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Research Paper

Investigation of Some Rational Recursive Sequences

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ABSTRACT. In this article, we study some properties of the solutions of the following difference equation: $u_{n+1} = au_n + \dfrac{bu_n u_{n-7}}{cu_{n-6} + du_{n-7}}, \quad n = 0, 1, \cdots,$ assuming positive initial conditions $u_{-7}, u_{-6}, \cdots, u_0 > 0$ and a, b, c, d are positive constants. Furthermore, we give some specific forms of the solutions for several special cases of the equation.

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I. Introduction

Difference equations are mathematical relations that describe the evolution of a variable over discrete time steps. They are the discrete-time analogs of differential equations and are widely used in modeling systems where changes occur at specific intervals. Several applications of difference equations that can be found either in economics and finance (Compound interest and loan repayment models that use recurrence relations for amortization schedules), in population dynamics (exponential and logistic growths), in engineering and control systems (discrete-time filters and modeling RLC circuits in discrete time) and in computer science (recurrence relations for divide-and-conquer algorithms, transition probabilities between states). Difference equations provide a powerful framework for modeling discrete-time systems across disciplines. From finance to biology, they offer insights into dynamic behavior, stability, and long-term trends.

Our aim in this paper is to investigate the behavior of the solution of the following nonlinear difference equation

$$u_{n+1} = au_n + \frac{bu_n u_{n-7}}{cu_{n-6} + du_{n-7}}, \quad n = 0, 1, \cdots, \tag{1.1}$$

where the initial conditions $u_{-7}, u_{-6}, \cdots, u_0 > 0$ are arbitrary positive real numbers and a, b, c and d are positive constants.

Recently there has been a great interest in studying the qualitative properties of rational difference equations. Some prototypes for the development of the basic theory of the global behavior of nonlinear difference equations of order greater than one come from the results for rational difference equations.
However, there have not been any effective general methods to deal with the global behavior of rational difference equations of order greater than one so far. From the known work, one can see that it is extremely difficult to understand thoroughly the global behaviors of solutions of rational difference equations although they have simple forms (or expressions). One can refer to [1–22,31–35,37–41] for examples to illustrate this. Therefore, the study of rational difference equations of order greater than one is worth further consideration.

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Many researchers have investigated the behavior of the solution of difference equations, for example, Elsayed et al. [22] has obtained results concerning the dynamics and global attractivity of the rational difference equation

$$u_{n+1} = \frac{a u_n u_{n-2}}{b u_{n-2} + c u_{n-3}}.$$

Aloqeili [3] has obtained the solutions of the difference equation

$$u_{n+1} = \frac{u_{n-1}}{a - u_n u_{n-1}}.$$

Simsek et al. [37] obtained the solution of the difference equation

$$u_{n+1} = \frac{u_{n-3}}{1 + u_{n-1}}$$

Çinar [7–9] got the solutions of the following difference equations

$$u_{n+1} = \frac{u_{n-1}}{1 + a u_n u_{n-1}}, u_{n+1} = \frac{u_{n-1}}{-1 + a u_n u_{n-1}}, u_{n+1} = \frac{a u_{n-1}}{1 + b u_n u_{n-1}}$$

In [28], Ibrahim got the form of the solution of the rational difference equation

$$u_{n+1} = \frac{u_n u_{n-2}}{u_{n-1}(a + b u_n u_{n-2})}$$

Karatas et al. [40] got the solution of the difference equation

$$u_{n+1} = \frac{u_{n-5}}{1 + u_{n-2} u_{n-5}}$$

Here, we recall some notations and results which will be useful in our investigation. Let I be some interval of real numbers and let

$$f : I^{k+1} \to I,$$

be a continuously differentiable function. Then for every set of initial conditions $u_{-k}, u_{-k+1}, u_{-k+2}, ..., u_0 \in I$, the difference equation

$$u_{n+1} = f(u_n, u_{n-1}, ..., u_{n-k}), \quad n = 0, 1, ..., \tag{1.2}$$

has a unique solution $\{u_n\}_{n=-k}^{\infty}$.

Definition 1.1. A point $\bar{u} \in I$ is called an equilibrium point of Eq. (1.2) if $\bar{x} = f(\bar{u}, \bar{u}, ..., \bar{u})$. That is, $u_n = \bar{u}$ for $n \geq 0$, is a solution of Eq. (1.2), or equivalently, $\bar{u}$ is a fixed point of f.

Definition 1.2.
- The equilibrium point $\bar{u}$ of Eq. (1.2) is locally stable if for every $\varepsilon > 0$, there exists $\delta > 0$ such that for all $u_{-k}, u_{-k+1}, u_{-k+2}, ..., u_0 \in I$, with

$$|u_{-k}-\bar{u}|+|u_{-k+1}-\bar{u}|+|u_{-k+2}-\bar{u}|+...+|u_0-\bar{u}|<\delta,$$

we have $|u_n-\bar{u}|<\varepsilon$, for all $n\geq -k$.

- The equilibrium point $\bar{u}$ of Eq. (1.2) is locally asymptotically stable if $\bar{u}$ is locally stable solution of Eq. (1.2) and there exists $\gamma>0$, such that for all $u_{-k},u_{-k+1},u_{-k+2},...,u_0\in I$, with

$$|u_{-k}-\bar{u}|+|u_{-k+1}-\bar{u}|+|u_{-k+2}-\bar{u}|+...+|u_0-\bar{u}|<\delta,$$

we have $\lim\limits_{n\to\infty} u_n=\bar{u}$.

- The equilibrium point $\bar{u}$ of Eq. (1.2) is global attractor if for all $u_{-k},u_{-k+1},...,u_0\in I$ we have

$$\lim_{n\to\infty} u_n=\bar{u}.$$

- The equilibrium point $\bar{u}$ of Eq. (1.2) is globally asymptotically stable if $\bar{u}$ is locally stable, and $\bar{u}$ is also a global attractor of Eq. (1.2).
- The equilibrium point $\bar{u}$ of Eq. (1.2) is unstable if $\bar{u}$ is not locally stable.

The linearized equation of Eq. (1.2) about the equilibrium $\bar{u}$ is the linear difference equation

$$y_{n+1}=\sum_{i=0}^{k}\frac{\partial f(\bar{u},\bar{u},...,\bar{u})}{\partial u_{n-i}}y_{n-i}$$

Theorem 1.3. *Assume that $p,q\in\mathbb{R}$ and $k\in\{0,1,2,...\}$. Then $|p|+|q|<1$ is a sufficient condition for the asymptotic stability of the difference equation*

$$u_{n+1}+pu_n+qu_{n-k}=0, n=0,1,...$$

Remark 1.4. The theorem can be easily extended to a general linear equations of the form

$$u_{n+k}+p_1u_{n+k-1}+...+p_ku_n=0,\quad n=0,1,... \tag{1.3}$$

where $p_1,p_2,...,p_k\in\mathbb{R}$ and $k\geq 0$. Then Eq. (1.3) is asymptotically stable provided that $\sum\limits_{i=0}^{k}|p_i|<1$.

Consider the following equation

$$u_{n+1}=g(u_n,u_{n-6},u_{n-7}). \tag{1.4}$$

The following theorem will be useful for the proof of our results in this paper.

Theorem 1.5. *Let $[a,b]$ be an interval of real numbers and assume that*

$$g:[a,b]^3\to[a,b],$$

is a continuous function satisfying the following properties:

(a) *$g(x,y,z)$ is nondecreasing in x and z in $[a,b]$ for each $y\in[a,b]$, and is nonincreasing in $y\in[a,b]$ for each x and z in $[a,b]$*
(b) *if $(m,M)\in[a,b]\times[a,b]$ is a solution of the system*

$$M=g(M,m,M),\qquad m=g(m,M,m),$$

then $m=M$.

Then (1.4) has a unique equilibrium point $\bar{u}\in[a,b]$ and every solution of (1.4) converges to $\bar{u}$.

2. Local Stability of Eq. (1.1)

In this section we investigate the local stability character of the solutions of Eq. (1.1). Equation (1.1) has a unique equilibrium point and is given by

$$\bar{u} = a\bar{u} + \frac{b\bar{u}^2}{c\bar{u} + d\bar{u}},$$

or

$$\bar{u}^2(1-a)(c+d) = b\bar{u}^2,$$

then if $(1-a)(c+d) \neq b$, then the unique equilibrium point is $\bar{u} = 0$.

Define the following function

$$f : (0,\infty)^3 \to (0,\infty)$$
$$f(u,v,w) = au + \frac{buw}{cv + dw}.$$

It follows that

$$f_u(u,v,w) = a + \frac{bw}{cv+dw}, \quad f_v(u,v,w) = -\frac{bcuw}{(cv+dw)^2}, \quad f_w(u,v,w) = \frac{bcuv}{(cv+dw)^2}.$$

Then

$$f_u(\bar{u},\bar{u},\bar{u}) = a + \frac{b}{c+d}, \quad f_v(\bar{u},\bar{u},\bar{u}) = -\frac{bc}{(c+d)^2}, \quad f_w(\bar{u},\bar{u},\bar{u}) = \frac{bc}{(c+d)^2}.$$

Then

$$f_u(\bar{u},\bar{u},\bar{u}) = a + \frac{b}{c+d}, \quad f_v(\bar{u},\bar{u},\bar{u}) = -\frac{bc}{(c+d)^2}, \quad f_w(\bar{u},\bar{u},\bar{u}) = \frac{bc}{(c+d)^2}.$$

The linearized equation of Eq. (1.1) about $\bar{u}$ is

$$v_{n+1} - \left(a + \frac{b}{c+d}\right) v_n + \frac{bc}{(c+d)^2} v_{n-6} - \frac{bc}{(c+d)^2} v_{n-7} = 0. \tag{2.1}$$

Theorem 2.1. *Assume that*

$$b(d+3c) < (1-a)(c+d)^2.$$

Then the equilibrium point of Eq. (1.1) is locally asymptotically stable.

Proof. It follows from Theorem 1.3 that Eq. (2.1) is asymptotically stable if

$$\left|a + \frac{b}{c+d}\right| + \left|\frac{bc}{(c+d)^2}\right| + \left|\frac{bc}{(c+d)^2}\right| < 1,$$

or

$$a + \frac{b}{c+d} + \frac{2bc}{(c+d)^2} < 1,$$

and so,

$$\frac{b(d+3c)}{(c+d)^2} < (1-a).$$

The proof is complete. □

3. Global Attractor of the Equilibrium Point of Eq. (1.1)

In this section we investigate the global attractivity character of solutions of Eq. (1.1).

Theorem 3.1. *The equilibrium point $\bar{u}$ of Eq. (1.1) is global attractor if $d(1-a) \neq b$*

Proof. Let p, q be real numbers and assume that $g : [p,q]^3 \to [p,q]$ is a function defined by $g(u,v,w) = au + \frac{buw}{cv+dw}$, then we can easily see that the function $g(u,v,w)$ is increasing in u, w and decreasing in v. Suppose that (m, M) is a solution of the system

$$M = g(M, m, M), \qquad m = g(m, M, m).$$

Then from Eq. (1.1), we see that

$$M = aM + \frac{bM^2}{cm+dM}, \qquad m = am + \frac{bm^2}{cM+dm},$$

or

$$M(1-a) = \frac{bM^2}{cm+dM}, \qquad m(1-a) = \frac{bm^2}{cM+dm},$$

then

$$c(1-a)mM + d(1-a)M^2 = bM^2, \qquad c(1-a)mM + d(1-a)m^2 = bm^2,$$

subtracting, we obtain

$$d(1-a)(M^2 - m^2) = b(M^2 - m^2).$$

Since $d(1-a) \neq b$ then

$$M = m.$$

It follows from Theorem 1.5 that $\bar{u}$ is a global attractor of Eq. (1.1), and then the proof is complete. □

4. Boundedness of Solutions of Eq. (1.1)

In this section we study the boundedness of solutions of Eq. (1.1).

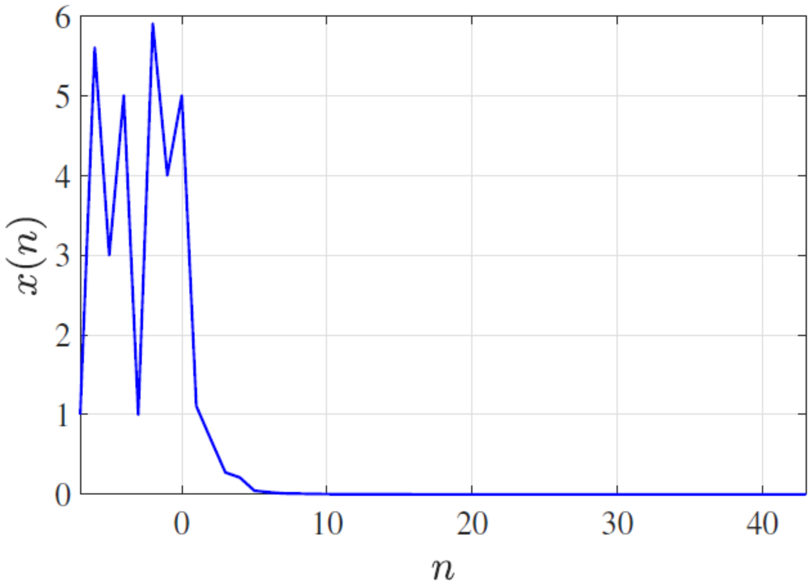
Theorem 4.1. *Every solution of Eq. (1.1) is bounded if $a + \frac{b}{d} < 1$.*

Proof. Let $\{u_n\}_{n=-4}^{\infty}$ be a solution of Eq. (1.1). It follows from Eq. (1.1) that

$$u_{n+1} = au_n + \frac{bu_n u_{n-7}}{cu_{n-6} + du_{n-7}} \leq au_n + \frac{bu_n u_{n-7}}{du_{n-7}} = \left(a + \frac{b}{d}\right) u_n.$$

Then $u_{n+1} \leq u_n, \quad \forall n \geq 0$. Then the sequences $\{u_n\}_{n=-7}^{\infty}$ is decreasing and so is bounded from above by $M = \max\{u_{-7}, u_{-6}, \cdots, u_0\}$. □

For confirming the results of this section, we consider numerical example for $u_{-7} = 1, u_{-6} = 5.6, u_{-5} = 3, u_{-4} = 5, u_{-3} = 1, u_{-2} = 5.9, u_{-1} = 4, u_0 = 5$ (See Figure 1). Difference equations are solved iteratively, making them ideal for computational modeling (e.g., Euler's method for discretizing ODEs).



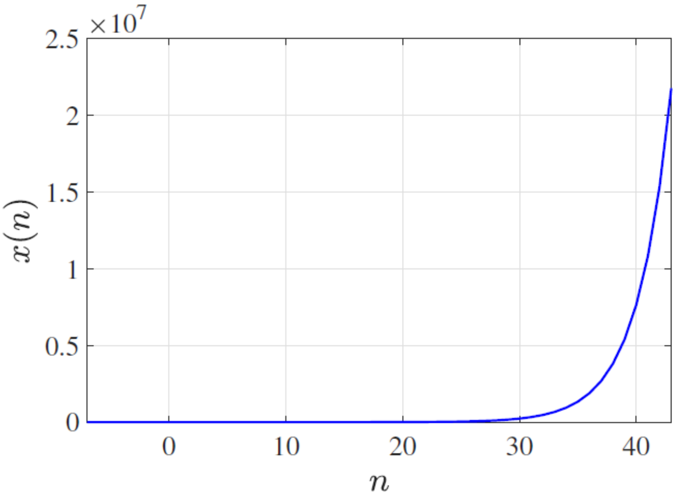


FIGURE 1. Left: $a = 0.1, b = 0.8, c = 1, d = 1$ which satisfy the boundedness conditions (the solution is bounded). Right: $a = 1, b = 0.8, c = 1, d = 0.5$ which don't satisfy the boundedness conditions (the solution is unbounded).

5. Periodicity of Solutions of Eq. (1.1)

Given the complexity of the recurrence relation and the lack of a straightforward periodic pattern for arbitrary initial conditions and parameters, we conclude that the sequence does not exhibit a universal periodic behavior. However, specific choices of parameters and initial conditions may lead to periodic solutions.

This is a recurrence of order 8 (since u_{n+1} depends on u_{n-7}). To find periodic solutions, we look for fixed points or cycles.

Theorem 5.1. *The solutions of Eq. (1.1) converges to a constant*

$$a + \frac{b}{c+d} = 1.$$

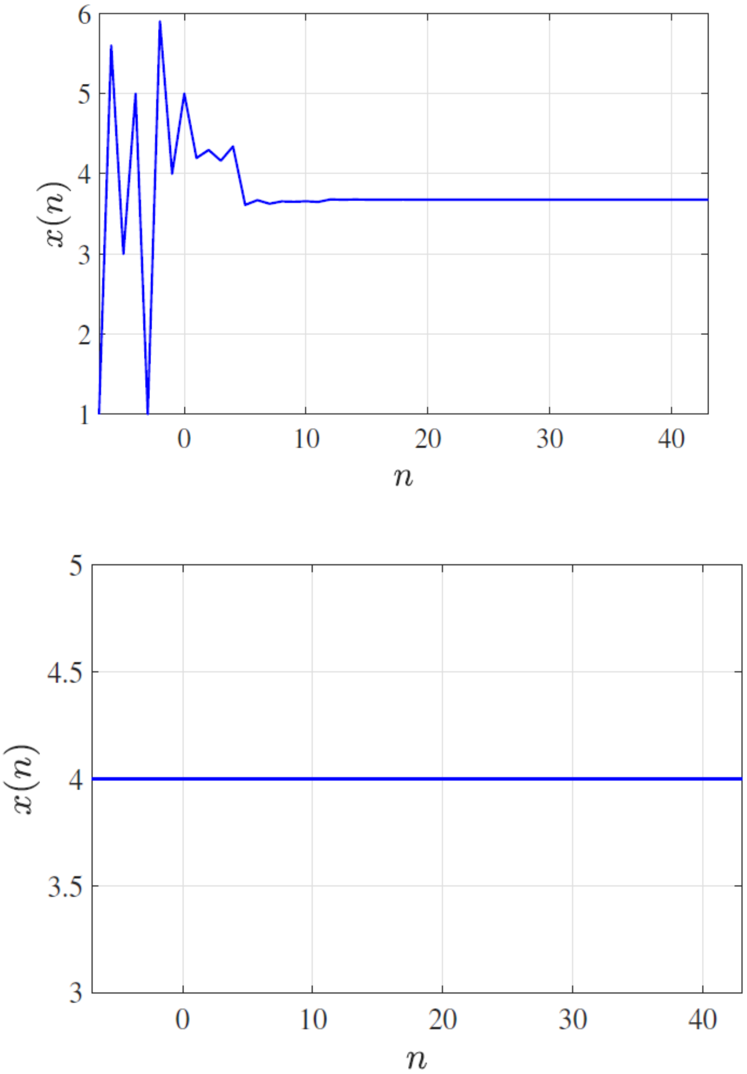


FIGURE 2. $a = 0.45455, b = 6, c = 1, d = 10$ which satisfy the condition $a + \dfrac{b}{c+d} = 1$ and the solution converges to a constant. Up: $u_{-7} = 1, u_{-6} = 5.6, u_{-5} = 3, u_{-4} = 5, u_{-3} = 1, u_{-2} = 5.9, u_{-1} = 4, u_0 = 5$. Down: $u_{-7} = u_{-6} = \cdots = u_0 = 4$ where the solution remains constant.

Under general parameters, the sequence u_n does not necessarily exhibit periodicity. Thus, the sequence can be periodic under certain conditions, but the general solution does not guarantee periodicity for arbitrary parameters.

Theorem 5.2. *In the special case where $a = 0$, $b = c$, and $d = 0$, the solution of Eq. (1.1) is periodic with period 9.*

This is a special case of the "Lyness cycle" or "Todd's recurrence," which is known to be periodic (Please see Figure 3). For this specific recurrence, the period is 9 (for the sequence $\{u_n\}$). For other parameter choices, the period (if it exists) depends on the specific values of a, b, c, d.

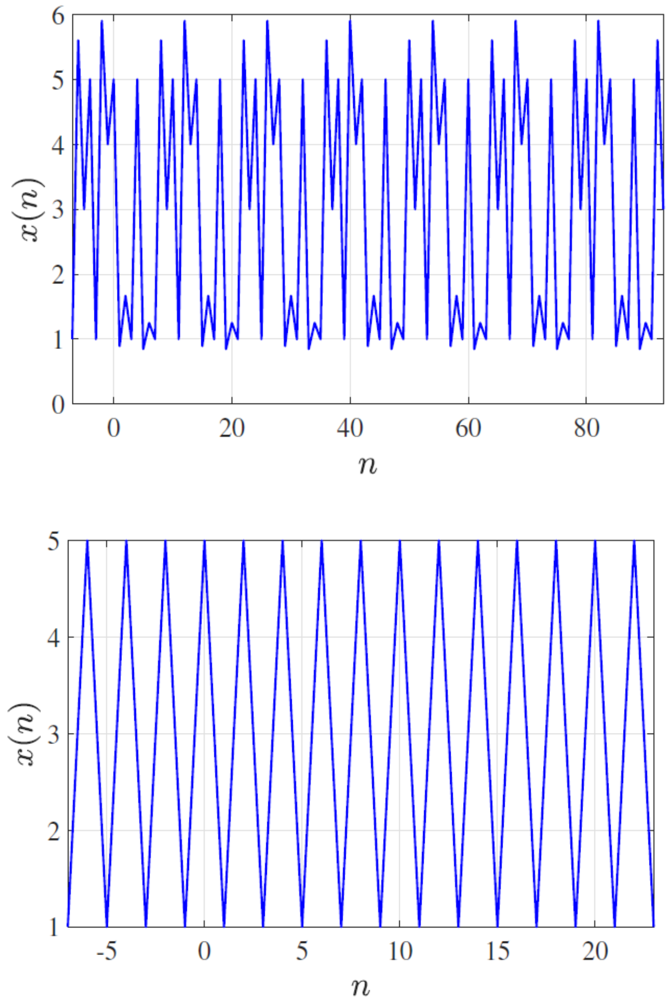


FIGURE 3. $a = 0.45455, b = 6, c = 1, d = 10$ which satisfy the condition $a + \dfrac{b}{c+d} = 1$ and the solution converges to a constant. Up: $u_{-7} = 1, u_{-6} = 5.6, u_{-5} = 3, u_{-4} = 5, u_{-3} = 1, u_{-2} = 5.9, u_{-1} = 4, u_0 = 5$. Down: $u_{-7} = 1, u_{-6} = 5, u_{-5} = 1, u_{-4} = 5, u_{-3} = 1, u_{-2} = 5, u_{-1} = 1, u_0 = 5$.

6. ON THE DIFFERENCE EQUATION $u_{n+1} = \dfrac{u_n u_{n-7}}{-u_{n-6} + u_{n-7}}$

In this subsection we study the following special case of Eq. (1.1):

$$u_{n+1} = \frac{u_n u_{n-7}}{-u_{n-6} + u_{n-7}}, \quad n = 0, 1, \cdots, \tag{6.1}$$

where the initial conditions $u_{-7}, u_{-6}, \cdots, u_0 > 0$ are arbitrary positive real numbers.

First, we need to understand what periodicity means in this context. A sequence is periodic if there exists a positive integer T such that $u_{n+T} = u_n$ for all n where the sequence is defined. In our case, the recurrence has the $u_{n+1} = \dfrac{u_n u_{n-k}}{-u_{n-k+1} + u_{n-k}}$ which is periodic of a period $2(k+1)$. The period is related to the least common multiple (LCM) of the differences. Therefore, the solutions of Eq. (1.2) is periodic of period 16 (Please see Figure 4).

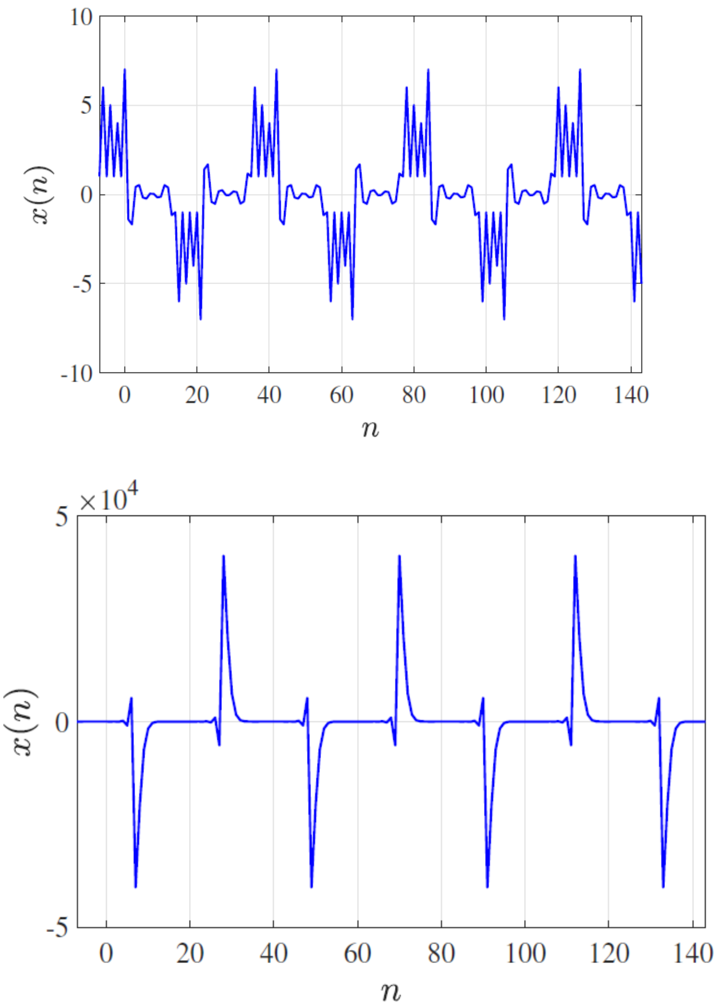


FIGURE 4. Up: $u_{-7} = 1, u_{-6} = 6, u_{-5} = 1, u_{-4} = 5, u_{-3} = 1, u_{-2} = 4, u_{-1} = 1, u_0 = 7$. Down: $u_{-7} = 1, u_{-6} = 2, u_{-5} = 3, u_{-4} = 4, u_{-3} = 5, u_{-2} = 6, u_{-1} = 7, u_0 = 8$.

7. On the Difference Equation $u_{n+1} = \frac{u_n u_{n-7}}{u_{n-6} - u_{n-7}}$

In this subsection we study the following special case of Eq. (1.1):

$$u_{n+1} = \frac{u_n u_{n-7}}{u_{n-6} - u_{n-7}}, \quad n = 0, 1, \cdots, \tag{7.1}$$

where the initial conditions $u_{-7}, u_{-6}, \cdots, u_0 > 0$ are arbitrary positive real numbers.
Given the complexity, we recall that for such recurrences, the general solution is periodic with period $3(k+1)$, where k is the delay. In this problem, $k = 7$, so the period is 24. Thus, the sequence u_n is periodic with period 24 (Please see Figure 5).

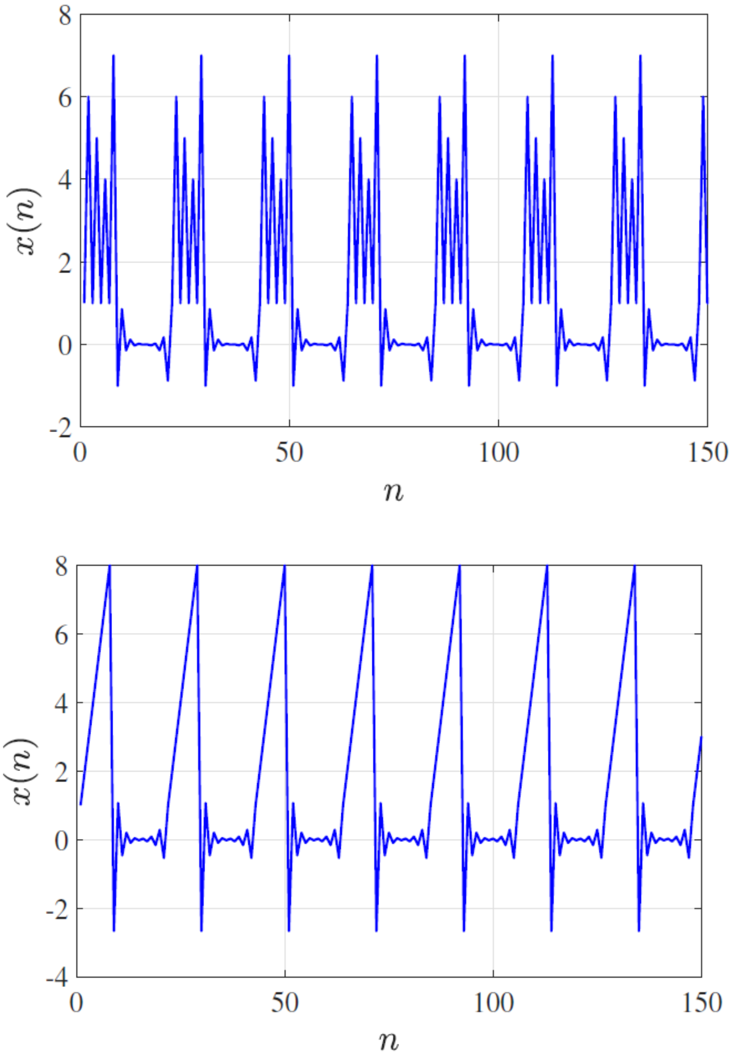


FIGURE 5. Up: $u_{-7} = 1, u_{-6} = 6, u_{-5} = 1, u_{-4} = 5, u_{-3} = 1, u_{-2} = 4, u_{-1} = 1, u_0 = 7$. Down: $u_{-7} = 1, u_{-6} = 2, u_{-5} = 3, u_{-4} = 4, u_{-3} = 5, u_{-2} = 6, u_{-1} = 7, u_0 = 8$.

8. CONCLUSION

This work discussed the global stability, the boundedness, and the solutions of some special cases of Eq. (1.1). We proved that if $b(d + 3c) < (1 - a)(c + d)^2$ then the equilibrium point of Eq. (1.1) is locally asymptotically stable. We showed that the unique equilibrium of Eq. (1.1) is globally asymptotically stable if $d(1 - a) \neq b$. We proved that the solution of Eq. (1.1) is bounded if $a + \frac{b}{d} < 1$. The findings of this study were validated by some numerical examples.

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