Quest Journals
Journal of Software Engineering and Simulation
Volume 9 ~ Issue 9 (2023) pp: 01-07
ISSN(Online) :2321-3795 ISSN (Print):2321-3809
www.questjournals.org

Research Paper

Formation Control of Multi-agent System Based on a Novel FTSMC Method

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ABSTRACT: *Based on a novel fast terminal sliding-mode control (FTSMC) method, this paper studies the finite-time formation tracking control of leader-following multi-agent system under directed graph. The goal is to control all followers to converge to the desired formation at a faster speed in a finite-time, while the centroid of the followers tracks the leader's trajectory. Firstly, in response to the problems of chattering and long convergence time in the fast terminal sliding-mode control with a constant velocity reaching law, a piecewise power reaching law is designed and a novel fast terminal sliding-mode control method is proposed. Then, combining the nearest neighbor principle, the property of Laplacian matrix, and the characteristics of sign function, a distributed finite-time formation tracking control protocol is designed. Using algebraic graph theory and Lyapunov stability theory, it is proved that the multi-agent system can realize the finite-time formation tracking control under the control protocol, and the upper bound of the convergence time is analysed in detail. Finally, the effectiveness of the proposed method is verified through simulation.*

KEYWORDS: *multi-agent system, formation tracking, piecewise power reaching law, fast terminal sliding-mode control, finite-time*

R*eceived 02 Sep, 2023;* ***R****evised 12 Sep., 2023;* ***A****ccepted 14 Sep., 2023 © The author(s) 2023.*
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I. INTRODUCTION

In recent years, the formation tracking control of multi-agent systems has received a lot of attention and research due to its high degree of self-organization, robustness, fault tolerance, and adaptability, and has been widely applied in multiple fields such as unmanned aerial vehicle transportation [1], satellite formation [3], and regional patrol and detection of aircraft [4].

At present, there are multiple control methods for formation control of multi-agent systems. Reference [6] studied the distributed time-varying formation tracking control problem of linear multi-agent systems based on adaptive methods. Reference [7] studied the formation control problem of multi-agent systems under directed communication topology based on backstepping method. Reference [8] studied the formation control problem of second-order multi-agent systems for collision avoidance based on the artificial potential field method. Reference [9] studied the distributed formation event triggering control problem of multi-agent systems in a directed communication topology based on the leader follower method. Reference [10] studied the bidirectional formation control problem of multi-agent systems under hybrid pulse control based on pulse control method. However, the above literature only studied the asymptotic convergence problem of multi-agent systems and did not consider finite-time convergence. Reference [11] studied the finite-time formation tracking control problem of multi-agent systems using a fast terminal sliding-mode control method based on the constant velocity reaching law. Reference [12] studied the finite-time formation tracking control problem of second-order multi-agent systems in directed graphs. Although references [11] and [12] have studied the finite-time formation tracking control problem, the system suffers from chattering and long convergence time issues.

Based on the above analysis, this paper puts forward a fast terminal sliding-mode control method based on piecewise power reaching law, and designs a distributed control protocol by combining the nearest neighbor principle, Laplacian matrix property and symbolic function property, so that the system can realize finite-time formation tracking control, and the convergence time and chattering phenomenon of the system are improved.

The structure of this paper is described as follows. Section 2 briefly reviews the knowledge points of algebraic graph theory. Section 3 designs the piecewise power reaching law, presents a novel fast terminal sliding-mode control method based on the law, analyses the upper bound value of finite time, and proves the stability of the system. Section 4 shows the related numerical simulation. Conclusions are drawn in Section 5.

II. KNOWLEDGE RELATED TO GRAPH THEORY

This article uses a weighted directed graph $G=\{V,\varepsilon,A\}$ to describe the communication relationship between multi-agent systems, where $V=\{1,...,N\}$ represents the node set, $\varepsilon\subset V\times V$ represents the directed edge set, and $A=[a_{ij}]\in\mathbb{R}^{N\times N}$ represents the non-negative weighted adjacency matrix. The directed edge $e_{ij}=(i,j)$ indicates that the node i can receive information from the node j, when $e_{ij}=(i,j)\in\varepsilon$, $a_{ij}>0$,otherwise $a_{ij}=0$. Assuming there is no self-loop in the graph, that is $a_{ii}=0$. Let $D=diag(d(1),d(2),\cdots,d(N))$, where $d(i)=\sum_{j=1}^{N}a_{ij}$ represents the in-degree of node i, then the Laplace matrix of a directed graph is $L=l_{ij}=D-A$, where $l_{ij}=\sum\limits_{j=1,j\neq i}^{N}a_{ij}$, $i=j$; $l_{ij}=-a_{ij}$, $i\neq j$.

III. FORMATION MOLDEL AND CONTROL PROTOCOL DESIGN

3.1 PROBLEM DESCRIPTION

Assuming a multi-agent system consists of $N+1$ agents, where the second-order differential dynamic equation of the i th follower is as follows

$$\left.\begin{aligned}&\dot{x}_i(t)=v_i(t)\\&\dot{v}_i(t)=u_i(t)\qquad i=1,2,\cdots,N\end{aligned}\right\}\rightarrow(1)$$

where $x_i\in\mathbb{R}^3$ represents the position vector, $v_i\in\mathbb{R}^3$ represents the velocity vector, and $u_i\in\mathbb{R}^3$ represents the control input. The second-order differential dynamics equation of the leader is as follows

$$\left.\begin{aligned}&\dot{x}_0(t)=v_0(t)\\&\dot{v}_0(t)=u_0(t)\end{aligned}\right\}\rightarrow(2)$$

where x_0、v_0、$u_0\in\mathbb{R}^3$ represents the position vector, velocity vector, and control input, respectively.

Assumption 1: Assuming that the directed graph G contains a directed spanning tree, then matrix $H=L+B$ is invertible, where $B=diag(b_1,b_2,\cdots,b_N)$ is the communication weight matrix between the leader and followers [13].

Definition 1: Multi-agent systems (1) and (2) implement finite-time formation tracking control if and only if there is a distributed control protocol and a finite-time $T>0$ that may depend on the initial value, so that all followers can form and maintain the desired formation, and their geometric center can track the leader's motion trajectory, when $t\rightarrow T$

$$\left.\begin{aligned}&\lim_{t\rightarrow T}\left\|(x_j(t)-h_j)-(x_i(t)-h_i)\right\|=0\\&\lim_{t\rightarrow T}\left\|v_j(t)-v_i(t)\right\|=0\\&\lim_{t\rightarrow T}\left\|\frac{1}{N}\sum_{i=1}^{N}x_i(t)-x_0(t)\right\|=0\\&\lim_{t\rightarrow T}\left\|v_i(t)-v_0(t)\right\|=0\qquad i,j=1,2,\cdots N\end{aligned}\right\}\rightarrow(3)$$

and $t\geq T$

$$\left.\begin{aligned}&(x_j(t)-h_j)-(x_i(t)-h_i)=0,v_j(t)-v_i=0\\&\frac{1}{N}\sum_{i=1}^{N}x_i(t)-x_0(t)=0,v_i(t)-v_0(t)=0\qquad i,j=1,2,\cdots,N\end{aligned}\right\}\rightarrow(4)$$

where $[h^T,0_{3N}^T]\in\mathbb{R}^{6N}$ with $h=[h_1^T,\cdots,h_N^T]^T$ denoting a desired geometric formation.

Lemma 1 [14]**:** For any $x_i \in \mathbb{R}, i = 1,2,\ldots n$, if $x_1, x_2, \cdots, x_n \geq 0, 0 < r \leq 1$ is a real number, then inequality $\left(\sum_{i=1}^{n} x_i\right)^r \leq \sum_{i=1}^{n} x_i^r$ holds.

Lemma 2 [15]**:** Fast terminal sliding mode is described by first-order dynamic equation.

$$s(t) = \dot{x}(t) + dx(t) + c(x(t))^{m/q}$$

where $x(t) \in \mathbb{R}$ is a scalar, $c, d > 0$, $q, m (q > m)$ are positive odd integers, and for any real number x, $(x(t))^{m/q}$ is also a positive real number. When $s = 0$, $\dot{x}(t) = -dx - c(x(t))^{m/q}$. Assuming the initial state $x(0) \neq 0$, state $(x(t), \dot{x}(t))$ will approach $(0,0)$ within finite-time T_0, and the finite-time T_0 satisfies condition $T_0 \leq \dfrac{q}{d(q-m)} \ln \dfrac{d(x(0))^{(q-m)/q} + c}{c}$.

3.2 Design of Control Protocol

This article uses the fast terminal sliding-mode control method to design a finite-time formation tracking control protocol. Firstly, a sliding-mode surface based on the formation tracking error is designed. Secondly, based on the formation tracking error and sliding-mode surface, a finite-time formation tracking control protocol is proposed for each follower.

The finite-time formation tracking error based on neighbor information is as follows:

$$\left.\begin{aligned} e_{ix}(t) &= \sum_{j=1}^{N} a_{ij}\left[(x_j(t) - h_j) - (x_i(t) - h_i)\right] + b_i(x_0(t) - (x_i(t) - h_i)) \\ e_{iv}(t) &= \sum_{j=1}^{N} a_{ij}(v_j(t) - v_i(t)) + b_i(v_0(t) - v_i(t)) \end{aligned}\right\} \rightarrow (5)$$

based on the formation tracking error, the following fast terminal sliding-mode vector is constructed for each follower:

$$S_i(t) = e_{iv}(t) + de_{ix}(t) + c(e_{ix}(t))^{m/q} \rightarrow (6)$$

where $i = 1,2,\cdots,N, c、d > 0, q > m > 0$ are positive odd integers.

Considering the problems of chattering and long convergence time in the fast terminal sliding-mode control system based on the constant velocity reaching law, this paper designs a piecewise power reaching law, which enables the system to approach the sliding-mode surface at a faster speed when away from the sliding-mode surface, and to approach the sliding-mode surface at a relatively small speed when approaching, achieving the goal of reducing convergence time while avoiding significant chattering. Its structure is as follows:

$$\dot{S} = \begin{cases} -\varepsilon_1 |S|^{\alpha} \operatorname{sign}(S), |S| > 1 \\ -\varepsilon_2 |S|^{\beta} \operatorname{sign}(S), |S| \leq 1 \end{cases} \rightarrow (7)$$

where $\varepsilon_1 > 0, \varepsilon_2 > 0, \alpha > 1, 0 < \beta < 1$.

According to the piecewise power reaching law designed above, the following control protocols based on a novel fast terminal sliding-mode control method are designed for systems (1) and (2).

$$u_i = \left(\sum_{j=1}^{N} a_{ij} + b_i\right)^{-1} \left\{\sum_{j=1}^{N} a_{ij}u_j + b_i u_0 + \left[d + c\frac{m}{q}\left(\sum_{j=1}^{N} a_{ij}\left[(x_j(t) - h_j) - (x_i(t) - h_i)\right] b_i(x_0(t) - (x_i(t) - h_i))\right)^{m/q-1}\right]\right.$$
$$\left. \times \left[\sum_{j=1}^{N} a_{ij}(v_j(t) - v_i(t)) + b_i(v_0(t) - v_i(t))\right] + \varepsilon |S_i(t)|^{\gamma} \operatorname{sign}(S_i(t))\right\} \rightarrow (8)$$

combination equation (5) can be simplified as:

$$u_i = \left(\sum_{j=1}^{N} a_{ij} + b_i\right)^{-1} \left\{\sum_{j=1}^{N} a_{ij}u_j + b_i u_0 + \left[d + c\frac{m}{q}\left(e_{ix}(t)\right)^{m/q-1}\right] \times e_{iv}(t) + \varepsilon |S_i(t)|^{\gamma} \operatorname{sign}(S_i(t))\right\} \rightarrow (9)$$

where $i = 1,2,\cdots,N$, $(\varepsilon, \gamma) = \begin{cases} (\varepsilon_1, \alpha), |S| > 1 \\ (\varepsilon_2, \beta), |S| \leq 1 \end{cases}$.

Theorem 1: For second-order multi-agent systems (1) and (2), if hypothesis 1 holds, the system can realize finite-time formation tracking control under the control protocol (9).

Proof: In order to prove the above theorem, it needs to be divided into the following two steps.

Step 1: Prove that the synovial surface of the system is equal to 0 in a finite-time. By deriving the synovial volume, we can get

$$\dot{S}_i(t) = \dot{e}_{iv}(t) + (d + c\frac{m}{q}(e_{ix}(t))^{m/q-1})e_{iv}(t)$$

Let $e_x(t) = [e_{1x}^T(t),\cdots,e_{Nx}^T(t)]^T$, $e_v(t) = [e_{1v}^T(t),\cdots,e_{Nv}^T(t)]^T$, $S(t) = [S_1^T(t),\cdots,S_N^T(t)]^T$, $u = [u_1^T,\cdots,u_N^T]^T$, $\mathrm{sign}(S(t)) = [\mathrm{sign}(S_1^T(t)),\cdots,\mathrm{sign}(S_N^T(t))]^T$. The control protocol u_i and $\dot{S}_i(t)$ can be written in the following matrix form

$$u(t) = (H \otimes I)\left\{BI \otimes u_0 + [dI + c\frac{m}{q}diag((e_x(t))^{m/q-1})]e_v(t) + \varepsilon diag(|S|^{\gamma})\mathrm{sign}(S_i(t))\right\}$$

$$\begin{aligned}\dot{S}(t) &= \dot{e}_v(t) + de_v(t) + c\frac{m}{q}diag((e_x(t))^{m/q-1})e_v(t)\\ &= \dot{e}_v(t) + [dI + c\frac{m}{q}diag((e_x(t))^{m/q-1})]e_v(t)\end{aligned}$$

therefore, the formation tracking error can be expressed as

$$\left.\begin{aligned}&\dot{e}_x(t) = e_v(t)\\ &\dot{e}_v(t) = -(H \otimes I)(u - 1 \otimes u_0)\end{aligned}\right\}$$

define the Lyapunov function $V(t) = \frac{1}{2}S^T(t)S(t)$, it can be obtained by derivation

$$\begin{aligned}\dot{V}(t) &= S^T(t)\dot{S}(t)\\ &= S^T(t)[\dot{e}_v(t) + (dI + c\frac{m}{q}(e_x(t))^{m/q-1})e_v(t)]\\ &= S^T(t)[-(H \otimes I)(u - 1 \otimes u_0) + (dI + c\frac{m}{q}(e_x(t))^{m/q-1})e_v(t)]\\ &= S^T(t)[-((L+B) \otimes I)u + B1 \otimes u_0 I + (dI + c\frac{m}{q}(e_x(t))^{m/q-1})e_v(t)]\\ &= -\varepsilon S^T(t)sign(S(t))|S(t)|^{\gamma}\\ &= -\varepsilon|S(t)|^{\gamma}(|S_1| + |S_2| + \cdots + |S_N|)\\ &\le -\varepsilon|S(t)|^{\gamma}(|S_1|^2 + |S_2|^2 + \cdots + |S_N|^2)^{\frac{1}{2}}\\ &= -\varepsilon|2V(t)|^{\frac{1}{2}\gamma + \frac{1}{2}} \to (10)\end{aligned}$$

Therefore, the sliding-mode can reach the equilibrium 0 point in a finite-time, which means that if $V(0) \neq 0$, the synovial surface $S(t) = 0$ can be achieved in a finite-time. Assuming that the initial state of the system $V(0) > 1$ is $S(0) > 1$, the finite-time T_1 calculation is carried out in two stages.

(1) $V(0) \to V = \frac{1}{2}$ (i.e. $S(0) \to |S| = 1$), at this time $\gamma = \alpha > 1$, let $\dot{V}(t) = -\varepsilon|2V(t)|^{\frac{1}{2}\alpha + \frac{1}{2}}$ and integrate it on both sides to obtain

$$V^{\frac{1-\alpha}{2}} = -\frac{1-\alpha}{2}\sqrt{2}^{\alpha+1}t_1 + V(0)^{\frac{1-\alpha}{2}}, \text{ so } t_1 = \frac{\frac{1}{2}^{\frac{1-\alpha}{2}} - V(0)^{\frac{1-\alpha}{2}}}{(\alpha-1)\sqrt{2}^{\alpha-1}\varepsilon}.$$

(2) $V = \frac{1}{2} \to V = 0$ (i.e. $|S| = 1 \to S = 0$), at this time $\gamma = \beta(0 < \beta < 1)$, let $\dot{V}(t) = -\varepsilon|2V(t)|^{\frac{1}{2}\beta + \frac{1}{2}}$ and integrate it on both sides to obtain

$$V^{\frac{1-\beta}{2}} = -\frac{1-\beta}{2}\sqrt{2}^{\beta+1}t_2 + \frac{1}{2}, \text{ so } t_2 = \frac{1}{(1-\beta)\sqrt{2}^{\beta+1}}.$$

So $T_1 = t_1 + t_2 \le \dfrac{\dfrac{1}{2}^{\frac{1-\alpha}{2}} - V(0)^{\frac{1-\alpha}{2}}}{(\alpha-1)\sqrt{2}^{\alpha-1}\varepsilon} + \dfrac{1}{(1-\beta)\sqrt{2}^{\beta+1}}$.

Step 2: When the sliding-mode surface is 0, it is proved that the system error converges to 0 in finite-time.

When the sliding-mode surface $S(t)=0$, according to lemma 2, the equilibrium point can be reached in finite-time, that is, the formation tracking error of the system converges to 0 in finite-time, and its finite-time satisfies the condition $T \le T_1 + \dfrac{q}{d(q-m)} \ln \dfrac{d(\max_{1\le i\le N} e_{ix}(T_1))^{(q-m)/q} + c}{c}$.

From the above analysis, it can be seen that under the control protocol (9), the formation tracking control of multi-agent systems (1) and (2) in finite-time can be realized.

IV. SIMULATION

Consider a multi-agent system consisting of one leader and eight followers. The communication topology of the system is shown in Figure 1.

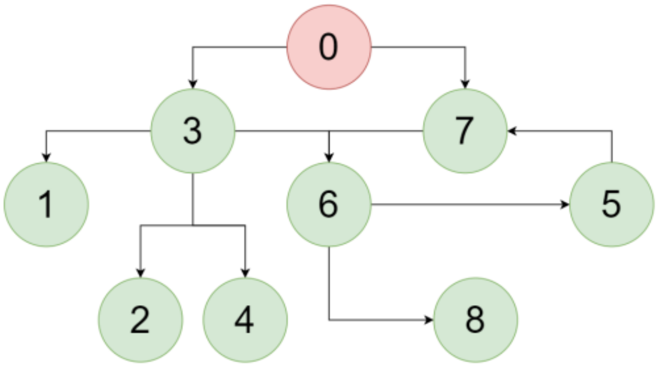


Figure 1: the communication topology diagram of the multi-agent system

According to the second-order multi-agent system, set the leader trajectory as $x_0 = [-50\sin(0.1t), -50\cos(0.1t), -50\sin(0.1t)]^T$, the leader speed as $v_0 = [-5\cos(0.1t), 5\sin(0.1t), -5\cos(0.1t)]^T$, and the control input as $u_0 = [0.5\sin(0.1t), 0.5\cos(0.1t), 0.5\sin(0.1t)]^T$. Set the parameters of the control protocol as follows: $c=0.2, d=0.5, q=5, m=3, \varepsilon_1=0.8, \varepsilon_2=1.6,\ \alpha=1.3, \beta=0.5$. The motion trajectory of the multi-agent system under the control protocol u_i is shown in Figure 2.

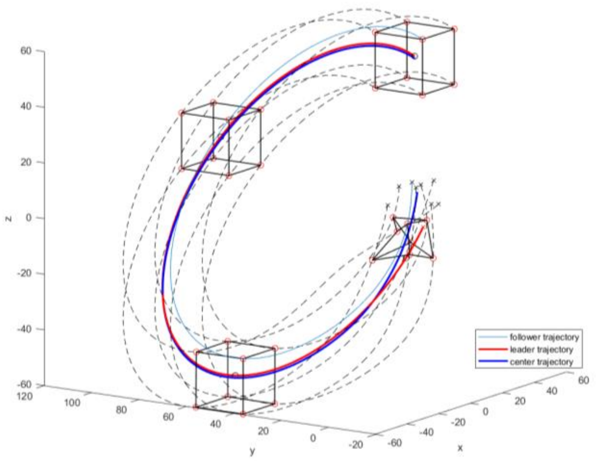
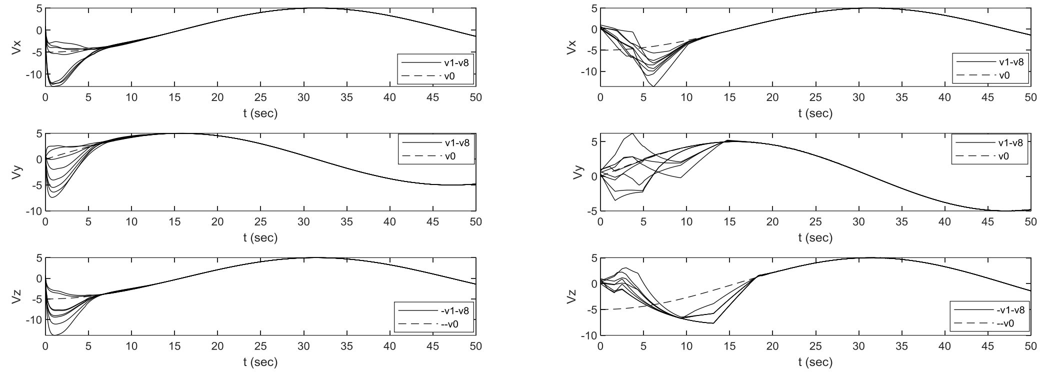
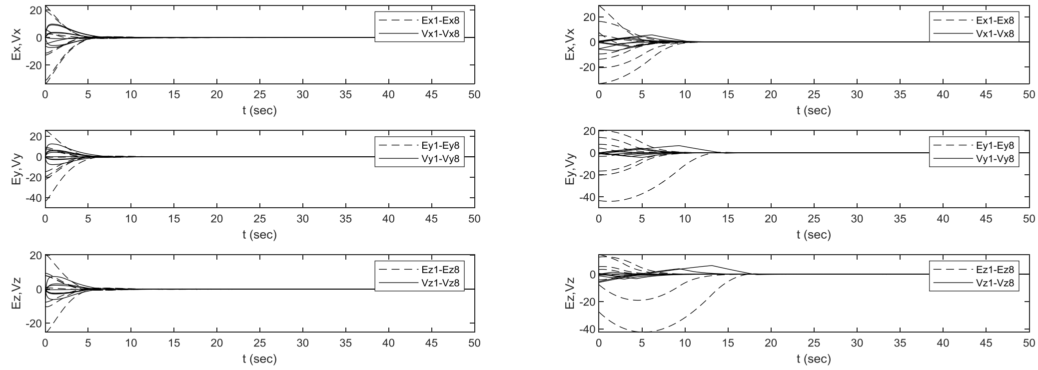


Figure 2: the motion trajectory diagram of the multi-agent system



(a) (b)

Figure 3: (a) the velocity trajectory of the agents under the action of u_i
(b) the velocity trajectory of the agents under the control protocol proposed in reference [11]



(c) (d)

Figure 4: (c) the position and velocity tracking error trajectory of the followers under the control protocol (9)
(d) the position and velocity tracking error trajectories of the followers under the control protocol proposed in reference [11]

It can be seen from figures 3 and 4 that, compared with the control effect of the control protocol proposed in reference [11], the multi-agent system under the control protocol (9) designed in this paper has shorter convergence time, smoother convergence rate curve and no chattering phenomenon.

V. CONCLUSION

This paper has studied the formation tracking control problem of multi-agent system under directed graph in finite-time. A novel piecewise power reaching law has been proposed, and a novel fast terminal sliding-mode control method based on the law for this system, which effectively eliminates chattering and shortens the convergence time. By using algebraic graph theory and Lyapunov stability theory, the stability of the system has been proved, and the upper bound of convergence time has been studied in detail. Finally, the simulations have been provided to verify the effectiveness of the theoretical results.

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