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Research Paper

COVID-19 Spreading of SELIQHR Model in Complex Social Networks with Isolating Mechanism

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ABSTRACT: *In this paper, we propose a new SELIQHR (susceptible-exposed-latent-infected-quarantined-hospitalized- recovered) COVID-19 spreading model with isolating mechanism. The global stability of the COVID-19 free equilibrium is proved in detail, and the basic reproduction number R_0 of the model is obtained. The existence of COVID-19 equilibrium and the dynamic behavior of the model are determined by the basic reproduction number R_0. The global attractivity of the COVID-19 prevailing equilibrium is proved by monotone iterative technique. Numerical simulation verifies the analysis results.*
KEYWORDS: *COVID-19; Virus spreading model; Isolating mechanism; Equilibrium; Global stability*

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I. INTRODUCTION

Coronavirus is a new, fatal and highly transmitted infectious disease, which is widely spread all over the world. Coronavirus is a kind of related viruses that cause diseases in birds and mammals. In humans, respiratory infections induced by coronavirus may be negligible, such as the common cold. But other viruses can be lethal, such as SARS, MERS and the novel COVID-19. So far, seven kinds of human coronavirus have appeared in the world: human coronavirus OC43(HCoV-OC43), human coronavirus 229E(HCoV-229E), severe acute respiratory syndrome coronavirus(SARS-CoV), human coronavirus NL63(HCoV-NL63), human coronavirus HKU1(HCoV-HKU1), Middle East respiratory syndrome associated coronavirus(MERS-CoV) and novel coronavirus (COVID-19) which is a worldwide epidemic disease[1].

Since people have different understanding of COVID-19, and different preventive measures have been taken, the effect of epidemic prevention and control is also different. According to official information, the initial symptoms of COVID-19 infection are fever and dry cough, similar to the common influenza. However, COVID-19 can develop into pneumonia, dyspnea and even death. Some patients infected with COVID-19 virus have no obvious symptoms, and the incubation period of the virus is about 14 days, but asymptomatic infected people are also infectious[2,3]. COVID-19 virus is mainly transmitted through the air. When people are in close contact with virus carriers (within 2 meters), they may be infected with COVID-19. At present, maintaining social distance and reducing the flow of people is considered to be the most effective epidemic prevention measures[4,5].

Mathematical models describing infectious diseases play an important role in theory and practice[6]. The establishment and analysis of these models will help us to understand the transmission mechanism and characteristics of diseases, so as to put forward effective strategies for disease prediction, prevention and suppression[7,8]. Recently, the mathematical model of COVID-19 epidemic has attracted the attention of many scholars, and many significant and important results have been obtained [9-14]. It can be considered as an effective way to study, simulate and predict the mechanism and transmission of COVID-19.

At present, there is no specific drug for the COVID-19 virus. In order to effectively deal with COVID-19, the first step is to speed up the detection of COVID-19, detect the infected person as soon as possible, and effectively isolate the close contacts; Secondly, we should reduce the gathering of personnel and impose strict restrictions on the population flow in the areas where the epidemic spread. In this paper, a new mathematical model of COVID-19 is established. The whole society is regarded as a complex network. The community members are divided into seven categories: S (susceptible), E(exposed), L(latent asymptomatic infected), I(symptomatic infected), Q(quarantined), H (hospitalized) and R(recovered). Each community is

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interconnected. The purpose of the model is to study the impact of isolation measures on the COVID-19, and put forward effective strategies for epidemic prevention and control.

The rest of the paper is organized as follows: In Sect. 2, we present a new SELIQHR model in social networks. In Sect. 3, it shows that COVID-19 free equilibrium exists and proves the COVID-19 free equilibrium is globally stable. In Sect. 4, the system has a COVID-19 prevailing equilibrium, and proves the COVID-19 prevailing equilibrium is globally asymptotically attractive. In Sect. 5, numerical simulations are given to illustrate the main results. Finally, the conclusions are given in Sect. 6.

II. MODEL FORMULATION

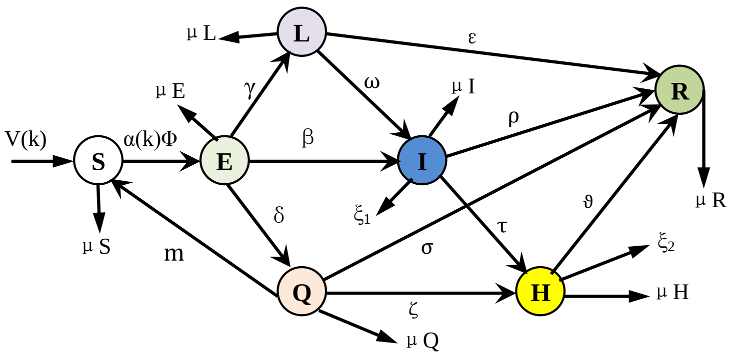

Figure 1 Transfer diagram for COVID-19 spreading model

In this paper, it is assumed that the whole population is an associated social network. The nodes in the network represent the individuals, and the edges connecting the nodes represent the connections between the individuals. The community members are divided into seven categories: S (susceptible), E(exposed), L(latent), I(infected), Q(quarantined), H (hospitalized) and R(recovered). Each community is interconnected. Each individual adopts one of the seven states of S, E, L, I, Q, H and R. S refers to people who have never contacted COVID-19 without immunity (susceptible); E refers to the close contacts of the infected individuals with COVID-19, which should be isolated and observed for 14 days(exposed); L refers to asymptomatic infection without isolation, which is in latent state and may develop into symptomatic infection(latent); I refers to non isolated patients with clinical symptoms who need to be hospitalized(infected); Q refers to the close contacts in the isolation period, which need to be observed for 14 days to remove the isolation(quarantined); H refers to COVID-19 patients admitted to hospital(hospitalized); R refers to the recovered and has immunity to COVID-19 (recovered). $S_k(t)$, $E_k(t)$, $L_k(t)$, $I_k(t)$, $Q_k(t)$, $H_k(t)$ and $R_k(t)$ express the relative density of susceptible, exposed, latent, infected, quarantined, hospitalized and recovered nodes of the community k at time t, respectively. The COVID-19 propagation model is shown in Fig. 1.

In the SELIQHR model, COVID-19 spread according to the following rules: The parameter $\alpha(k) > 0$ is the degree dependent rate, which indicates the accessibility of community k to COVID-19. Close contacts who have been found will be isolated for 14 days at rate δ, some of close contacts who have not been found become asymptomatic infected individuals at rate γ, some of close contacts who have not been found become infected individuals with clinical symptoms at rate β. Some of quarantined individuals infected with virus are hospitalized at rate ζ, part of the quarantined individuals recover from the virus infection during the isolation period, and become into recovered individuals at rate σ, some of the quarantines terminate isolation 14 days later and become into susceptible individuals at rate m. Latent individuals become into recovered individuals at rate ε, and become into infected individuals at rate ω. Infected individuals become into recovered individuals at rate ρ, become into hospitalized individuals at rate τ, and the case fatality rate of infected individuals was ξ_1 due to COVID-19. Hospitalized individuals become into recovered individuals at rate ϑ, and the case fatality rate of infected individuals was ξ_2 due to COVID-19.

The degree-dependent parameter $V(k)>0$ represents the number of newly immigrated individuals in community K per unit time, and each newly immigrated individual is susceptible, the reconnection of these nodes follows the above propagation rules. This type of rewiring preserves the network mean degree(the total number of links remains constant) but changes the mean degree of susceptible and infected nodes. The natural mortality rate in community K was μ. The parameters are all nonnegative. The model can be described by the following system of ordinary differential equations.

$$\begin{cases} \frac{dS_k(t)}{dt} = V(k) - \alpha(k)\Phi(t)S_k(t) - \mu S_k(t) + mQ_k(t) \\ \frac{dE_k(t)}{dt} = \alpha(k)\Phi(t)S_k(t) - (\gamma + \beta + \delta + \mu)E_k(t) \\ \frac{dL_k(t)}{dt} = \gamma E_k(t) - (\omega + \varepsilon + \mu)L_k(t) \\ \frac{dI_k(t)}{dt} = \beta E_k(t) + \omega L_k(t) - (\xi_1 + \tau + \rho + \mu)I_k(t) \\ \frac{dQ_k(t)}{dt} = \delta E_k(t) - (\zeta + \sigma + m + \mu)Q_k(t) \\ \frac{dH_k(t)}{dt} = \tau I_k(t) + \zeta Q_k(t) - (\xi_2 + \vartheta + \mu)H_k(t) \\ \frac{dR_k(t)}{dt} = \rho I_k(t) + \varepsilon L_k(t) + \sigma Q_k(t) + \vartheta H_k(t) - \mu R_k(t) \end{cases} \tag{2.1}$$

$\Phi(t)$ denotes the probability of susceptible individuals contacts with a virus carrier(infected individual or latent individual) at time t, which satisfies the relation:

$$\Phi(t) = \sum_{i=1}^{n} \frac{\varphi_1(i)}{i} P(i\square k)\frac{I_i(t)}{N_i(t)} + \frac{\varphi_2(i)}{i} P(i\square k)\frac{L_i(t)}{N_i(t)}$$

Here P(i|k) is the conditional probability of a node with degree k connecting to a node with degree i. Because of the uncorrelated network [15], $P(i|k) = iP(i)/\langle k\rangle$. The probability of a randomly selected node with degree k is P(k), thus $\sum_{k=1}^{n} P(k) = 1$, $\langle k\rangle = \sum_{k=1}^{n} kP(k)$ denotes the average degree, and $\varphi_1(i)$ indicates the probability of infection of a node with degree i caused by infected individuals; $\varphi_2(i)$ indicates the probability of infection of a node with degree i caused by latent individuals. 1/i denotes the probability of contacting an infected neighbor node with degree i at the present time step. The size of the population is constant.

$$S_k(t) + E_k(t) + L_k(t) + I_k(t) + Q_k(t) + H_k(t) + R_k(t) = N_k(t) = \eta_K = V(k)/\mu \tag{2.2}$$

So we can obtain:

$$\Phi(t) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} \frac{\varphi_1(k)}{\eta_K} P(k)I_k(t) + \frac{1}{\langle k\rangle}\sum_{k=1}^{n} \frac{\varphi_2(k)}{\eta_K} P(k)L_k(t) \tag{2.3}$$

The initial conditions for system can be given as follows:

$$S_k(0) = \eta_K - E_k(0) - L_k(0) - I_k(0) - Q_k(0) - H_k(0) - R_k(0) > 0$$

$$E_k(0) \geq 0 \quad L_k(0) \geq 0 \quad I_k(0) \geq 0 \quad Q_k(0) \geq 0 \quad H_k(0) \geq 0 \quad R_k(0) \geq 0 \quad \Phi(0) > 0$$

The parameters of the system are all nonnegative.

III. THE BASIC REPRODUCTION NUMBER AND STABILITY ANALYSIS OF COVID-19 FREE EQUILIBRIUM

We define the solution vector of the system (2.1) as:

$$X_k(t) = [S_k(t), E_k(t), L_k(t), I_k(t), Q_k(t), H_k(t), R_k(t)]^T \tag{3.1}$$

The basic reproduction number is defined as:

$$R_0 = \frac{\beta(\omega+\varepsilon+\mu)+\gamma\omega}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)} \frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} + \frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)} \frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle} \tag{3.2}$$

Theorem1 If $R_0 < 1$, the COVID-19 free equilibrium $X_0 = [\eta_K, 0, 0, 0, 0, 0, 0]^T$ of system (2.1) is locally asymptotically stable, and it is unstable when $R_0 > 1$.

Proof:

Obviously, X_0 is a special solution of the equation(2.1), which is an equilibrium state. Next, we will prove that X_0 is locally asymptotically stable by Lyapunov's first method.

Jacobian matrix of the equation at $X_0=[\eta_K, 0, 0, 0,0, 0,0]^T$ as follows:

$$J = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}_{7n\times 7n}$$

$$A_{kk} = \begin{bmatrix} -\mu & 0 & -\frac{\alpha(k)\varphi_2(k)P(k)}{\langle k\rangle} & -\frac{\alpha(k)\varphi_1(k)P(k)}{\langle k\rangle} & m & 0 & 0 \\ 0 & -(\gamma+\beta+\delta+\mu) & \frac{\alpha(k)\varphi_2(k)P(k)}{\langle k\rangle} & \frac{\alpha(k)\varphi_1(k)P(k)}{\langle k\rangle} & 0 & 0 & 0 \\ 0 & \gamma & -(\omega+\varepsilon+\mu) & 0 & 0 & 0 & 0 \\ 0 & \beta & \omega & -(\xi_1+\tau+\rho+\mu) & 0 & 0 & 0 \\ 0 & \delta & 0 & 0 & -(\sigma+\zeta+m+\mu) & 0 & 0 \\ 0 & 0 & 0 & \tau & \zeta & -(\xi_2+\vartheta+\mu) & 0 \\ 0 & 0 & \varepsilon & \rho & \sigma & \vartheta & -\mu \end{bmatrix}$$

$$\underset{(i\neq k)}{A_{ik}} = \begin{bmatrix} 0 & 0 & -\frac{\alpha(i)\varphi_2(k)P(k)}{\langle k\rangle} & -\frac{\alpha(i)\varphi_1(k)P(k)}{\langle k\rangle} & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha(i)\varphi_2(k)P(k)}{\langle k\rangle} & \frac{\alpha(i)\varphi_1(k)P(k)}{\langle k\rangle} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

i=1,2,3…n k=1,2,3…n

The characteristic equation of Jacobian matrix as follows：

$$(x+\mu)^{2n}(x+\sigma+\zeta+m+\mu)^n(x+\xi_2+\vartheta+\mu)^n(x+\gamma+\beta+\delta+\mu)^{(n-1)}(x+\omega+\varepsilon+\mu)^{(n-1)}\times (x+\xi_1+\tau+\rho+\mu)^{(n-1)}\{(x+\gamma+\beta+\delta+\mu)(x+\omega+\varepsilon+\mu)(x+\xi_1+\tau+\rho+\mu)- [\beta(x+\omega+\varepsilon+\mu)+\gamma\omega]\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}-\gamma(x+\xi_1+\tau+\rho+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}\}=0$$

Factors:

$$(x+\gamma+\beta+\delta+\mu)(x+\omega+\varepsilon+\mu)(x+\xi_1+\tau+\rho+\mu)-[\beta(x+\omega+\varepsilon+\mu)+\gamma\omega]\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}- \gamma(x+\xi_1+\tau+\rho+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}=0 \quad (3.3)$$

According to the Routh criterion, all characteristic roots of the system have negative real parts if the following conditions are satisfied:

$$(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)-[\beta(\omega+\varepsilon+\mu)+\gamma\omega]\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}- \gamma(\xi_1+\tau+\rho+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}>0$$

$$(\gamma+\beta+\delta+\mu+\omega+\varepsilon+\mu+\xi_1+\tau+\rho+\mu)[(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)+(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)+ (\gamma+\beta+\delta+\mu)(\xi_1+\tau+\rho+\mu)-\beta\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}-\gamma\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}]-\{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu) -[\beta(\omega+\varepsilon+\mu)+\gamma\omega]\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}-\gamma(\xi_1+\tau+\rho+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}\}>0$$

After simplification, there are:

$$R_0=\frac{\beta}{(\gamma+\beta+\delta+\mu)(\xi_1+\tau+\rho+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}+\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}+ \frac{\gamma\omega}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}<1 \quad (3.4)$$

$$R_1=\frac{\beta}{\Delta 1}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}+\frac{\gamma}{\Delta 2}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle})-\frac{\gamma\omega}{(\gamma+\beta+\delta+\mu+\xi_1+\tau+\rho+\mu)\Delta 1}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}<1 \quad (3.5)$$

Here:

$$\Delta 1=(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)+(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)+(\gamma+\beta+\delta+\mu)(\xi_1+\tau+\rho+\mu)+ (\omega+\varepsilon+\mu)^2$$

$$\Delta 2=(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)+(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)+(\gamma+\beta+\delta+\mu)(\xi_1+\tau+\rho+\mu)+ (\xi_1+\tau+\rho+\mu)^2$$

there are: $R_1 < R_0 < 1$

So just satisfy the following:

$$R_0=\frac{\beta(\omega+\varepsilon+\mu)+\gamma\omega}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}+\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}<1 \quad (3.6)$$

If $R_0 < 1$, all characteristic roots of the system have negative real parts, then the equilibrium state X_0 is asymptotically stable. If $R_0 > 1$, at least one characteristic root of the system has a positive real part, then the equilibrium state X_0 is unstable.
The proof is completed.

Theorem2 If $R_0 < 1$, the COVID-19 free equilibrium $X_0 = [\eta_K,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]^T$ of system (2.1) is globally asymptotically stable.
Proof:
We will prove that X_0 is globally asymptotically stable by Lyapunov's second method.
$S_k + E_k + I_k + L_k + Q_k + H_k + R_k = \eta_K$ is a constant, six equations can be taken

$$\begin{cases} \frac{dE_k(t)}{dt} = \alpha(k)\Phi(t)S_k(t) - (\gamma+\beta+\delta+\mu)E_k(t) \\ \frac{dL_k(t)}{dt} = \gamma E_k(t) - (\omega+\varepsilon+\mu)L_k(t) \\ \frac{dI_k(t)}{dt} = \beta E_k(t) + \omega L_k(t) - (\xi_1+\tau+\rho+\mu)I_k(t) \\ \frac{dQ_k(t)}{dt} = \delta E_k(t) - (\zeta+\sigma+m+\mu)Q_k(t) \\ \frac{dH_k(t)}{dt} = \tau I_k(t) + \zeta Q_k(t) - (\xi_2+\vartheta+\mu)H_k(t) \\ \frac{dR_k(t)}{dt} = \rho I_k(t) + \varepsilon L_k(t) + \sigma Q_k(t) + \vartheta H_k(t) - \mu R_k(t) \end{cases} \tag{3.7}$$

We define the solution vector of the system as:
$X = [E_k(t),\ L_k(t),\ I_k(t),\ Q_k(t),\ H_k(t),\ R_k(t)]^T$
Let's take the Lyapunov function:

$$V(x) = \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}E_k(t) + \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}{\beta(\omega+\varepsilon+\mu)+\omega\gamma}I_k(t) + \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{\omega(\gamma+\beta+\delta+\mu)}{\beta(\omega+\varepsilon+\mu)+\omega\gamma}L_k(t) + \frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}E_k(t) + \frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\frac{(\gamma+\beta+\delta+\mu)}{\gamma}L_k(t) \tag{3.8}$$

Since all parameters are positive, the function V(x) is a positive definite function.

$$\frac{dV(x)}{dt} = \frac{\partial V(x)}{\partial E_k(t)}\frac{dE_k(t)}{dt} + \frac{\partial V(x)}{\partial I_k(t)}\frac{dI_k(t)}{dt} + \frac{\partial V(x)}{\partial L_k(t)}\frac{dL_k(t)}{dt} + \frac{\partial V(x)}{\partial Q_k(t)}\frac{dQ_k(t)}{dt} + \frac{\partial V(x)}{\partial H_k(t)}\frac{dH_k(t)}{dt} + \frac{\partial V(x)}{\partial R_k(t)}\frac{dR_k(t)}{dt}$$

$$= \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\{\alpha(k)\Phi(t)S_k(t) - (\gamma+\beta+\delta+\mu)E_k(t)\} + \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}{\beta(\omega+\varepsilon+\mu)+\omega\gamma}\{\beta E_k(t) + \omega L_k(t) - (\xi_1+\tau+\rho+\mu)I_k(t)\} + \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{\omega(\gamma+\beta+\delta+\mu)}{\beta(\omega+\varepsilon+\mu)+\omega\gamma}\{\gamma E_k(t) - (\omega+\varepsilon+\mu)L_k(t)\} + \frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\{\alpha(k)\Phi(t)S_k(t) - (\gamma+\beta+\delta+\mu)E_k(t)\} + \frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\frac{(\gamma+\beta+\delta+\mu)}{\gamma}\{\gamma E_k(t) - (\omega+\varepsilon+\mu)L_k(t)\}$$

$$= \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)+\varphi_2(k)}{\eta_K}P(k)\alpha(k)\Phi(t)S_k(t) - \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)(\xi_1+\tau+\rho+\mu)}{\beta(\omega+\varepsilon+\mu)+\omega\gamma}I_k(t) - \frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}L_k(t)$$

$$= \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)+\varphi_2(k)}{\eta_K}P(k)\alpha(k)\Phi(t)S_k(t) - \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}I_k(t) - \frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}L_k(t) - \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)[\gamma(\xi_1+\tau+\rho+\mu)-\beta(\omega+\varepsilon+\mu)-\omega\gamma]}{\gamma(\beta(\omega+\varepsilon+\mu)+\omega\gamma)}I_k(t)$$

$$= \frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}\{\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)+\varphi_2(k)}{\eta_K}P(k)\alpha(k)\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}\Phi(t)S_k(t) - \Phi(t) - \frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{[\gamma(\xi_1+\tau+\rho+\mu)-\beta(\omega+\varepsilon+\mu)-\omega\gamma]}{(\beta(\omega+\varepsilon+\mu)+\omega\gamma)}I_k(t)\}$$

At the equilibrium point, the right side of equation(3.7) should be equal to 0.

$$\begin{cases} \alpha(k)\Phi(t)S_k - (\gamma+\beta+\delta+\mu)E_k = 0 \\ \gamma E_k - (\omega+\varepsilon+\mu)L_k = 0 \\ \beta E_k + \omega L_k - (\xi_1+\tau+\rho+\mu)I_k = 0 \\ \delta E_k - (\zeta+\sigma+m+\mu)Q_k = 0 \\ \tau I_k + \zeta Q_k - (\xi_2+\vartheta+\mu)H_k = 0 \\ \rho I_k + \varepsilon L_k + \sigma Q_k + \vartheta H_k - \mu R_k = 0 \end{cases}$$

We can get the expression: $I_k = \frac{\alpha(k)\Phi(\beta(\omega+\varepsilon+\mu)+\gamma\omega)}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}S_k$
Substituting it into the following:

$$\frac{dV(x)}{dt}=\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}\{\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)+\varphi_2(k)}{\eta_K}P(k)\alpha(k)\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}\Phi(t)S_k(t)-\Phi(t)-\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)\alpha(k)P(k)}{\eta_K}\frac{[\gamma(\xi_1+\tau+\rho+\mu)-\beta(\omega+\varepsilon+\mu)-\omega\gamma]}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}\Phi S_k\}$$

$$=\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}\{\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)}{\eta_K}P(k)\alpha(k)\frac{\beta(\omega+\varepsilon+\mu)+\omega\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}\Phi(t)S_k(t)+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)}{\eta_K}P(k)\alpha(k)\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}\Phi(t)S_k(t)-\Phi(t)\}$$

$$\le\Phi(t)\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}\{\frac{1}{\langle k\rangle}\sum\varphi_1(k)P(k)\alpha(k)\frac{\beta(\omega+\varepsilon+\mu)+\omega\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}+\frac{1}{\langle k\rangle}\sum\varphi_2(k)P(k)\alpha(k)\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}-1\}$$

$$=\Phi(t)\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}\{\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}\frac{\beta(\omega+\varepsilon+\mu)+\omega\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}+\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}-1\}$$

$$=\Phi(t)\frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\gamma}\{R_0-1\}$$

If $R_0 < 1$, $dV(x)/dt \le 0$, and only if $x = 0$, $\Phi(t) = 0$, $dV(x)/dt = 0$.
Therefore, the equilibrium state of origin X = (0, 0, 0, 0, 0, 0) is asymptotically stable.
When ‖ x ‖→∞, V(x)→∞, then the system(3.7) is globally asymptotically stable at the origin $X = (0, 0, 0, 0, 0, 0)^T$. Because $S_k + E_k + L_k + I_k + Q_k + H_k + R_k = \eta_K$ is a constant, the system(2.1) is globally asymptotically stable at $X_0 = [\eta_K, 0, 0, 0,0, 0,0]^T$
The proof is completed.

IV. EXISTENCE OF COVID-19 PREVAILING EQUILIBRIUM

Theorem3 When $R_0 > 1$, the system (2.1) has a COVID-19 prevailing equilibrium:
$X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, Q_k^*, H_k^*, R_k^*)^T$.

Proof:

$$\begin{cases}\frac{dS_k(t)}{dt}=V(k)-\alpha(k)\Phi(t)S_k(t)-\mu S_k(t)+mQ_k(t)\\ \frac{dE_k(t)}{dt}=\alpha(k)\Phi(t)S_k(t)-(\gamma+\beta+\delta+\mu)E_k(t)\\ \frac{dL_k(t)}{dt}=\gamma E_k(t)-(\omega+\varepsilon+\mu)L_k(t)\\ \frac{dI_k(t)}{dt}=\beta E_k(t)+\omega L_k(t)-(\xi_1+\tau+\rho+\mu)I_k(t)\\ \frac{dQ_k(t)}{dt}=\delta E_k(t)-(\zeta+\sigma+m+\mu)Q_k(t)\\ \frac{dH_k(t)}{dt}=\tau I_k(t)+\zeta Q_k(t)-(\xi_2+\vartheta+\mu)H_k(t)\\ \frac{dR_k(t)}{dt}=\rho I_k(t)+\varepsilon L_k(t)+\sigma Q_k(t)+\vartheta H_k(t)-\mu R_k(t)\end{cases}\tag{4.1}$$

At the equilibrium point, the right side of equation should be equal to 0.

$$\begin{cases}V(k)-\alpha(k)\Phi^*S_k^*-\mu S_k^*+mQ_k^*=0\\ \alpha(k)\Phi^*S_k^*-(\gamma+\beta+\delta+\mu)E_k^*=0\\ \gamma E_k^*-(\omega+\varepsilon+\mu)L_k^*=0\\ \beta E_k^*+\omega L_k^*-(\xi_1+\tau+\rho+\mu)I_k^*=0\\ \delta E_k^*-(\zeta+\sigma+m+\mu)Q_k^*=0\\ \tau I_k^*+\zeta Q_k^*-(\xi_2+\vartheta+\mu)H_k^*=0\\ \rho I_k^*+\varepsilon L_k^*+\sigma Q_k^*+\vartheta H_k^*-\mu R_k^*=0\\ S_k^*+E_k^*+I_k^*+L_k^*+Q_k^*+H_k^*+R_k^*=\eta_K\end{cases}$$

So we get the following：

$$\begin{cases} S_k^* = \frac{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}{\alpha(k)\Phi^*\gamma} L_k^* \\ E_k^* = \frac{(\omega+\varepsilon+\mu)}{\gamma} L_k^* \\ I_k^* = \frac{\beta(\omega+\varepsilon+\mu)+\gamma\omega}{\gamma(\xi_1+\tau+\rho+\mu)} L_k^* \\ Q_k^* = \frac{\delta(\omega+\varepsilon+\mu)}{\gamma(\zeta+\sigma+m+\mu)} L_k^* \\ H_k^* = \frac{\tau(\beta(\omega+\varepsilon+\mu)+\gamma\omega)(\zeta+\sigma+m+\mu)+\zeta\delta(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)}{\gamma(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu)} L_k^* \\ R_k^* = \frac{\Delta 3}{\mu\gamma(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu)} L_k^* \end{cases} \quad (4.2)$$

$$\Delta 3 = \varepsilon\gamma(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu) + \rho(\beta(\omega+\varepsilon+\mu)+\gamma\omega)(\zeta+\sigma+m+\mu)\times (\xi_2+\vartheta+\mu) + \sigma\delta(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu) + \vartheta\tau(\beta(\omega+\varepsilon+\mu)+\gamma\omega)(\zeta+\sigma+m+\mu) + \vartheta\zeta\delta(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)$$

$$L_k^* = \frac{\eta_K\gamma\mu(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu)\alpha(k)\Phi^*}{\Delta 4} \quad (4.3)$$

$$I_k^* = \frac{\eta_K\mu(\zeta+\sigma+m+\mu)(\xi_2+\vartheta+\mu)\alpha(k)\Phi^*[\beta(\omega+\varepsilon+\mu)+\gamma\omega]}{\Delta 4} \quad (4.4)$$

$$\Delta 4 = \alpha(k)\Phi^*\{\varepsilon\gamma(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu) + \rho(\beta(\omega+\varepsilon+\mu)+\gamma\omega)(\zeta+\sigma+m+\mu)\times (\xi_2+\vartheta+\mu) + \sigma\delta(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu) + \vartheta\tau(\beta(\omega+\varepsilon+\mu)+\gamma\omega)\times (\zeta+\sigma+m+\mu) + \vartheta\zeta\delta(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)\} + (\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)\mu(\zeta+\sigma+m+\mu)\times (\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu) + \alpha(k)\Phi^*\mu(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu)(\omega+\varepsilon+\mu) + \alpha(k)\Phi^*\mu(\zeta+\sigma+m+\mu)(\xi_2+\vartheta+\mu)(\beta(\omega+\varepsilon+\mu)+\gamma\omega) + \gamma\mu(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)\times (\xi_2+\vartheta+\mu)\alpha(k)\Phi^* + \delta(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)\alpha(k)\Phi^*\mu(\xi_2+\vartheta+\mu) + \alpha(k)\Phi^*\mu\times [\tau(\beta(\omega+\varepsilon+\mu)+\gamma\omega)(\zeta+\sigma+m+\mu) + \zeta\delta(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)]$$

$$\Phi^* = \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_1(k)}{\eta_K}P(k)I_k^* + \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_2(k)}{\eta_K}P(k)L_k^*$$
$$= \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_1(k)}{\eta_K}P(k)\frac{\eta_K\mu(\zeta+\sigma+m+\mu)(\xi_2+\vartheta+\mu)\alpha(k)[\beta(\omega+\varepsilon+\mu)+\gamma\omega]\Phi^*}{\Delta 4} + \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_2(k)}{\eta_K}P(k)\frac{\eta_K\gamma\mu(\zeta+\sigma+m+\mu)(\xi_1+\tau+\rho+\mu)(\xi_2+\vartheta+\mu)\alpha(k)\Phi^*}{\Delta 4} := F(\Phi^*) \quad (4.5)$$

Apparently, $\Phi^*=0$ is a trivial solution of (4.5), i.e., $F(0) = 0$. In order to let (4.5) have a non-trivial solution, i.e., $0<\Phi^*<1$, the right side of (4.5) must satisfy the following conditions:

$$\frac{dF(\Phi^*)}{d\Phi^*}\Big|_{\Phi^*=0} > 1$$

$$\frac{dF(\Phi^*)}{d\Phi^*} = \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\varphi_1(k)P(k)\frac{\alpha(k)[\beta(\omega+\varepsilon+\mu)+\gamma\omega]}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)(\xi_1+\tau+\rho+\mu)} + \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\varphi_2(k)P(k)\frac{\alpha(k)\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}$$
$$= \frac{\langle\varphi_1(k)\alpha(k)\rangle}{\langle k\rangle}\frac{\beta(\omega+\varepsilon+\mu)+\gamma\omega}{(\gamma+\beta+\delta+\mu)(\xi_1+\tau+\rho+\mu)(\omega+\varepsilon+\mu)} + \frac{\langle\varphi_2(k)\alpha(k)\rangle}{\langle k\rangle}\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)} = R_0 > 1$$

So, a nontrivial solution exists if and only if $R_0 > 1$.

Inserting the nontrivial solution into Eq(4.3, 4.4), we can obtain I_k^* and L_k^*. By I_k^* and L_k^* we can easily get:

$0 < S_k^* < \eta_k$, $0< E_k^* < \eta_k$, $0< L_k^* < \eta_k$, $0< I_k^* < \eta_k$, $0< Q_k^* < \eta_k$, $0< H_k^* < \eta_k$, $0< R_k^* < \eta_k$

Thus, the COVID-19 prevailing equilibrium $X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, Q_k^*, H_k^*, R_k^*)^T$ is well-defined.

Hence, when $R_0 > 1$, only one positive equilibrium $X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, Q_k^*, H_k^*, R_k^*)^T$ of system (2.1) exists.

The proof is completed.

Remark The basic reproduction number R_0 depends on some model parameters and the fluctuations of the degree distribution. Next, we discuss the influence of network topology and model parameters on the basic reproduction number R_0.

$$R_0 = \frac{\langle\varphi_1(k)\alpha(k)\rangle}{\langle k\rangle}\frac{\beta(\omega+\varepsilon+\mu)+\gamma\omega}{(\gamma+\beta+\delta+\mu)(\xi_1+\tau+\rho+\mu)(\omega+\varepsilon+\mu)} + \frac{\langle\varphi_2(k)\alpha(k)\rangle}{\langle k\rangle}\frac{\gamma}{(\gamma+\beta+\delta+\mu)(\omega+\varepsilon+\mu)}$$

$$\frac{\partial R_0}{\partial\delta} = -\frac{\langle\varphi_1(k)\alpha(k)\rangle}{\langle k\rangle}\frac{(\beta(\omega+\varepsilon+\mu)+\gamma\omega)}{(\gamma+\beta+\delta+\mu)^2(\xi_1+\tau+\rho+\mu)(\omega+\varepsilon+\mu)} - \frac{\langle\varphi_2(k)\alpha(k)\rangle}{\langle k\rangle}\frac{\gamma}{(\gamma+\beta+\delta+\mu)^2(\omega+\varepsilon+\mu)} < 0$$

$\frac{\langle\varphi_1(k)\alpha(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\varphi_2(k)\alpha(k)\rangle}{\langle k\rangle}$ reflect the complexity of the network. The larger $\frac{\langle\varphi_1(k)\alpha(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\varphi_2(k)\alpha(k)\rangle}{\langle k\rangle}$, the more network connections, the greater population density, the more contacts between people, and the larger R_0, which is conducive to the spread of the COVID-19. Therefore, reducing the gathering and flow of people is conducive to reducing the spread of the COVID-19.

Among all the parameters, the most important parameter is the isolation rate δ. β and γ denote the infection rate of close contacts without isolation, so $\delta + \beta + \gamma < 1$. If the government takes compulsory measures, the isolation rate may reach 0.95, then β and γ are relatively small, R_0 is also very small, and the COVID-19 transmission will be blocked ; When an epidemic occurs, if the government does not take compulsory measures, δ will be very small, and some people will isolate themselves to make $\delta \geq 0.2$. β and γ will increase, and R_0 will increase, accelerating the spread of the COVID-19. Therefore, the key to block the spread of the COVID-19 is to increase the isolation ratio.

Next, the global attractivity of the COVID-19 prevailing equilibrium is discussed. The main result is given in the following theorem.

Lemma1 ([16])
If $a > 0$, $b > 0$, and $dx(t)/dt \leq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have

$$\lim_{t\to\infty} \sup x(t) \leq \frac{b}{a}$$

If $a > 0$, $b > 0$, and $dx(t)/dt \geq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have

$$\lim_{t\to\infty} \inf x(t) \geq \frac{b}{a}$$

Theorem4 $X_k = (S_k, E_k, L_k, I_k, Q_k, H_k, R_k)^T$ is a solution of system (4.1) satisfying initial conditions $0< E_k < \eta_k$, $0< L_k < \eta_k$, $0< I_k < \eta_k$. If $R_0 > 1$, then
Lim $t\to\infty$ $X_k = (S_k, E_k, L_k, I_k, Q_k, H_k, R_k)^T = X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, Q_k^*, H_k^*, R_k^*)^T$
where $X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, Q_k^*, H_k^*, R_k^*)^T$ is the COVID-19 prevailing equilibrium of (4.1) satisfying (4.2) for k = 1, 2, . . . , n.

Proof:
In the following, k is fixed to be any integer in (1, 2, . . . , n)
By Theorem 4 $0< E_k < \eta_k$, $0< L_k < \eta_k$, $0< I_k < \eta_k$
there exists a sufficiently small constant $\rho (0 <\rho< 1)$ and a larger enough constant $T > 0$
such that $I_k(t) \geq \rho$ $L_k(t) \geq \rho$ for $t > T$.
Thus for $t > T$

$$\Phi(t) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_1(k)}{\eta_k}P(k)I_k(t) + \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_2(k)}{\eta_k}P(k)L_k(t) \geq \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_1(k)+\varphi_2(k)}{\eta_k}P(k)\rho = \Upsilon\rho > 0 \quad (4.6)$$

$$\Upsilon = \sum_{k=1}^{n}\frac{\varphi_1(k)+\varphi_2(k)}{\eta_k}P(k)$$

Step 1:
Submitting this into the equation of (4.1)

$$\frac{dS_k(t)}{dt} \leq V(k) - \alpha(k)S_k(t)\Upsilon\rho - \mu S_k(t) + m(\eta_k - S_k(t)) \qquad t > T$$

By Lemma 1 $\lim_{t\to\infty} \sup S_k(t) \leq \frac{V(k)+m\eta_k}{\alpha(k)\Upsilon\rho+m+\mu}$

For any given constant $0 < \rho_1 \leq \frac{V(k)+m\eta_k}{2[\alpha(k)\Upsilon\rho+m+\mu]}$

There exists $t_1>T$, for $t>t_1$ $S_k(t) \leq X_k^{(1)} - \rho_1$

$$X_k^{(1)} \leq \frac{V(k)+m\eta_k}{\alpha(k)\Upsilon\rho+m+\mu} + 2\rho_1 < \eta_k \quad (4.7)$$

Step 2:

$$\Phi(t) \leq \frac{1}{\langle k\rangle}\sum(\varphi_1(k) + \varphi_2(k))P(k) := \lambda \quad (4.8)$$

we obtain from the second equation of system (4.1) that

$$\frac{dE_k(t)}{dt} \leq \alpha(k)\lambda\big(\eta_k - E_k(t)\big) - (\gamma + \beta + \delta + \mu)E_k(t) \qquad t > t_1$$

By Lemma 1 $\lim_{t\to\infty} \sup E_k(t) \leq \frac{\alpha(k)\lambda\eta_k}{\alpha(k)\lambda+\gamma+\beta+\delta+\mu} < \eta_k$

For any given constant $0 < \rho_2 < min\left\{\frac{1}{2},\ \rho_1,\ \frac{\alpha(k)\lambda\eta_k}{2[\alpha(k)\lambda+\gamma+\beta+\delta+\mu]}\right\}$

There exists $t_2> t_1$, for $t>t_2$ $E_k(t) \leq Y_k^{(1)} - \rho_2 < \eta_k$

$$Y_k^{(1)} = \frac{\alpha(k)\lambda\eta_k}{\alpha(k)\lambda+\gamma+\beta+\delta+\mu} + 2\rho_2 < \eta_k \quad (4.9)$$

Step 3:
Then it follows from the third equation of (4.1) that

$$\frac{dL_k(t)}{dt} \leq \gamma\big(\eta_k - L_k(t)\big) - (\omega + \varepsilon + \mu)L_k(t) \qquad t > t_2$$

By Lemma 1 $\lim_{t\to\infty} \sup L_k(t) \leq \frac{\gamma\eta_k}{\gamma+\omega+\varepsilon+\mu} < \eta_k$

For any given constant $0 < \rho_3 < min\left\{\frac{1}{3},\ \rho_2,\ \frac{\gamma\eta_k}{2(\gamma+\omega+\varepsilon+\mu)}\right\}$

There exists t$_3$> t$_2$, for t>t$_3$ $L_k(t) \le Z_k^{(1)} - \rho_3 < \eta_k$

$$Z_k^{(1)} = \frac{\gamma\eta_k}{\gamma+\omega+\varepsilon+\mu} + 2\rho_3 < \eta_k \qquad (4.10)$$

Step 4:

Then it follows from the fourth equation of (4.1) that

$\frac{dI_k(t)}{dt} \le \beta(\eta_k - I_k(t)) + \omega(\eta_k - I_k(t)) - (\xi_1 + \tau + \rho + \mu)I_k(t) \qquad t > t_3$

By Lemma 1 $\lim_{t\to\infty} \sup I_k(t) \le \frac{(\beta+\omega)\eta_k}{\beta+\omega+\xi_1+\tau+\rho+\mu} < \eta_k$

For any given constant $0 < \rho_4 < min\left\{\frac{1}{4},\ \rho_3,\ \frac{(\beta+\omega)\eta_k}{2(\beta+\omega+\xi_1+\tau+\rho+\mu)}\right\}$

There exists t$_4$> t$_3$, for t>t$_4$ $I_k(t) \le W_k^{(1)} - \rho_4 < \eta_k$

$$W_k^{(1)} = \frac{(\beta+\omega)\eta_k}{\beta+\omega+\xi_1+\tau+\rho+\mu} + 2\rho_4 < \eta_k \qquad (4.11)$$

Step 5:

Then it follows from the fifth equation of (4.1) that

$\frac{dQ_k(t)}{dt} \le \delta(\eta_k - Q_k(t)) - (\zeta + \sigma + m + \mu)Q_k(t) \qquad t > t_4$

By Lemma 1 $\lim_{t\to\infty} \sup Q_k(t) \le \frac{\delta\eta_k}{\delta+\zeta+\sigma+m+\mu} < \eta_k$

For any given constant $0 < \rho_5 < min\left\{\frac{1}{5},\ \rho_4,\ \frac{\delta\eta_k}{2(\delta+\zeta+\sigma+m+\mu)}\right\}$

There exists t$_5$> t$_4$, for t>t$_5$ $Q_k(t) \le B_k^{(1)} - \rho_5 < \eta_k$

$$B_k^{(1)} = \frac{\delta\eta_k}{\delta+\zeta+\sigma+m+\mu} + 2\rho_5 < \eta_k \qquad (4.12)$$

Step 6:

Then it follows from the sixth equation of (4.1) that

$\frac{dH_k(t)}{dt} \le \tau(\eta_k - H_k(t)) + \zeta(\eta_k - H_k(t)) - (\xi_2 + \vartheta + \mu)H_k(t) \qquad t > t_5$

By Lemma 1 $\lim_{t\to\infty} \sup H_k(t) \le \frac{(\tau+\zeta)\eta_k}{\tau+\zeta+\xi_2+\vartheta+\mu} < \eta_k$

For any given constant $0 < \rho_6 < min\left\{\frac{1}{6},\ \rho_5,\ \frac{(\tau+\zeta)\eta_k}{2(\tau+\zeta+\xi_2+\vartheta+\mu)}\right\}$

There exists t$_6$> t$_5$, for t>t$_6$ $H_k(t) \le G_k^{(1)} - \rho_6 < \eta_k$

$$G_k^{(1)} = \frac{(\tau+\zeta)\eta_k}{\tau+\zeta+\xi_2+\vartheta+\mu} + 2\rho_6 < \eta_k \qquad (4.13)$$

Step 7:

On the other hand, we substitute this into the first equation of (4.1)

$\frac{dS_k(t)}{dt} \ge V(k) - \alpha(k)\lambda S_k(t) - \mu S_k(t) + m(\eta_k - S_k(t)) \qquad t > t_6$

By Lemma 1 $\lim_{t\to\infty} \inf S_k(t) \ge \frac{V(k)+m\eta_k}{\alpha(k)\lambda+m+\mu}$

For any given constant $0 < \rho_7 < min\left\{\frac{1}{7},\ \rho_6,\ \frac{V(k)+m\eta_k}{2[\alpha(k)\lambda+m+\mu]}\right\}$

There exists t$_7$> t$_6$, for t>t$_7$ $S_k(t) \ge x_k^{(1)} + \rho_7$

$$x_k^{(1)} = \frac{V(k)+m\eta_k}{\alpha(k)\lambda+m+\mu} - 2\rho_7 \qquad (4.14)$$

Step 8:

It follows that

$\frac{dE_k(t)}{dt} \ge \alpha(k)\Upsilon\rho x_k^{(1)} - (\gamma + \beta + \delta + \mu)E_k(t) \qquad t > t_7$

By Lemma 1 $\lim_{t\to\infty} \inf E_k(t) \ge \frac{\alpha(k)\Upsilon\rho x_k^{(1)}}{\gamma+\beta+\delta+\mu}$

For any given constant $0 < \rho_8 < min\left\{\frac{1}{8},\ \rho_7,\ \frac{\alpha(k)\Upsilon\rho x_k^{(1)}}{2(\gamma+\beta+\delta+\mu)}\right\}$

There exists t$_8$> t$_7$, for t>t$_8$ $E_k(t) \ge y_k^{(1)} + \rho_8$

$$y_k^{(1)} = \frac{\alpha(k)\Upsilon\rho x_k^{(1)}}{\gamma+\beta+\delta+\mu} - 2\rho_8 \qquad (4.15)$$

Step 9:

The third equation of (4.1) implies that

$\frac{dL_k(t)}{dt} \ge \gamma y_k^{(1)} - (\omega + \varepsilon + \mu)L_k(t) \qquad t > t_8$

By Lemma 1 $\lim_{t\to\infty} \inf L_k(t) \geq \frac{\gamma y_k^{(1)}}{\omega+\varepsilon+\mu}$

For any given constant $0 < \rho_9 < min\left\{\frac{1}{9},\ \rho_8,\ \frac{\gamma y_k^{(1)}}{2(\omega+\varepsilon+\mu)}\right\}$

There exists $t_9 > t_8$, for $t > t_9$ $L_k(t) \geq z_k^{(1)} + \rho_9$

$$z_k^{(1)} = \frac{\gamma y_k^{(1)}}{\omega+\varepsilon+\mu} - 2\rho_9 \tag{4.16}$$

Step 10:

It follows that $\frac{dI_k(t)}{dt} \geq \beta y_k^{(1)} + \omega z_k^{(1)} - (\xi_1 + \tau + \rho + \mu)I_k(t) \qquad t > t_9$

By Lemma 1 $\lim_{t\to\infty} \inf I_k(t) \geq \frac{\beta y_k^{(1)}+\omega z_k^{(1)}}{\xi_1+\tau+\rho+\mu}$

For any given constant $0 < \rho_{10} < min\left\{\frac{1}{10},\ \rho_9,\ \frac{\beta y_k^{(1)}+\omega z_k^{(1)}}{2(\xi_1+\tau+\rho+\mu)}\right\}$

There exists $t_{10} > t_9$, for $t > t_{10}$ $I_k(t) \geq w_k^{(1)} + \rho_{10}$

$$w_k^{(1)} = \frac{\beta y_k^{(1)}+\omega z_k^{(1)}}{\xi_1+\tau+\rho+\mu} - 2\rho_{10} \tag{4.17}$$

Step 11:

It follows that $\frac{dQ_k(t)}{dt} \geq \delta y_k^{(1)} - (\zeta + \sigma + m + \mu)Q_k(t) \qquad t > t_{10}$

By Lemma 1 $\lim_{t\to\infty} \inf Q_k(t) \geq \frac{\delta y_k^{(1)}}{\zeta+\sigma+m+\mu}$

For any given constant $0 < \rho_{11} < min\left\{\frac{1}{11},\ \rho_{10},\ \frac{\delta y_k^{(1)}}{2(\zeta+\sigma+m+\mu)}\right\}$

There exists $t_{11} > t_{10}$, for $t > t_{11}$ $Q_k(t) \geq b_k^{(1)} + \rho_{11}$

$$b_k^{(1)} = \frac{\delta y_k^{(1)}}{\zeta+\sigma+m+\mu} - 2\rho_{11} \tag{4.18}$$

Step 12:

It follows that $\frac{dH_k(t)}{dt} \geq \tau w_k^{(1)} + \zeta\, b_k^{(1)} - (\xi_2 + \vartheta + \mu)H_k(t) \qquad t > t_{11}$

By Lemma 1 $\lim_{t\to\infty} \inf H_k(t) \geq \frac{\tau w_k^{(1)}+\zeta\, b_k^{(1)}}{\xi_2+\vartheta+\mu}$

For any given constant $0 < \rho_{12} < min\left\{\frac{1}{12},\ \rho_{11},\ \frac{\tau w_k^{(1)}+\zeta\, b_k^{(1)}}{2(\xi_2+\vartheta+\mu)}\right\}$

There exists $t_{12} > t_{11}$, for $t > t_{12}$ $H_k(t) \geq g_k^{(1)} + \rho_{12}$

$$g_k^{(1)} = \frac{\tau w_k^{(1)}+\zeta\, b_k^{(1)}}{\xi_2+\vartheta+\mu} - 2\rho_{12} \tag{4.19}$$

Step 13:

Due to ρ being a small positive constant, we can derive that

$$0 < x_k^{(1)} < X_k^{(1)},\quad 0 < y_k^{(1)} < Y_k^{(1)},\quad 0 < z_k^{(1)} < Z_k^{(1)},$$
$$0 < w_k^{(1)} < W_k^{(1)},\quad 0 < b_k^{(1)} < B_k^{(1)},\quad 0 < g_k^{(1)} < G_k^{(1)}$$

$$h^{(j)} = \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}w_i^{(j)} + \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}z_i^{(j)} \qquad H^{(j)} = \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}W_i^{(j)} + \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}Z_i^{(j)} \tag{4.20}$$

We can easily get $0 < h^{(j)} < \Phi(t) < H^{(j)} < \lambda$ (4.21)

Again, from the first equation of (4.1), we have

$$\frac{dS_k(t)}{dt} \leq V(k) - \alpha(k)h^{(1)}S_k(t) - \mu S_k(t) + m(\eta_k - S_k(t)) \qquad t > t_{12}$$

By Lemma 1 $\lim_{t\to\infty} \sup S_k(t) \leq \frac{V(k)+m\eta_k}{\alpha(k)h^{(1)}+m+\mu}$

For any given constant $0 < \rho_{13} < min\{1/13,\ \rho_{12}\}$

There exists $t_{13} > t_{12}$, for $t > t_{13}$ $S_k(t) \leq X_k^{(2)}$

$$X_k^{(2)} = \min\left\{X_k^{(1)} - \rho_1\ ,\ \frac{V(k)+m\eta_k}{\alpha(k)h^{(1)}+m+\mu} + \rho_{13}\right\} \tag{4.22}$$

Step 14:

Then, from the second equation of (4.1), we have

$$\frac{dE_k(t)}{dt} \leq \alpha(k)H^{(1)}X_k^{(2)} - (\gamma + \beta + \delta + \mu)E_k(t) \qquad t > t_{13}$$

By Lemma 1 $\lim_{t\to\infty} \sup E_k(t) \le \frac{\alpha(k)H^{(1)}X_k^{(2)}}{\gamma+\beta+\delta+\mu}$

For any given constant $0 < \rho_{14} < min\{1/14,\ \rho_{13}\}$

There exists $t_{14} > t_{13}$, for $t > t_{14}$ $E_k(t) \le Y_k^{(2)} = \min\left\{Y_k^{(1)} - \rho_2\ ,\ \frac{\alpha(k)H^{(1)}X_k^{(2)}}{\gamma+\beta+\delta+\mu} + \rho_{14}\right\}$ (4.23)

Step 15:

Consequently, from the third equation of (4.1), we have

$$\frac{dL_k(t)}{dt} \le \gamma Y_k^{(2)} - (\omega+\varepsilon+\mu)L_k(t) \qquad t > t_{14}$$

By Lemma 1 $\lim_{t\to\infty} \sup L_k(t) \le \frac{\gamma Y_k^{(2)}}{\omega+\varepsilon+\mu}$

For any given constant $0 < \rho_{15} < min\{1/15,\ \rho_{14}\}$

There exists $t_{15} > t_{14}$, for $t > t_{15}$ $L_k(t) \le Z_k^{(2)} = \min\left\{Z_k^{(1)} - \rho_3\ ,\ \frac{\gamma Y_k^{(2)}}{\omega+\varepsilon+\mu} + \rho_{15}\right\}$ (4.24)

Step 16:

from the fourth equation of (4.1), we have

$$\frac{dI_k(t)}{dt} \le \beta Y_k^{(2)} + \omega Z_k^{(2)} - (\xi_1+\tau+\rho+\mu)I_k(t) \qquad t > t_{15}$$

By Lemma 1 $\lim_{t\to\infty} \sup I_k(t) \le \frac{\beta Y_k^{(2)} + \omega Z_k^{(2)}}{\xi_1+\tau+\rho+\mu}$

For any given constant $0 < \rho_{16} < min\{1/16,\ \rho_{15}\}$

There exists $t_{16} > t_{15}$, for $t > t_{16}$

$$I_k(t) \le W_k^{(2)} = \min\left\{W_k^{(1)} - \rho_4\ ,\ \frac{\beta Y_k^{(2)} + \omega Z_k^{(2)}}{\xi_1+\tau+\rho+\mu} + \rho_{16}\right\} \qquad (4.25)$$

Step 17:

from the fifth equation of (4.1), we have

$$\frac{dQ_k(t)}{dt} \le \delta Y_k^{(2)} - (\zeta+\sigma+m+\mu)Q_k(t) \qquad t > t_{16}$$

By Lemma 1 $\lim_{t\to\infty} \sup Q_k(t) \le \frac{\delta Y_k^{(2)}}{\zeta+\sigma+m+\mu}$

For any given constant $0 < \rho_{17} < min\{1/17,\ \rho_{16}\}$

There exists $t_{17} > t_{16}$, for $t > t_{17}$

$$Q_k(t) \le B_k^{(2)} = \min\left\{B_k^{(1)} - \rho_5\ ,\ \frac{\delta Y_k^{(2)}}{\zeta+\sigma+m+\mu} + \rho_{17}\right\} \qquad (4.26)$$

Step 18:

from the sixth equation of (4.1), we have

$$\frac{dH_k(t)}{dt} \le \tau W_k^{(2)} + \zeta B_k^{(2)} - (\xi_2+\vartheta+\mu)H_k(t) \qquad t > t_{17}$$

By Lemma 1 $\lim_{t\to\infty} \sup H_k(t) \le \frac{\tau W_k^{(2)} + \zeta B_k^{(2)}}{\xi_2+\vartheta+\mu}$

For any given constant $0 < \rho_{18} < min\{1/18,\ \rho_{17}\}$

There exists $t_{18} > t_{17}$, for $t > t_{18}$

$$H_k(t) \le G_k^{(2)} = \min\left\{G_k^{(1)} - \rho_6\ ,\ \frac{\tau W_k^{(2)} + \zeta B_k^{(2)}}{\xi_2+\vartheta+\mu} + \rho_{18}\right\} \qquad (4.27)$$

Step 19:

Turning back, one has

$$\frac{dS_k(t)}{dt} \ge V(k) - \alpha(k)H^{(2)}S_k(t) - \mu S_k(t) + m(\eta_k - S_k(t)) \qquad t > t_{18}$$

By Lemma 1 $\lim_{t\to\infty} \inf S_k(t) \ge \frac{V(k)+m\eta_k}{\alpha(k)H^{(2)}+m+\mu}$

For any given constant $0 < \rho_{19} < min\left\{\frac{1}{19},\ \rho_{18}\ ,\ \frac{V(k)+m\eta_k}{2[\alpha(k)H^{(2)}+m+\mu]}\right\}$

There exists $t_{19} > t_{18}$, for $t > t_{19}$ $S_k(t) \ge x_k^{(2)} = \max\left\{x_k^{(1)} + \rho_7\ ,\ \frac{V(k)+m\eta_k}{\alpha(k)H^{(2)}+m+\mu} - \rho_{19}\right\}$ (4.28)

Step 20:

It follows that

$$\frac{dE_k(t)}{dt} \ge \alpha(k)h^{(1)}x_k^{(2)} - (\gamma+\beta+\delta+\mu)E_k(t) \qquad t > t_{19}$$

By Lemma 1 $\lim_{t\to\infty} \inf E_k(t) \ge \frac{\alpha(k)h^{(1)}x_k^{(2)}}{\gamma+\beta+\delta+\mu}$

For any given constant $0 < \rho_{20} < min\left\{\frac{1}{20}, \ \rho_{19} \ , \ \frac{\alpha(k)h^{(1)}x_k^{(2)}}{2[\gamma+\beta+\delta+\mu]}\right\}$

There exists $t_{20} > t_{19}$, for $t > t_{20}$ $\quad E_k(t) \geq y_k^{(2)} = \max\left\{y_k^{(1)} + \rho_8 \ , \ \frac{\alpha(k)h^{(1)}x_k^{(2)}}{\gamma+\beta+\delta+\mu} - \rho_{20}\right\}$ (4.29)

Step 21:

The third equation of (4.1) implies that

$\frac{dL_k(t)}{dt} \geq \gamma y_k^{(2)} - (\omega + \varepsilon + \mu)L_k(t) \qquad t > t_{20}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf L_k(t) \geq \frac{\gamma y_k^{(2)}}{\omega+\varepsilon+\mu}$

For any given constant $0 < \rho_{21} < min\left\{\frac{1}{21}, \ \rho_{20} \ , \ \frac{\gamma y_k^{(2)}}{2(\omega+\varepsilon+\mu)}\right\}$

There exists $t_{21} > t_{20}$, for $t > t_{21}$ $\quad L_k(t) \geq z_k^{(2)} = \max\left\{z_k^{(1)} + \rho_9 \ , \ \frac{\gamma y_k^{(2)}}{\omega+\varepsilon+\mu} - \rho_{21}\right\}$ (4.30)

Step 22:

It follows that

$\frac{dI_k(t)}{dt} \geq \beta y_k^{(2)} + \omega z_k^{(2)} - (\xi_1 + \tau + \rho + \mu)I_k(t) \qquad t > t_{21}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf I_k(t) \geq \frac{\beta y_k^{(2)}+\omega z_k^{(2)}}{\xi_1+\tau+\rho+\mu}$

For any given constant $0 < \rho_{22} < min\left\{\frac{1}{22}, \ \rho_{21} \ , \ \frac{\beta y_k^{(2)}+\omega z_k^{(2)}}{2(\xi_1+\tau+\rho+\mu)}\right\}$

There exists $t_{22} > t_{21}$, for $t > t_{22}$ $\quad I_k(t) \geq w_k^{(2)} = \max\left\{w_k^{(1)} + \rho_{10} \ , \ \frac{\beta y_k^{(2)}+\omega z_k^{(2)}}{\xi_1+\tau+\rho+\mu} - \rho_{22}\right\}$ (4.31)

Step 23:

It follows that

$\frac{dQ_k(t)}{dt} \geq \delta y_k^{(2)} - (\zeta + \sigma + m + \mu)Q_k(t) \qquad t > t_{22}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf Q_k(t) \geq \frac{\delta y_k^{(2)}}{\zeta+\sigma+m+\mu}$

For any given constant $0 < \rho_{23} < min\left\{\frac{1}{23}, \ \rho_{22} \ , \ \frac{\delta y_k^{(2)}}{2(\zeta+\sigma+m+\mu)}\right\}$

There exists $t_{23} > t_{22}$, for $t > t_{23}$ $\quad Q_k(t) \geq b_k^{(2)} = \max\left\{b_k^{(1)} + \rho_{11} \ , \ \frac{\delta y_k^{(2)}}{\zeta+\sigma+m+\mu} - \rho_{23}\right\}$ (4.32)

Step 24:

It follows that

$\frac{dH_k(t)}{dt} \geq \tau w_k^{(2)} + \zeta b_k^{(2)} - (\xi_2 + \vartheta + \mu)H_k(t) \qquad t > t_{23}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf H_k(t) \geq \frac{\tau w_k^{(2)}+\zeta b_k^{(2)}}{\xi_2+\vartheta+\mu}$

For any given constant $0 < \rho_{24} < min\left\{\frac{1}{24}, \ \rho_{23} \ , \ \frac{\tau w_k^{(2)}+\zeta b_k^{(2)}}{2(\xi_2+\vartheta+\mu)}\right\}$

There exists $t_{24} > t_{23}$, for $t > t_{24}$ $\quad H_k(t) \geq g_k^{(2)} = \max\left\{g_k^{(1)} + \rho_{12} \ , \ \frac{\tau w_k^{(2)}+\zeta b_k^{(2)}}{\xi_2+\vartheta+\mu} - \rho_{24}\right\}$ (4.33)

Repeating the above analyses and calculations, we get twelve sequences:

$x_k^{(l)} \quad y_k^{(l)} \quad z_k^{(l)} \quad w_k^{(l)} \quad b_k^{(l)} \quad g_k^{(l)} \quad X_k^{(l)} \quad Y_k^{(l)} \quad Z_k^{(l)} \quad W_k^{(l)} \quad B_k^{(l)} \quad G_k^{(l)} \qquad l = 1,2,3 \ldots\ldots$

Due to the first six being monotone increasing sequences and the last six being monotone decreasing ones there exists a sufficiently large positive integer L≥2, such that $l \geq$ L:

$$X_k^{(l)} = \frac{V(k)+m\eta_k}{\alpha(k)h^{(l-1)}+m+\mu} + \rho_{12l-11} \qquad Y_k^{(l)} = \frac{\alpha(k)H^{(l-1)}X_k^{(l)}}{\gamma+\beta+\delta+\mu} + \rho_{12l-10}$$
$$Z_k^{(l)} = \frac{\gamma Y_k^{(l)}}{\omega+\varepsilon+\mu} + \rho_{12l-9} \qquad W_k^{(l)} = \frac{\beta Y_k^{(l)}+\omega Z_k^{(l)}}{\xi_1+\tau+\rho+\mu} + \rho_{12l-8}$$
$$B_k^{(l)} = \frac{\delta Y_k^{(l)}}{\zeta+\sigma+m+\mu} + \rho_{12l-7} \qquad G_k^{(l)} = \frac{\tau W_k^{(l)}+\zeta B_k^{(l)}}{\xi_2+\vartheta+\mu} + \rho_{12l-6} \tag{4.34}$$
$$x_k^{(l)} = \frac{V(k)+m\eta_k}{\alpha(k)H^{(l)}+m+\mu} - \rho_{12l-5} \qquad y_k^{(l)} = \frac{\alpha(k)h^{(l-1)}x_k^{(l)}}{\gamma+\beta+\delta+\mu} - \rho_{12l-4}$$
$$z_k^{(l)} = \frac{\gamma y_k^{(l)}}{\omega+\varepsilon+\mu} - \rho_{12l-3} \qquad w_k^{(l)} = \frac{\beta y_k^{(l)}+\omega z_k^{(l)}}{\xi_1+\tau+\rho+\mu} - \rho_{12l-2}$$

$b_k^{(l)} = \frac{\delta y_k^{(l)}}{\zeta+\sigma+m+\mu} - \rho_{12l-1}$ $\qquad$ $g_k^{(l)} = \frac{\tau w_k^{(l)}+\zeta b_k^{(l)}}{\xi_2+\vartheta+\mu} - \rho_{12l}$

We can easily get that

$$x_k^{(l)} \le S_k(t) \le X_k^{(l)} \quad y_k^{(l)} \le E_k(t) \le Y_k^{(l)} \quad z_k^{(l)} \le L_k(t) \le Z_k^{(l)}$$
$$w_k^{(l)} \le I_k(t) \le W_k^{(l)} \quad b_k^{(l)} \le Q_k(t) \le B_k^{(l)} \quad g_k^{(l)} \le H_k(t) \le G_k^{(l)} \qquad (4.35)$$

Noting that $0 < \rho_l < 1/l$, one has: $l\to\infty$, $\rho\to 0$

taking $l\to\infty$, it follows from (4.34) that

$$X_k = \frac{V(k)+m\eta_k}{\alpha(k)h+m+\mu} \qquad Y_k = \frac{\alpha(k)HX_k}{\gamma+\beta+\delta+\mu}$$
$$Z_k = \frac{\gamma Y_k}{\omega+\varepsilon+\mu} \qquad W_k = \frac{\beta Y_k+\omega Z_k}{\xi_1+\tau+\rho+\mu}$$
$$B_k = \frac{\delta Y_k}{\zeta+\sigma+m+\mu} \qquad G_k = \frac{\tau W_k+\zeta B_k}{\xi_2+\vartheta+\mu} \qquad (4.36)$$
$$x_k = \frac{V(k)+m\eta_k}{\alpha(k)H+m+\mu} \qquad y_k = \frac{\alpha(k)hx_k}{\gamma+\beta+\delta+\mu}$$
$$z_k = \frac{\gamma y_k}{\omega+\varepsilon+\mu} \qquad w_k = \frac{\beta y_k+\omega z_k}{\xi_1+\tau+\rho+\mu}$$
$$b_k = \frac{\delta y_k}{\zeta+\sigma+m+\mu} \qquad g_k = \frac{\tau w_k+\zeta b_k}{\xi_2+\vartheta+\mu}$$

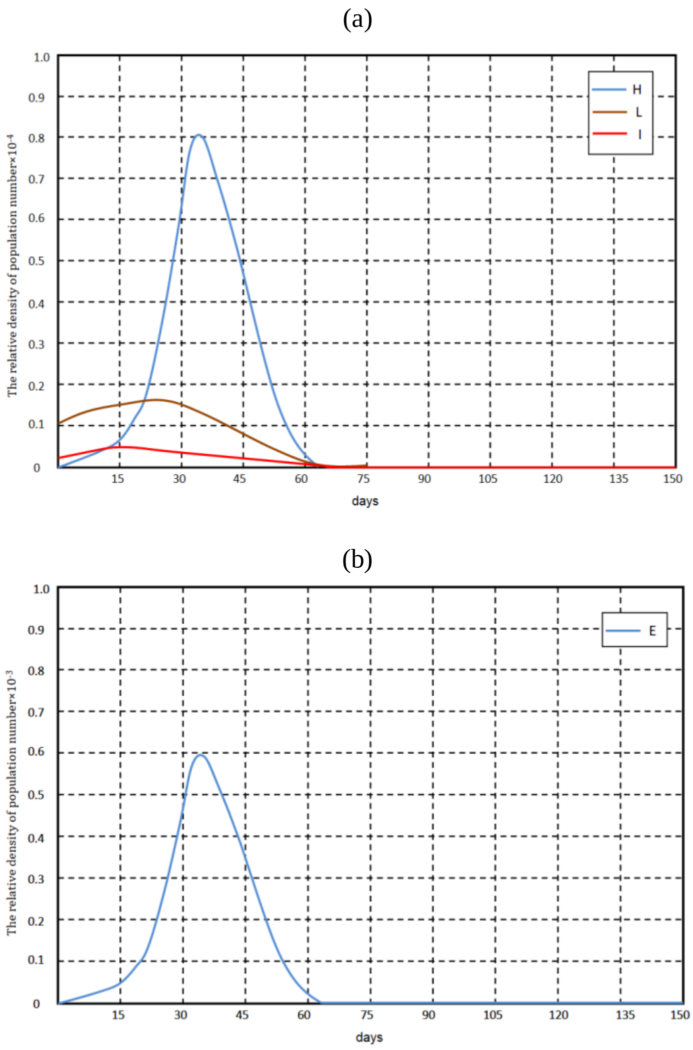

Figure2 Density of H_k, L_k, I_k and E_k versus time(δ=0.8, R_0=0.49<1)

$$h=\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}w_i+\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}z_i \qquad H=\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}W_i+\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}Z_i \tag{4.37}$$

Further

$$w_k=\frac{\beta}{(\xi_1+\tau+\rho+\mu)}\frac{\alpha(k)h}{(\gamma+\beta+\delta+\mu)}\frac{(V(k)+m\eta_k)}{(\alpha(k)H+m+\mu)}+\frac{\omega}{(\xi_1+\tau+\rho+\mu)}\frac{\gamma}{(\omega+\varepsilon+\mu)}\frac{\alpha(k)h}{(\gamma+\beta+\delta+\mu)}\frac{(V(k)+m\eta_k)}{(\alpha(k)H+m+\mu)} \tag{4.38}$$

$$W_k=\frac{\beta}{(\xi_1+\tau+\rho+\mu)}\frac{\alpha(k)H}{(\gamma+\beta+\delta+\mu)}\frac{(V(k)+m\eta_k)}{(\alpha(k)h+m+\mu)}+\frac{\omega}{(\xi_1+\tau+\rho+\mu)}\frac{\gamma}{(\omega+\varepsilon+\mu)}\frac{\alpha(k)H}{(\gamma+\beta+\delta+\mu)}\frac{(V(k)+m\eta_k)}{(\alpha(k)h+m+\mu)} \tag{4.39}$$

$$z_k=\frac{\gamma}{(\omega+\varepsilon+\mu)}\frac{\alpha(k)h}{(\gamma+\beta+\delta+\mu)}\frac{(V(k)+m\eta_k)}{(\alpha(k)H+m+\mu)} \qquad Z_k=\frac{\gamma}{(\omega+\varepsilon+\mu)}\frac{\alpha(k)H}{(\gamma+\beta+\delta+\mu)}\frac{(V(k)+m\eta_k)}{(\alpha(k)h+m+\mu)} \tag{4.40}$$

Substituting (4.38, 4.39, 4.40) into h and H(4.37), respectively, one has

$$1=\frac{1}{\langle k\rangle}\frac{\beta}{\eta_k(\xi_1+\tau+\rho+\mu)(\gamma+\beta+\delta+\mu)}\sum\frac{\alpha(k)\varphi_1(k)P(k)(V(k)+m\eta_k)}{(\alpha(k)H+m+\mu)}+\frac{1}{\langle k\rangle}\frac{\omega\gamma}{\eta_k(\xi_1+\tau+\rho+\mu)(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}\times$$
$$\sum\frac{\alpha(k)\varphi_1(k)P(k)(V(k)+m\eta_k)}{(\alpha(k)H+m+\mu)}+\frac{1}{\langle k\rangle}\frac{\gamma}{\eta_k(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}\sum\frac{\alpha(k)\varphi_2(k)P(k)(V(k)+m\eta_k)}{(\alpha(k)H+m+\mu)}$$

$$1=\frac{1}{\langle k\rangle}\frac{\beta}{\eta_k(\xi_1+\tau+\rho+\mu)(\gamma+\beta+\delta+\mu)}\sum\frac{\alpha(k)\varphi_1(k)P(k)(V(k)+m\eta_k)}{(\alpha(k)h+m+\mu)}+\frac{1}{\langle k\rangle}\frac{\omega\gamma}{\eta_k(\xi_1+\tau+\rho+\mu)(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}\times$$
$$\sum\frac{\alpha(k)\varphi_1(k)P(k)(V(k)+m\eta_k)}{(\alpha(k)h+m+\mu)}+\frac{1}{\langle k\rangle}\frac{\gamma}{\eta_k(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}\sum\frac{\alpha(k)\varphi_2(k)P(k)(V(k)+m\eta_k)}{(\alpha(k)h+m+\mu)}$$

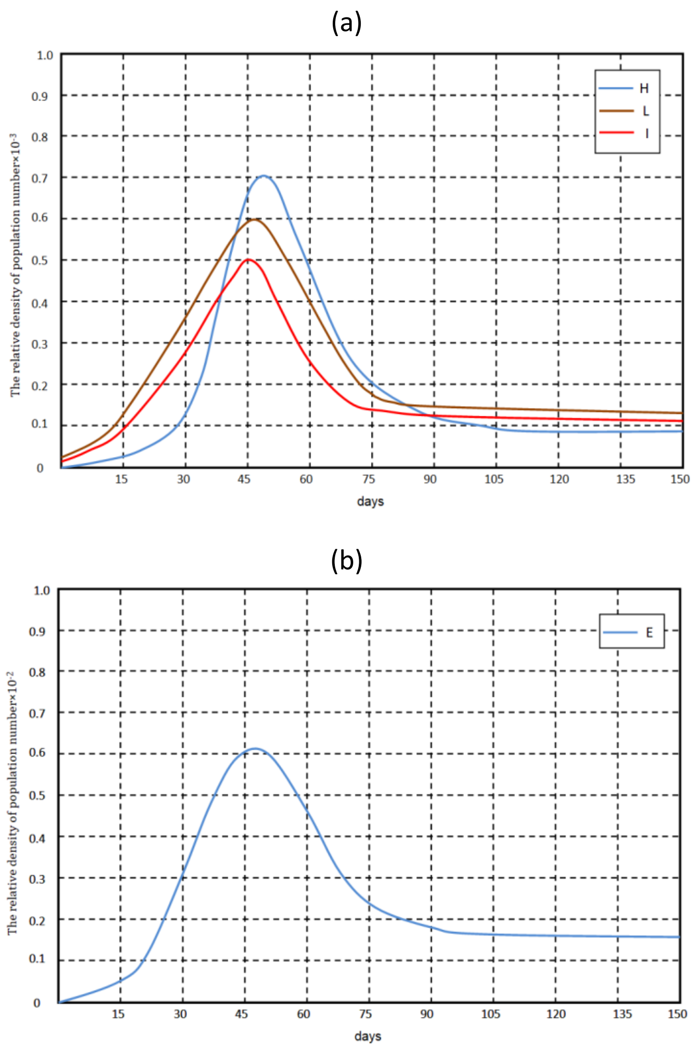

Figure3 Density of H_k, L_k, I_k and E_k versus time(δ=0.5, R_0=1.61>1)

By subtracting the above two equations, we arrive at

$$0=\frac{1}{\langle k\rangle}\frac{\beta}{\eta_k(\xi_1+\tau+\rho+\mu)(\gamma+\beta+\delta+\mu)}\sum\frac{\alpha(k)^2\varphi_1(k)P(k)(V(k)+m\eta_k)(h-H)}{(\alpha(k)H+m+\mu)(\alpha(k)h+m+\mu)}+\frac{1}{\langle k\rangle}\frac{\omega\gamma}{\eta_k(\xi_1+\tau+\rho+\mu)(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}\times$$

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$$\sum \frac{\alpha(k)^2\varphi_1(k)P(k)(V(k)+m\eta_k)(h-H)}{(\alpha(k)H+m+\mu)(\alpha(k)h+m+\mu)} + \frac{1}{\langle k\rangle}\frac{\gamma}{\eta_k(\omega+\varepsilon+\mu)(\gamma+\beta+\delta+\mu)}\sum \frac{\alpha(k)^2\varphi_2(k)P(k)(V(k)+m\eta_k)(h-H)}{(\alpha(k)H+m+\mu)(\alpha(k)h+m+\mu)}$$

It is obvious that: H=h

$$H-h=\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}(W_i-w_i)+\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}(Z_i-z_i)=0$$

That $W_i = w_i \quad Z_i = z_i \quad i = 1,2,3,\ldots n$

we can arrive at $W_i = w_i \quad X_i = x_i \quad Y_i = y_i \quad Z_i = z_i \quad B_i = b_i \quad G_i = g_i \quad i = 1,2,3\cdots n$

It follows that:

$$\lim_{t\to\infty} S_k(t) = X_k = x_k \quad \lim_{t\to\infty} E_k(t) = Y_k = y_k \quad \lim_{t\to\infty} L_k(t) = Z_k = z_k$$
$$\lim_{t\to\infty} I_k(t) = W_k = w_k \quad \lim_{t\to\infty} Q_k(t) = B_k = b_k \quad \lim_{t\to\infty} H_k(t) = G_k = g_k$$

Finally, substituting h = H into (4.35), in view of (4.2) and (4.36), we obtain:

$$S_k = S_k^* \quad E_k = E_k^* \quad L_k = L_k^* \quad I_k = I_k^* \quad Q_k = Q_k^* \quad H_k = H_k^* \quad R_k = R_k^*$$

that is: $X_k = (S_k, E_k, L_k, I_k, Q_k, H_k, R_k)^T = X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, Q_k^*, H_k^*, R_k^*)^T$

The proof is completed.

V. SIMULATION RESULTS AND ANALYSIS

In this section, we present several numerical simulations to illustrate the analysis results. We take the community degree distribution to be $P(k) = ck^{-l}$ $(2 < l \leq 3)$, in which $l = 3$ and c satisfies $\sum_{k=1}^{n} P(k) = 1$, n = 1000. We choose $\alpha(k) = \alpha k$, $\varphi_1(k) = k$, $\varphi_2(k) = 0.7k$, $V(k) = v/n$.

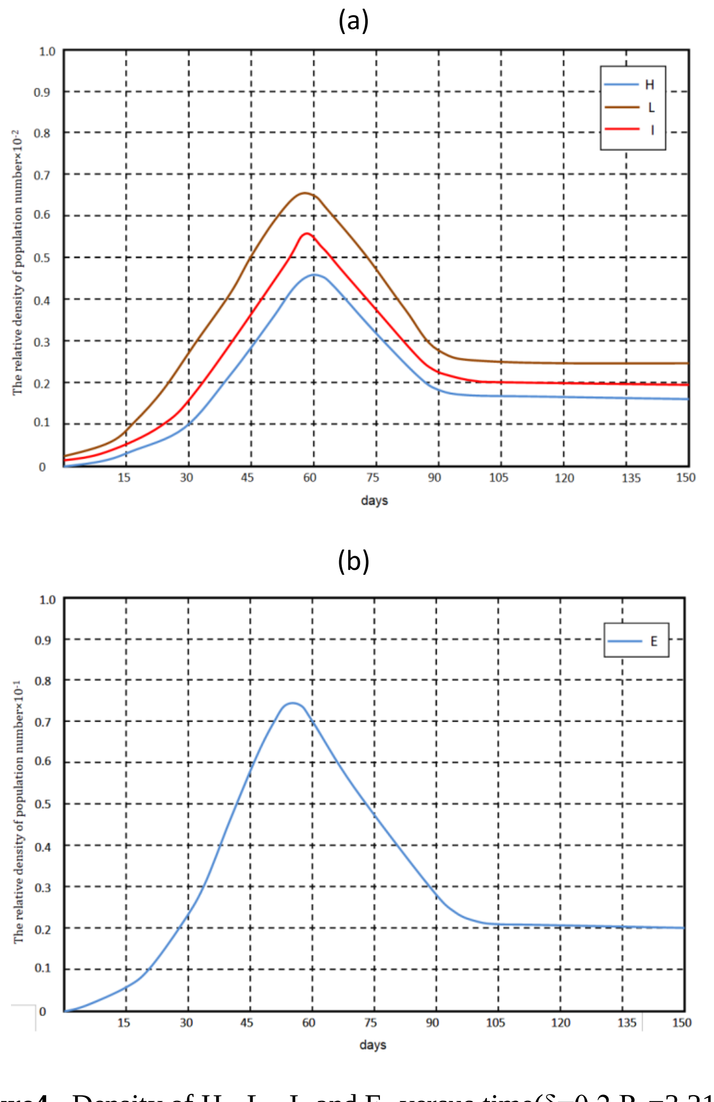

Figure4 Density of H_k, L_k, I_k and E_k versus time(δ=0.2 R_0=3.21>1)

In Fig. 2, the parameters are chosen as, γ= 0.06, β= 0.03, δ= 0.8, μ=0.007, ξ_1=0.08, ξ_2=0.04, τ=0.5, ρ=0.3, ω=0.15, ε=0.8, ϑ = 0.9, ζ=0.08, σ=0.005, m = 0.9, R_0 = 0.49 < 1. In the early stage of the outbreak, strict isolation measures are taken. The isolation ratio reaches 80%, R_0 = 0.49 < 1. The H_k peak value of hospitalized patients reaches 0.8 × 10^{-4}, the L_k peak value of asymptomatic infection without isolation reaches 0.17 × 10^{-4}, and the I_k peak value of patients without isolation reaches 0.05 × 10^{-4}. After 70 days, all the three cases are cleared and achieve good epidemic prevention effect.

In Fig. 3, the parameters are chosen as, γ= 0.17, β= 0.08, δ= 0.5, μ=0.007, ξ_1=0.08, ξ_2=0.04, τ=0.5, ρ=0.3, ω=0.15, ε=0.8, ϑ = 0.9, ζ=0.08, σ=0.005, m = 0.9, R_0 = 1.61 > 1. In the early stage of the outbreak, no strict isolation measures are taken. The isolation rate is 50%, which is a medium level of isolation. R_0 = 1.61 > 1, the H_k peak value of hospitalized patients reaches 0.7 × 10^{-3}, the L_k peak value of asymptomatic infection without isolation reaches 0.6 × 10^{-3}, and the I_k peak value of patients without isolation reaches 0.5 × 10^{-3}, Infected people always exist, and finally reach equilibrium. The effect of epidemic prevention is not satisfactory. Because the infected people always exist, once the epidemic prevention is relaxed, the epidemic will break out again.

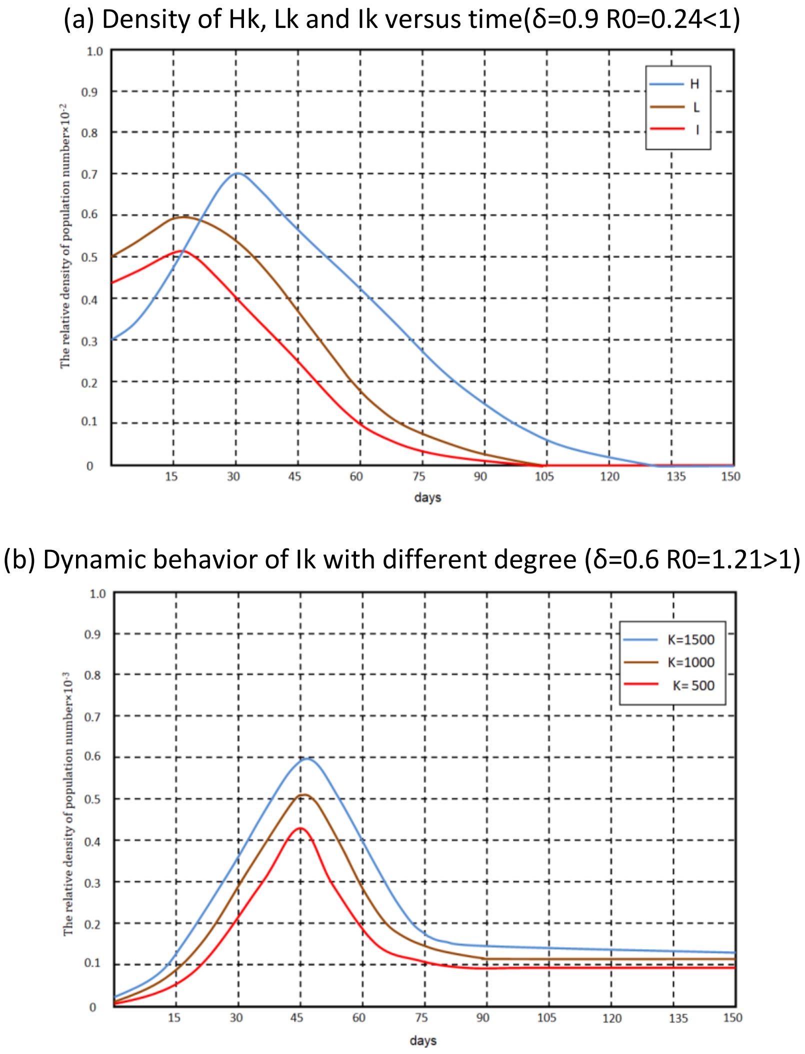

Figure5 Dynamic behavior of H_k, L_k and I_k versus time

In Fig. 4, the parameters are chosen as, γ= 0.26, β= 0.14, δ= 0.2, μ=0.007, ξ_1=0.08, ξ_2=0.04, τ=0.5, ρ=0.3, ω=0.15, ε=0.8, ϑ = 0.9, ζ=0.08, σ=0.005, m = 0.9, R_0 = 3.21 > 1. Relatively loose isolation measures have been adopted, the isolation ratio is 20%, which is a low level of isolation. R_0 = 3.21 > 1, the H_k peak value of hospitalized patients reaches 0.45 × 10^{-2}, the L_k peak value of asymptomatic infection without isolation reaches 0.65×10^{-2}, and the I_k peak value of patients without isolation reaches 0.55×10^{-2}, Infected people always exist. Finally, a balance state of high infection rate is achieved.

In Fig. 5(a), the parameters are chosen as, $\gamma= 0.03$, $\beta= 0.016$, $\delta= 0.9$, $\mu=0.007$, $\xi_1=0.08$, $\xi_2=0.04$, $\tau=0.5$, $\rho=0.3$, $\omega=0.15$, $\varepsilon=0.8$, $\vartheta = 0.9$, $\zeta=0.08$, $\sigma=0.005$, $m = 0.9$, $R_0 = 0.24 < 1$. At first, the COVID-19 is not well understood, resulting in a high infection rate. The initial infection is very high, the initial value of H_k is 0.3×10^{-2}, the initial value of I_k is 0.44×10^{-2}, and the initial value of L_k is 0.5×10^{-2}. Strict isolation measures are taken, the isolation ratio reaches 90%, $R_0 = 0.24 < 1$. The L_k and I_k of infection cases reach the peak in about two weeks, and reach the peak after 30 days of hospitalization. The L_k and I_k cases are cleared after 100 days, and all cases are cleared after 135 days, satisfactory results have been achieved.

In Fig. 5(b), the parameters are chosen as, $\gamma= 0.13$, $\beta= 0.07$, $\delta= 0.6$, $\mu=0.007$, $\xi_1=0.08$, $\xi_2=0.04$, $\tau=0.5$, $\rho=0.3$, $\omega=0.15$, $\varepsilon=0.8$, $\vartheta = 0.9$, $\zeta=0.08$, $\sigma=0.005$, $m = 0.9$, $R_0 = 1.21 > 1$. The Fig. 5(b) describe the time series of the infected individuals I_k with different degree. Obviously, we can see that the positive level will be higher with the increase of degree. The more network connections, the more contacts between people, and the larger R_0, which is conducive to the spread of the COVID-19. We find that the larger degree leads to a larger value of the spreading level. Therefore, reducing the gathering and flow of people is conducive to reducing the spread of the COVID-19.

VI. CONCLUSION

Considering isolating mechanism, this paper proposes a new SELIQHR model of COVID-19 spreading in complex social networks. With the mean field theory, the spreading dynamics of the model is analyzed in detail. By Lyapunov's first method, it is proved that the COVID-19 free equilibrium X_0 is locally asymptotically stable, and the basic reproduction number R_0 is obtained. The global asymptotic stability of the COVID-19 free equilibrium X_0 is proved with Lyapunov's second method. The basic reproduction number R_0 not only determines the existence of COVID-19 equilibrium, but also determines the global dynamic behavior of the model. If $R_0 < 1$, then X_0 is globally asymptotically stable. No matter what the initial value of the infected individuals, the infected individuals will disappear eventually. If $R_0 > 1$, then the COVID-19 prevailing equilibrium X_k^* is globally asymptotically attractive, it is that the infected individuals will continue to exist and converge to a positive stable level.

We also investigate the influence of some model parameters on COVID-19 spreading. With the increase of degree, the COVID-19 spreading level will be higher. The larger isolating factor δ can weaken the spread of COVID-19. The larger infection factor β and γ will lead to the enhancement of COVID-19 spreading. These results will help to formulate policies to block the COVID-19 spreading. The study has important significance for effectively predicting and preventing COVID-19 spreading.

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REFERENCES

[1]. Muhammad, A., Abdon, A., Ebraheem, A., Fatmawati.: The dynamics of COVID-19 with quarantined and isolation. Adv. Differ. Equ. 425 (2020)
[2]. Abdo, M.S., Hanan, K.S., Satish, A.W., Pancha, K.: On a comprehensive model of the novel coronavirus (COVID-19) under Mittag-Leffler derivative. Chaos Solitons Fractals 135, 109867 (2020)
[3]. Ming, W., Huang, J.V., Zhang, C.J.P.: Breaking down of the healthcare system: mathematical modelling for controlling the novel coronavirus (2019-nCoV) outbreak in Wuhan, China (2020). https://doi.org/10.1101/2020.01.27.922443
[4]. Bekiros, S., Kouloumpou, D., SBDiEM: a new mathematical model of infectious disease dynamics. Chaos Solitons Fractals 136, 109828 (2020)
[5]. Cakan, S.: Dynamic analysis of a mathematical model with health care capacity for COVID-19 pandemic. Chaos Solitons Fractals 139, 110033 (2020)
[6]. Bocharov, G., Volpert, V., Ludewig, B., Meyerhans, A.: Mathematical Immunology of Virus Infections. Springer, Berlin (2018)
[7]. Brauer, F.: Mathematical epidemiology: past, present, and future. Infect. Dis. Model. 2, 113–127 (2017)
[8]. Brauer, F., van den Driessche, P., Wu, J.: Mathematical Epidemiology. Springer, Berlin (2008)
[9]. Higazy, M.: Novel fractional order SIDARTHE mathematical model of COVID-19 pandemic. Chaos Solitons Fractals 138, 110007 (2020)
[10]. Kumar, S., Cao, J., Abdel-Aty, M.: A novel mathematical approach of COVID-19 with non-singular fractional derivative. Chaos Solitons Fractals 139, 110048 (2020)
[11]. Okuonghae, D., Omame, A.: Analysis of a mathematical model for COVID-19 population dynamics in Lagos, Nigeria. Chaos Solitons Fractals 139, 110032 (2020)
[12]. Postnikov, E.B.: Estimation of COVID-19 dynamics "on a back-of-envelope": does the simplest SIR model provide quantitative parameters and predictions? Chaos Solitons Fractals 135, 109841 (2020)

[13]. Torrealba-Rodriguez, O., Conde-Gutiérrez, R.A., Hernández-Javiera, A.L.: Modeling and prediction of COVID-19 in Mexico applying mathematical and computational models. Chaos Solitons Fractals 138, 109946 (2020)
[14]. Ud Din, R., Shah, K., Ahmad, I., Abdeljawad, T.: Study of transmission dynamics of novel COVID-19 by using mathematical model. Adv. Differ. Equ. 2020, 323 (2020)
[15]. Centola.D.: The spread of behavior in an online social network experiment. Science, 329, 1194–1197, 2010.
[16]. Chen, F.: On a nonlinear nonautonomous predator-prey model with diffusion and distributed delay. Comput. Appl. Math. 180, 33–49 (2005)

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