required by the SSY reference stress field.

(O'dowd, 1995; O'dowd & Shih, 1991, 1992, 1994)used several FEA results to demonstrate that the J-Q approach holds nearly in the area of interest ($1 \leq r/(J/\sigma_0) \leq 5$)for most laboratory fracture test specimens up to some realistic deformation level. Under the small scale yielding conditions, the MBL analysis indicated that J-Q approach and J-T approach are found to be equivalent under the state where both approaches apply. However, this equivalence condition does not apply under large scale yielding or fully yielding conditions. This parameter is not suitableto describe the constraint effecton the crack growth J-R curve, based on the observation of (Faleskog, 1995) that indicated the Q parameter fluctuates on the J-R curve while in ductile crack growth. Generally, the Q parameter can efficiently define the constraint effect atthevicinity of crack tip for diverse geometries under a range of deformation stages. J-Q approach has got recognition for application in structural integrity assessment standard by European Engineering Programs such as (SINTAP, 1999) and (FITNET, 2006). The Q-stress is used to construct the fracture toughness criterion and also for the fracture toughness assessment in structural parts. Hence, several fracture criterions based on O'Dowd's theory has emerged and has practical applications in engineering problems.

(O'dowd, 1995) suggested a fracture criterion based on the J-Q theory as shown in the Equation (24)

$$J_C = J_{IC}\left(1 - \frac{Q}{\sigma_c/\sigma_0}\right)^{n+1} \quad \text{……………………………………………..Equation 24}$$

Where Jc represents the real fracture toughness for structural part measured by geometrical constraint described by Q stress (of smaller value usually < 0), Jic denotes plain strain fracture toughness (Q=0) while $\sigma_c$represents the critical stress according to the Richie-Knott-Rice hypothesis proposed in(Ritchie et al., 1973). This criterion was further studied by(Neimitz et al., 2007) where they suggested another criterion by modification of equation (24) by simply replacing the critical stress σ_c by σ_{max} representing maximum opening stress, which is numerically calculated using large strain formulation. The modified fracture criterion as suggested by (Neimitz et al.,) is of the form:

$$J_C = J_{IC}\left(1 - \frac{Q}{\sigma_{max}/\sigma_0}\right)^{n+1} \quad \text{…………………………………………………..Equation 25}$$

Accordingly, Q stress has been widely applied to characterized the constraint effect on fracture toughness parameters J_c and J_{Ic}as reported in the work of (O'dowd & Shih, 1994)(Nevalainen & Dodds, 1995)(Joyce & Link, 1997, 1995)(Gao & Dodds Jr, 2001)

2.4 J-A_2 Approach

As described above, the weakness of J-Q approach is that it was actually a typical numerical solution that was built based on FEA results. As load increases, the stresses around the crack tip in low constraint geometry slowly diverge from the HRR solution. An in-depth theoretical analysis was conducted by (Yang et al., 1993)(Chao et al., 1994)working from the University of South Carolina, they developed the higher order crack tip field for materials that are elastic-plastic and power –law hardening compliance. Applying the theory of deformation plasticity under plain strain condition, these researchers revealed that the first three terms of the stress asymptotic expansion can be defined by only two parameters, where J-integral is the crack driving force and a second parameter termed as A_2.This three term asymptotic stress solution is given as:

$$\frac{\sigma_{ij}}{\sigma_0} = A_1 \left[\left(\frac{r}{L}\right)^{s_1} \widetilde{\sigma_{ij}}^{(1)}(\theta, n) + A_2 \left(\frac{r}{L}\right)^{s_2} \widetilde{\sigma_{ij}}^{(2)}(\theta, n) + A_2^2 \left(\frac{r}{L}\right)^{s_3} \widetilde{\sigma_{ij}}^{(3)}(\theta, n) \right] \text{…….Equation 26}$$

Where the stress power exponent $s_k (s_1 < s_2 < s_3 = 2s_2 - s_1)$and stress angular function $\widetilde{\sigma_{ij}}^{(k)}(\theta,)$ $(k = 1,2,3)$ depends only on the strain hardening exponent *n*(S. Yang et al., 1993a) L is the characteristic length parameter and mostly taken as 1mm. the A_1 *and* s_1 parameter are related to the HRR field given by:

$$A_1 = \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I_n r}\right)^{-s_1} \text{………………………………………………………Equation 27}$$

$$s_1 = -1/(1+n) \text{……………………………………………………….Equation 28}$$

From equation (26) above, the first termis a singular HRR field, while the other two terms are higher order terms, with amplitudes specified in terms of A_2constraint parameter. (Chao et al., 1994; S. Yang et al., 1993b)determined the A_2 parameter applying a simple point matching technique at $r=2J/\sigma_0$and $\theta = 0$ as applied in equation (23) in the definition of Q. other more accurate techniques applied in the determination of A_2 are the least square fitting reported in (Nikishkov et al., 1995)or a simple weight average method by (Chao & Zhu, 2000)

(S. Yang, 1993) demonstrated numerically that the parameter A_2changes to constant under LSY or fully plastic state when the load applied is greater than 1.2 times the limit load for three point bending (3PB) specimen or SENT specimens. A study by (Chao & Zhu, 2000)later provided a hard theoretical evidence that the A_2 parameter is independent of the applied J-integral under LSY or fully plastic deformation. This finding of been load independent makes A_2 more importantsince its value determined at a ductile crack initiation load measured by $J_i$may remain constant for an applied J larger than J_i. Therefore, A_2 is a suitable parameter to define the constraint effect on both J_c orJ_{Ic}fracture toughness parameters and J-R curve during ductile crack tearing. The Q parameter in contrast is almost independent of the applied J

2.5 J-A Approach

The J-A_2 two parameter approach combining J and A_2 parameters, was first suggested by (S. Yang et al., 1993b) which is centered on a three term asymptotic expansion.

(Nikishkov, 1995; Nikishkov et al., 1995) developed a similar series expansion. They applied the term A to represent the second constraint parameter, which is actually a different normalized method of A_2 in the J-A_2 approach. Hence, the term J-A approach is applied instead of J-A_2 approach.

The deformation for the material of the cracked part is in conformity with the Ramberg-Osgood power-law strain hardening curve Equation (1). (Hutchinson, 1968) and (Rice & Rosengren, 1968) unraveled an asymptotic problem for elastic-plastic crack and revealed that within the framework of small strains, the stresses in the vicinity of the crack tip referred to as HRR field are singular. From Equation (2) the stress can be expressed in the form

$$\frac{\sigma_{ij}}{\sigma_Y} = \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I_n r}\right)^{\frac{1}{(n+1)}} \widetilde{\sigma_{ij}}^{(0)}(\theta) \qquad \text{…………………………………………….Equation 29}$$

From the above expression r is the distance from the crack tip, and J is as specified in Equation (3). The yield criterion is written in the form given in equation (30) this HRR field does not properly define stresses in the region $1 < r\frac{\sigma_Y}{J} < 5$ that is important for fracture process.

$$(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + (\tau_{xy}{}^2 \tau_{yz}{}^2 \tau_{zx}{}^2) = 2\sigma_Y^2 \text{…………Equation 30}$$

More descriptive stress field in this region can be obtained with three term asymptotic expansion suggested by (S. Yang et al., 1993b) and improved further by (Nikishkov et al., 1995)

$$\frac{\sigma_{ij}}{\sigma_Y} = A_0 \rho^s \tilde{\sigma}_{ij}^{(0)}(\theta) - A \rho^t \tilde{\sigma}_{ij}^{(1)}(\theta) + \frac{A^2}{A_0} \rho^{2t-s} \tilde{\sigma}_{ij}^{(2)}(\theta) \text{……………………………………Equation 31}$$

In this equation (31), the second fracture parameter is denoted by A, whileσ_{ij} are components of stress $\sigma_r, \sigma_\theta$and $\sigma_{r\theta}$ in the polar coordinate system $r\theta$ with origin at the crack tip, dimensionless angular stress functions denoted by $\tilde{\sigma}_{ij}^{(k)}$ gained from solution of asymptotic problem of order 0,1 and 2. Angular stress functions $\tilde{\sigma}_{ij}^{(0)}$ and $\tilde{\sigma}_{ij}^{(1)}$ are gauged to achieved a unity equivalent Mises stress, expressed as $\max_\theta \tilde{\sigma}_e^{(0)} = \max_\theta \tilde{\sigma}_e^{(1)} = 1$. Amplitude of stress angular functions for the order 2 problem are determined by on the solutions of the order 0 and 1 problems. While s is an exponent that has a closed form expression given in equation (28). Exponent t is a numerically calculated eigenvalue that relies on the strain hardening exponent n. A_0 is a coefficient expressed mathematically as in equation (32)

$$A_0 = (\alpha \varepsilon_0 I_n)^s \quad \text{.........Equation 32}$$

ρ is a dimensionless radius given by the following expression, equation (33)

$$\rho = \frac{r}{J/\sigma_0} \quad \text{.........Equation 33}$$

comparing equation (29) and (31) it can simply be seen that the first term of the asymptotic expansion in (31) is precisely the HRR field in (29). The three term of the expansion in equation (31) are governed by two parameters, J and A respectively. The parameter A is a degree of stress field deviation from the HRR field.

(Matvienko, 2020)(Matvienko & Nikishkov, 2017)discussed the theoretical and numerical features of the two parameter J-A theory in elastic plastic fracture mechanics in relation to crack tip constraint, where the parameter A was reported to be capable of been introduced as a fracture criterion as a constraint parameter. The two parameter J-A fracture criterion permits approximating whether elastic approach is conservative or otherwise.

2.6 J- T_Z Approach

To assess the effect of out of plane stress constraint on the elastic-plastic crack tip fields, (Guo, 1993a, 1993b, 1995) has proposed an out of plane stress constraint factor T_Z. Neglecting the out of plane component of shear, the constraint parameter T_Z can be defined as the ratio of the out of plane stress $\sigma_{33}$to the sum of in-plane stress $(\sigma_{11} + \sigma_{22})$ it is expressed as:

$$T_z = \frac{\sigma_{33}}{\sigma_{11} + \sigma_{22}} \quad \text{.........Equation 34}$$

where subscripts 1, 2 and 3 represent x, y and z in the Cartesian coordinate or r, θ and z in the polar coordinate, respectively, with z axis along the direction tangential to the crack front line. $T_z = 0$ in plane stress state. While, in plane strain state T_z equals to Poisson's ratio ϑ for elastic materials and 0.5 for incompressible plastic materials, and may change from ϑ to 0.5 for elastic-plastic material. T_z normally ranges from 0 - 0.5, in the vicinity of the crack in a given finite parts. (Guo,) has extended the HRR solution from 2D solution with $T_z = 0$ for plane stress or $T_z = 0.5$ for plane strain to a modified HRR solution referred to as $J - T_z$ approach covering the range of T_z from 0 - 0.5 completely. (Neimitz, 2000, 2004)(Guo, 1993a)

$$\sigma_{ij} = \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I_n(T_z, n) r}\right)^{\frac{1}{(n+1)}} \widetilde{\sigma_{ij}}(T_z, n, \theta) \quad \text{.........Equation 35}$$

The three dimensionaleffect can be introduced into equation (35) through I_n *and*$\widetilde{\sigma_{ij}}$ that are functions of (T_z). The values of the function $I_n$and$\widetilde{\sigma_{ij}}$ can computed numerically as studied by (Galkiewicz & Graba, 2006). The application of J-Tz approach in measuring the stress distribution along the crack front was studied by (Guo, 1995). Hoop stress were computed applying T_z values from (Narasimhan & Rosakis, 1990) used in equation (35). (Guo, 1995) observed that the hoop stress, σ_{33} projected from the $J - T_z$ approach agreed with the findings from (Narasimhan & Rosakis, 1990)

To obtain the three dimensional crack tip stress field according to the $J - T_z$ approach, the exact value of T_z is usually obtained through a 3D finite element formulation. But, the values of T_z along the crack front tip are crack configurations dependent.Procedures to achieve an exact T_z distribution were studied and fitting equation were formulated for ease of T_z distribution along a 3D crack front configurations.(Zhao & Guo, 2012) derived fitting empirical solutions for T_z distribution for mixed mode elastic compact tension shear specimen. A double edged notched specimen in tension with numerous notch shape was studied in(Li et al., 2000)while a single edge cracked specimen under tension in creep was reported in (Xiang & Guo, 2013) and a three dimensional modified boundary layer formulation by(She et al., 2008). However, the usage of these empirical fitting solution were narrowed since they were proposedonly for specific crack configuration.

Three dimensional elastic –plastic crack according to (Guo, 1995),showed that the T_z distribution at an angle θ=0° along the crack front can be described by:

$$T_z = \frac{1}{2}\left[1 - (1-2v)\left(\frac{r}{r_p}\right)^{\frac{n-1}{2.3n+1}}\right]\left[1 - 1.218\left(\frac{r}{B}\right)^{1/2} - 0.395\left(\frac{r}{B}\right) + 0.361\left(\frac{r}{B}\right)^{3/2}\right] X \left[1 - \left|\frac{x_3}{h}\right|^{0.94\left(\frac{r}{B}\right) - 0.58}\right]^2 \quad \text{.........Equation 36}$$

where v, r_p, h, z and n are Poisson's ratio, plastic zone size, half thickness of specimen, distance measured from the mid-plane of the specimen, and strain hardening exponent respectively. Equation (36) is limited for a through thickness crack with a/W=0.5 under SSY condition only with loading level, $J/\sigma_0\varepsilon_0 B \leq 10$.The $\mathrm{T_z}$ distribution along a crack front projected from equation (36) was compared with the findings from (Nakamura & Parks, 1990). Conversely, it is imperative to note equation (36) does not distinguish between the effect of positive and negative T-stresses on the configuration of the plastic deformation at the crack tip fields as observed in (Du & Hancock, 1991)

An approach that combines $\mathrm{T_z}$ with the existing two parameter approaches was proposed in measuring the three dimensional crack tip field. $K - \mathrm{T_z}$ approach was proposed for elastic three dimensional crack (She et al., 2008) and that was followed by $K - T - \mathrm{T_z}$ approach as reported in (Zhao & Guo, 2012). An effort to link$J - Q$ approach with $\mathrm{T_z}$ was studied by (Neimitz & Graba, 2008)where it was shown that numerical effort is required to compute the modified Q parameter denoted by Q^* function, which was found to be dependent of several variables, it represent the difference between actual stress field and the reference field given from Guo's general asymptotic stress field at crack tip in a power law material.

The applicability of $T_Z$in defining the out of plane constraint effect was studied by several scholars. (Shlyannikov et al., 2011) discussed the through thickness distribution of $\mathrm{T_z}$in a biaxial loaded plate. $\mathrm{T_z}$ was found not sensitive to the change in load biaxiality due to applied T-stress. Evaluation of the influence of crack length andtest specimen thickness on the $\mathrm{T_z}$ through thickness distribution using compact tension (CT) and a single edge notched bend (SENB) specimens were conducted by (Shlyannikov et al., 2014) it was observed that despite $\mathrm{T_z}$ been an out of plane constraint parameter, it was delicate to changes in crack length in the CT specimen. This means that the $\mathrm{T_z}$ distribution along the crack front could be affected by both in plane and out of plane constraint loss. However, (Yusof & Leong, 2019) suggested that the measurement of the 3D crack tip stress in power law hardening material is still an unanswered problem thus requires other suitable ideas to resolve the problem.

(Wang et al., 2014a, 2014b)carried out a detailed study on the $\mathrm{T_z}$ distribution along the crack frontusing a single edge notched tensile (SENT) specimen. Diverse crack configurations were utilized in the FEA by altering the thickness to width ratio (*B/W*) crack length to width ratio (*a/W*) and strain hardening rate (*n*). It was observed that the through thickness distribution of $\mathrm{T_z}$ in SENT specimen was independent of the *n* and*a/W* . The through thickness distribution of $\mathrm{T_z}$ tends to exhibit more uniformity with increase in specimen thickness, this implied consistent value of $\mathrm{T_z}$ along the crack front and it value reduces at the area near the free surface.

III. Conclusion

Constraint-based fracture mechanics has emerged as a crucial advancement in the field of fracture mechanics, offering a more nuanced and accurate understanding of crack behavior. By considering factors beyond just crack size and applied load, such as stress fields and material properties, this approach has proven invaluable in assessing the criticality of cracks in various engineering applications.

The incorporation of parameters like T-stress and Q-stress has expanded the analytical toolbox available to researchers and engineers, enabling them to make more informed decisions about structural integrity and safety. This, in turn, has led to improved designs, reduced maintenance costs, and enhanced overall reliability in critical industries.

As technology continues to advance, and materials and structures become increasingly complex, the principles of constraint-based fracture mechanics will likely play an even more significant role in ensuring the safety and longevity of engineering systems. Researchers will continue to refine and expand upon these principles, ultimately contributing to safer and more efficient designs in the future.

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