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Research Paper

Fractional Derivatives of Implicit Fractional Functions

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ABSTRACT: *In this paper, we study the implicit fractional differential problems of two variables fractional equations based on the Jumarie type of Riemann-Liouville fractional derivatives. By using product rule, quotient rule and chain rule for fractional derivatives, we can obtain the expressions of the implicit fractional functions.*
KEYWORDS: *Jumarie Type of Riemann-Liouville Fractional Derivatives, Product Rule, Quotient Rule, Chain Rule, Implicit Fractional Functions*

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I. INTRODUCTION

Fractional calculus is the theory of derivative and integral of non-integer order, which can be traced back to Leibniz, Liouville, Grunwald, Letnikov and Riemann. Fractional calculus has been attracting the attention of scientists and engineers from long time ago, and has been widely used in physics, engineering, biology, economics and other fields [1-16].

In this article, we mainly find the fractional derivatives of the following four types of two variables fractional equations:

$$y_\alpha^{\otimes 5}+\frac{4}{\Gamma(2\alpha+1)}x^{2\alpha}\otimes y_\alpha^{\otimes 3}-\frac{6}{\Gamma(7\alpha+1)}x^{7\alpha}=0, \tag{1}$$

$$sin_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)-cos_\alpha(2x^\alpha)\otimes sin_\alpha(5y_\alpha)+3=0, \tag{2}$$

$$\frac{A}{\Gamma(2\alpha+1)}x^{2\alpha}+By_\alpha^{\otimes 2}=C, \tag{3}$$

$$\frac{1}{\Gamma(\alpha+1)}x^\alpha-y_\alpha+\frac{1}{2}sin_\alpha(y_\alpha)=0. \tag{4}$$

Where $0<\alpha\leq 1$, A,B,C are real numbers, and $B\neq 0$. E_α, cos_α, sin_α are α-fractional exponential function, cosine function, sine function respectively. Using product rule, quotient rule and chain rule for fractional derivatives, the fractional derivatives of the implicit fractional functions can be easily obtained.

II. DEFINITIONS AND PROPERTIES

Firstly, we introduce the fractional calculus used in this paper.

Definition 2.1: Let α be a real number and m be a positive integer, then the modified Riemann-Liouville fractional derivatives of Jumarie type ([ ]) is defined by

$$\left({}_{x_0}D_x^\alpha\right)[y(x)]=\begin{cases}\frac{1}{\Gamma(-\alpha)}\int_{x_0}^{x}(x-\tau)^{-\alpha-1}y(\tau)d\tau, & \text{if } \alpha<0\\ \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_{x_0}^{x}(x-\tau)^{-\alpha}[y(\tau)-y(a)]\,d\tau & \text{if } 0\leq\alpha<1\\ \frac{d^m}{dx^m}\left({}_{x_0}D_x^{\alpha-m}\right)[y(x)], & \text{if } m\leq\alpha<m+1\end{cases} \tag{5}$$

where $\Gamma(u)=\int_0^\infty t^{u-1}e^{-t}dt$ is the gamma function defined on $u>0$. If $\left({}_{x_0}D_x^\alpha\right)^n[y(x)]=\left({}_{x_0}D_x^\alpha\right)\left({}_{x_0}D_x^\alpha\right)\cdots\left({}_{x_0}D_x^\alpha\right)[y(x)]$ exists, then $y(x)$ is called n-th order α-fractional differentiable function, and $\left({}_{x_0}D_x^\alpha\right)^n[y(x)]$ is the n-th order α-fractional derivative of $y(x)$. We note that $\left({}_{x_0}D_x^\alpha\right)^n\neq{}_{x_0}D_x^{n\alpha}$ in general, and we have the following properties [18].

Proposition 2.2: *Let α,β,c be real numbers and $\beta\geq\alpha>0$, then*

$$\left({}_0D_x^\alpha\right)[x^\beta]=\frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}x^{\beta-\alpha}, \tag{6}$$

and

$${}_0D_x^\alpha[c]=0. \tag{7}$$

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Definition 2.3 ([19])**:** Let x, x_0 and a_n be real numbers, $x_0 \in (a,b)$, and $0 < \alpha \le 1$. If the function $f_\alpha: [a,b] \to R$ can be expressed as a α-fractional power series, that is, $f_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{a_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}$ on some open interval $(x_0 - r, x_0 + r)$, then we say that $f_\alpha(x^\alpha)$ is α-fractional analytic at x_0, where r is the radius of convergence about x_0. If $f_\alpha: [a,b] \to R$ is continuous on closed interval $[a,b]$ and is α-fractional analytic at every point in open interval (a,b), then we say that f_α is an α-fractional analytic function on $[a,b]$.

Definition 2.4: The Mittag-Leffler function is defined by

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+1)}, \tag{8}$$

where α is a real number, $\alpha > 0$, and z is a complex variable.

Definition 2.5: Assume that $0 < \alpha \le 1$ and x is a real variable. Then $E_\alpha(x^\alpha)$ is called α-fractional exponential function, and the α-fractional cosine and sine function are defined as follows:

$$cos_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n\alpha}}{\Gamma(2n\alpha+1)}, \tag{9}$$

and

$$sin_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)\alpha}}{\Gamma((2n+1)\alpha+1)}. \tag{10}$$

Next, we introduce a new multiplication of fractional analytic functions.

Definition 2.6 ([20])**:** Let $0 < \alpha \le 1$, $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ be two α-fractional analytic functions,

$$f_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{a_n}{\Gamma(n\alpha+1)} x^{n\alpha}, \tag{11}$$

$$g_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{b_n}{\Gamma(n\alpha+1)} x^{n\alpha}. \tag{12}$$

Then we define

$$\begin{aligned} & f_\alpha(x^\alpha) \otimes g_\alpha(x^\alpha) \\ &= \sum_{n=0}^{\infty} \frac{a_n}{\Gamma(n\alpha+1)} x^{n\alpha} \otimes \sum_{n=0}^{\infty} \frac{b_n}{\Gamma(n\alpha+1)} x^{n\alpha} \\ &= \sum_{n=0}^{\infty} \frac{1}{\Gamma(n\alpha+1)} \left(\sum_{m=0}^{n} \binom{n}{m} a_{n-m} b_m \right) x^{n\alpha}. \end{aligned} \tag{13}$$

Definition 2.7: Assume that $\left(f_\alpha(x^\alpha)\right)^{\otimes n} = f_\alpha(x^\alpha) \otimes \cdots \otimes f_\alpha(x^\alpha)$ is the n times product of the fractional function $f_\alpha(x^\alpha)$. If $f_\alpha(x^\alpha) \otimes g_\alpha(x^\alpha) = 1$, then $g(x^\alpha)$ is called the $\otimes$ reciprocal of $f_\alpha(x^\alpha)$, and is denoted by $\left(f_\alpha(x^\alpha)\right)^{\otimes -1}$.

Theorem 2.8 (product rule for fractional derivatives)([17])**:** *Let* $0 < \alpha \le 1$, *and* f_α, g_α *be* α *-fractional analytic function. Then*

$$\left({}_0D_x^\alpha\right)[f_\alpha(x^\alpha) \otimes g_\alpha(x^\alpha)] = \left({}_0D_x^\alpha\right)[f_\alpha(x^\alpha)] \otimes g_\alpha(x^\alpha) + f_\alpha(x^\alpha) \otimes \left({}_0D_x^\alpha\right)[g_\alpha(x^\alpha)]. \tag{14}$$

Theorem 2.9 (quotient rule for fractional derivatives) ([17])**:** *Assume that* $0 < \alpha \le 1$, *and* f_α, g_α *are* α *-fractional analytic functions,* $g_\alpha \ne 0$, *then*

$$\left({}_0D_x^\alpha\right)\left[f_\alpha(x^\alpha) \otimes g_\alpha^{\otimes -1}(x^\alpha)\right] = g_\alpha^{\otimes -2}(x^\alpha) \otimes \left(\left({}_0D_x^\alpha\right)[f_\alpha(x^\alpha)] \otimes g_\alpha(x^\alpha) - f_\alpha(x^\alpha) \otimes \left({}_0D_x^\alpha\right)[g_\alpha(x^\alpha)] \right). \tag{15}$$

Theorem 2.10 (chain rule for fractional derivatives)([17])**:** *If* $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $g_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{b_n}{\Gamma(n\alpha+1)} x^{n\alpha}$. *Let* $f_{\otimes\alpha}\left(g_\alpha(x^\alpha)\right) = \sum_{k=0}^{\infty} a_k \left(g_\alpha(x^\alpha)\right)^{\otimes k}$ *and* $f'_{\otimes\alpha}\left(g_\alpha(x^\alpha)\right) = \sum_{k=1}^{\infty} k a_k \left(g_\alpha(x^\alpha)\right)^{\otimes (k-1)}$, *then*

$$\left({}_0D_x^\alpha\right)\left[f_{\otimes\alpha}\left(g_\alpha(x^\alpha)\right)\right] = f'_{\otimes\alpha}\left(g_\alpha(x^\alpha)\right) \otimes \left({}_0D_x^\alpha\right)[g_\alpha(x^\alpha)]. \tag{16}$$

III. CALCULATION AND RESULTS

In this section, we will use the product rule, quotient rule and chain rule for fractional derivatives to evaluate the fractional derivatives of the equations discussed in this article.

Example 3.1: Let $0 < \alpha \le 1$. Find $\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]$ from the equation

$$y_\alpha^{\otimes 5} + \frac{4}{\Gamma(2\alpha+1)} x^{2\alpha} \otimes y_\alpha^{\otimes 3} - \frac{6}{\Gamma(7\alpha+1)} x^{7\alpha} = 0.$$

Solution: Taking the fractional derivatives on both sides of Eq. ( ) at the same time, and we get

$$\left({}_0D_x^\alpha\right)\left[y_\alpha^{\otimes 5} + \frac{4}{\Gamma(2\alpha+1)} x^{2\alpha} \otimes y_\alpha^{\otimes 3} - \frac{6}{\Gamma(7\alpha+1)} x^{7\alpha}\right] = 0. \tag{17}$$

Using chain rule and product rule for fractional derivatives yields

$$5y_\alpha^{\otimes 4} \otimes \left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)] + \frac{4}{\Gamma(\alpha+1)} x^{\alpha} \otimes y_\alpha^{\otimes 3} + \frac{4}{\Gamma(2\alpha+1)} x^{2\alpha} \otimes 3y_\alpha^{\otimes 2} \otimes \left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)] - \frac{6}{\Gamma(6\alpha+1)} x^{6\alpha} = 0. \tag{18}$$

Thus,

$$\left[5y_\alpha^{\otimes 4} + \frac{12}{\Gamma(2\alpha+1)} x^{2\alpha} \otimes y_\alpha^{\otimes 2}\right] \otimes \left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)] = \frac{6}{\Gamma(6\alpha+1)} x^{6\alpha} - \frac{4}{\Gamma(\alpha+1)} x^{\alpha} \otimes y_\alpha^{\otimes 3}. \tag{19}$$

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And hence,

$$\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]=\left[\frac{6}{\Gamma(6\alpha+1)}x^{6\alpha}-\frac{4}{\Gamma(\alpha+1)}x^\alpha\otimes y_\alpha{}^{\otimes 3}\right]\otimes\left[5y_\alpha{}^{\otimes 4}+\frac{12}{\Gamma(2\alpha+1)}x^{2\alpha}\otimes y_\alpha{}^{\otimes 2}\right]^{\otimes -1}. \quad (20)$$

Example 3.2: If $0<\alpha\le 1$**.** Evaluate $\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]$ from the equation

$$sin_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)-cos_\alpha(2x^\alpha)\otimes sin_\alpha(5y_\alpha)+3=0.$$

Solution: Since

$$\left({}_0D_x^\alpha\right)[sin_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)-cos_\alpha(2x^\alpha)\otimes sin_\alpha(5y_\alpha)+3]=0. \quad (21)$$

It follows that

$$\left({}_0D_x^\alpha\right)[sin_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)]-\left({}_0D_x^\alpha\right)[cos_\alpha(2x^\alpha)\otimes sin_\alpha(5y_\alpha)]=0. \quad (22)$$

And hence, by product rule for fractional derivatives,

$$\begin{aligned}&3cos_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)+sin_\alpha(3x^\alpha)\otimes 4E_\alpha(4y_\alpha)\otimes\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]\\&+2sin_\alpha(2x^\alpha)\otimes sin_\alpha(5y_\alpha)-cos_\alpha(2x^\alpha)\otimes 5cos_\alpha(5y_\alpha)\otimes\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]=0. \quad (23)\end{aligned}$$

Therefore,

$$\begin{aligned}&[4sin_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)-5cos_\alpha(2x^\alpha)\otimes cos_\alpha(5y_\alpha)]\otimes\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]\\&=-3cos_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)-2sin_\alpha(2x^\alpha)\otimes sin_\alpha(5y_\alpha). \quad (24)\end{aligned}$$

Thus,

$$\begin{aligned}\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]&=[-3cos_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)-2sin_\alpha(2x^\alpha)\otimes sin_\alpha(5y_\alpha)]\\&\otimes[4sin_\alpha(3x^\alpha)\otimes E_\alpha(4y_\alpha)-5cos_\alpha(2x^\alpha)\otimes cos_\alpha(5y_\alpha)]^{\otimes -1}. \quad (25)\end{aligned}$$

Example 3.3: Assume that $0<\alpha\le 1$**,** A,B,C are real numbers, and $B\neq 0$. Evaluate the second order α-fractional derivative $\left({}_0D_x^\alpha\right)^2[y_\alpha(x^\alpha)]$ from the equation

$$\frac{A}{\Gamma(2\alpha+1)}x^{2\alpha}+By_\alpha{}^{\otimes 2}=C.$$

Solution: Since

$$\left({}_0D_x^\alpha\right)\left[\frac{A}{\Gamma(2\alpha+1)}x^{2\alpha}+By_\alpha{}^{\otimes 2}\right]=0. \quad (26)$$

It follows that

$$\frac{A}{\Gamma(\alpha+1)}x^\alpha+2By_\alpha\otimes\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]=0. \quad (27)$$

And hence,

$$\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]=-\frac{A}{2B\cdot\Gamma(\alpha+1)}x^\alpha\otimes y_\alpha{}^{\otimes -1}. \quad (28)$$

Furthermore, by quotient rule for fractional derivatives,

$$\begin{aligned}&\left({}_0D_x^\alpha\right)^2[y_\alpha(x^\alpha)]\\&=\left({}_0D_x^\alpha\right)\left[-\frac{A}{2B\cdot\Gamma(\alpha+1)}x^\alpha\otimes y_\alpha{}^{\otimes -1}\right]\\&=y_\alpha{}^{\otimes -2}\otimes\left(-\frac{A}{2B}\cdot y_\alpha+\frac{A}{2B\cdot\Gamma(\alpha+1)}x^\alpha\otimes\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]\right)\\&=y_\alpha{}^{\otimes -2}\otimes\left(-\frac{A}{2B}\cdot y_\alpha-\frac{A^2}{2B^2\cdot\Gamma(2\alpha+1)}x^{2\alpha}\otimes y_\alpha{}^{\otimes -1}\right)\\&=y_\alpha{}^{\otimes -3}\otimes\left(-\frac{A}{2B}\cdot y_\alpha{}^{\otimes 2}-\frac{A^2}{2B^2\cdot\Gamma(2\alpha+1)}x^{2\alpha}\right)\\&=y_\alpha{}^{\otimes -3}\otimes\left(-\frac{A}{2B}\cdot\left[\frac{C}{B}-\frac{A}{B\cdot\Gamma(2\alpha+1)}x^{2\alpha}\right]-\frac{A^2}{2B^2\cdot\Gamma(2\alpha+1)}x^{2\alpha}\right)\\&=-\frac{AC}{2B^2}\cdot y_\alpha{}^{\otimes -3}. \quad (29)\end{aligned}$$

Example 3.4: Suppose that that $0<\alpha\le 1$**.** Find the second order α-fractional derivative $\left({}_0D_x^\alpha\right)^2[y_\alpha(x^\alpha)]$ from the equation

$$\frac{1}{\Gamma(\alpha+1)}x^\alpha-y_\alpha+\frac{1}{2}sin_\alpha(y_\alpha)=0.$$

Solution: Since

$$\left({}_0D_x^\alpha\right)\left[\frac{1}{\Gamma(\alpha+1)}x^\alpha-y_\alpha+\frac{1}{2}sin_\alpha(y_\alpha)\right]=0. \quad (30)$$

By chain rule for fractional derivatives, we obtain

$$1-\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]+\frac{1}{2}cos_\alpha(y_\alpha)\otimes\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]=0. \quad (31)$$

Hence,

$$\left({}_0D_x^\alpha\right)[y_\alpha(x^\alpha)]=2[2-cos_\alpha(y_\alpha)]^{\otimes -1}. \quad (32)$$

Furthermore, using quotient rule for fractional derivatives yields

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$$
\begin{aligned}
&\left({}_0D_x^{\alpha}\right)^2[y_\alpha(x^\alpha)] \\
&= \left({}_0D_x^{\alpha}\right)[2[2-cos_\alpha(y_\alpha)]^{\otimes -1}] \\
&= -2[2-cos_\alpha(y_\alpha)]^{\otimes -2} \otimes sin_\alpha(y_\alpha) \otimes \left({}_0D_x^{\alpha}\right)[y_\alpha(x^\alpha)] \\
&= -2[2-cos_\alpha(y_\alpha)]^{\otimes -2} \otimes sin_\alpha(y_\alpha) \otimes 2[2-cos_\alpha(y_\alpha)]^{\otimes -1} \\
&= -4 \cdot sin_\alpha(y_\alpha) \otimes [2-cos_\alpha(y_\alpha)]^{\otimes -3} \;. \qquad (33)
\end{aligned}
$$

IV. CONCLUSION

The fractional differential problem of implicit fractional functions is very important in fractional calculus. Using product rule, quotient rule and chain rule for fractional derivatives, we can easily obtain the solution of this problem. In the future, we will also use these methods to study the engineering mathematics problems and fractional differential equations based on the Jumarie type of Riemann-Liouville fractional derivatives.

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